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ASYMPTOTIC BEHAVIOR OF THE MAXIMUM LIKELIHOOD ESTIMATOR FOR GENERAL MARKOV SWITCHING MODELS

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Abstract: Motivated by studying the asymptotic properties of the parameter estimator in switching linear state space models, switching GARCH models, switching stochastic volatility models, and recurrent neural networks, we investigate the maximum likelihood estimator for general Markov switching models. To this end, we first propose an innovative matrix-valued Markovian iterated function system (MIFS) representation for the likelihood function. Then, we express the derivatives of the MIFS as a composition of random matrices. To the best of our knowledge, this is a new method in the literature. Using this useful device, we establish the strong consistency and asymptotic normality of the maximum likelihood estimator under some regularity conditions. Furthermore, we characterize the Fisher information as the inverse of the asymptotic variance.

Key words and phrases: Asymptotic normality, consistency, Markovian iterated function systems, recurrent neural networks, switching linear state space model.

1. Introduction

Motivated by studying the asymptotic properties of the parameter estimator in switching linear state space models, switching GARCH models, switching stochastic volatility (SV) models, and recurrent neural networks (RNNs), we investigate the maximum likelihood estimator (MLE) for general Markov switching models (GMSMs). Let $\{H_t, t \geq 0\}$ be an ergodic (aperiodic, irreducible, and positive recurrent) Markov chain on a finite state space $\mathcal{D} = \{1, \ldots, d\}$, and denote

$$Y_t = g_{H_t}(X_t, Y_{t-1}, \varepsilon_t; \theta),\ t \geq 1, \quad \text{with } Y_0 = \mathbf{0}, \tag{1.1}$$

$$X_t = f_{H_t}(X_{t-1}, \eta_t; \theta),\ t \geq 1, \quad \text{with } X_0 = \mathbf{0}, \tag{1.2}$$

where $Y_t \in \mathbf{R}^p$, for some $p \geq 1$, $X_t \in \mathbf{R}^m$, for some $m \geq 1$, $\{\varepsilon_t, t \geq 1\}$ is a sequence of independent and identically distributed (i.i.d.) $p \times 1$ random vectors, and $\{\eta_t, t \geq 1\}$ is a sequence of i.i.d. $m \times 1$ random vectors. Furthermore, we assume that $\{H_t, t \geq 0\}$ is a first-order Markov chain, and that $\{H_t, t \geq 0\}$, $\{\eta_t, t \geq 1\}$, and $\{\varepsilon_t, t \geq 1\}$ are independent. The GMSM is very flexible, and

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