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A UNIFIED INFERENCE FRAMEWORK FOR MULTIPLE IMPUTATION USING MARTINGALES

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Abstract: Multiple imputation is widely used to handle missing data. Although Rubin's combining rule is simple, it is not clear whether the standard multiple imputation inference is consistent when coupled with the commonly used full-sample estimators. Here, we establish a unified martingale representation of multiple imputation for a wide class of asymptotically linear full-sample estimators. This representation invokes the wild bootstrap inference to provide a consistent variance estimation under a correct specification of the imputation models. As a motivating application, we use the proposed method to estimate the average causal effect (ACE) with partially observed confounders in a causal inference. Our framework applies to asymptotically linear ACE estimators, including the regression imputation, weighting, and matching estimators. Lastly, we extend the proposed method to include scenarios in which both the outcome and the confounders are subject to missingness, and when the data are missing not at random.

Key words and phrases: Causality, congeniality, influence function, martingale representation, weighted bootstrap.

1. Introduction

Missing data are ubiquitous in practice. A widely used approach to handle incomplete/missing data is multiple imputation (MI). The National Research Council recommends MI as one of its preferred approaches to addressing missing data (National Research Council (2010)). The idea of MI is to fill the missing values multiple times by sampling from the posterior predictive distribution of the missing values, given the observed values. Then, we can apply full-sample analyses straightforwardly to the imputed data sets. These multiple results are summarized by an easy-to-implement combining rule for inference (Rubin (1987)). MI can provide valid frequentist inferences in various applications (e.g., Clogg et al. (1991)). However, some authors have found that Rubin's variance estimator is not always consistent (e.g., Fay (1992), Kott (1995), Fay (1996), Binder and Sun (1996), Wang and Robins (1998), Robins and Wang (2000), Nielsen (2003) and Kim et al. (2006)). To ensure the validity of Rubin's variance estimation, imputations must be proper (Rubin (1987)). A sufficient condition for proper imputation is the congeniality condition of Meng (1994), imposed on both the

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