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TRADE-OFF BETWEEN VALIDITY AND EFFICIENCY OF MERGING p-VALUES UNDER ARBITRARY DEPENDENCE

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Abstract: Various methods are widely used to combine individual p-values into one p-value in many areas of statistical applications. We say that a combining method is valid for arbitrary dependence if it does not require any assumption on the dependence structure of the p-values, whereas it is valid for some dependence if it requires some specific, perhaps realistic, but unjustifiable, dependence structures. The trade-off between the validity and efficiency of these methods is studied by analyzing the choices of critical values under different dependence assumptions. We introduce the notions of independence-comonotonicity balance (IC-balance) and the price for validity. In particular, IC-balanced methods always produce an identical critical value for independent and perfectly positively dependent p-values, a specific type of insensitivity to a family of dependence assumptions. We show that, among two very general classes of merging methods commonly used in practice, the Cauchy combination method and the Simes method are the only IC-balanced ones. Simulation studies and a real-data analysis are conducted to analyze the size and power of various combining methods in the presence of weak and strong dependence.

Key words and phrases: Efficiency, hypothesis testing, multiple hypothesis testing, validity.

1. Introduction

In many statistical applications in which multiple hypothesis testing is involved, the task of merging several p-values into one naturally arises. Depending on the specific application, these p-values may be from a single hypothesis or from multiple hypotheses, in small or large numbers, independent or correlated, and with sparse or dense signals, leading to different considerations when choosing merging procedures.

Let K be a positive integer, and $F : [0,1]^K \to [0,\infty)$ be an increasing Borel function used to combine K p-values, which we refer to as a *combining function.* In general, the combined value may not be a valid p-value itself, and

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