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SPARSE AND LOW-RANK MATRIX QUANTILE ESTIMATION WITH APPLICATION TO QUADRATIC REGRESSION

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Abstract: This study examines matrix quantile regression where the covariate is a matrix and the response is a scalar. Although the statistical estimation of matrix regression is an active field of research, few studies examine quantile regression with matrix covariates. We propose an estimation procedure based on convex regularizations in a high-dimensional setting. In order to reduce the dimensionality, the coefficient matrix is assumed to be low rank and/or sparse. Thus, we impose two regularizers to encourage different low-dimensional structures. We develop the asymptotic properties and an implementation based on the incremental proximal gradient algorithm. We then apply the proposed estimator to quadratic quantile regression, and demonstrate its advantages using simulations and a real-data analysis.

Key words and phrases: Dual norm, interaction effects, matrix regression, penalization.

1. Introduction

Quantile regression (Koenker and Bassett (1978)) is a useful statistical tool in data analysis. It provides a complement to a mean regression, allowing us to analyze the entire conditional distribution by modeling the covariate effects at different quantile levels. Despite there being a large body of literature on the theoretical and computational aspects of vector covariate quantile regression (Koenker (2005); Belloni and Chernozhukov (2011); Yu, Lin and Wang (2017); Yi and Huang (2017)), matrix quantile regression is rarely studied. However, matrix data arise frequently in fields such as digital image analysis (Zhou and Li (2014)), multi-task regression (Yuan et al. (2007); Argyriou, Evgeniou and Pontil (2008); Bunea, She and Wegkamp (2012)), matrix completion (Candes and Plan (2010); Koltchinskii, Lounici and Tsybakov (2011); Negahban and Wainwright (2012)), and quadratic regression (Bien, Taylor and Tibshirani (2013)).

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