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PARAMETRIC BOOTSTRAP INFERENCE FOR STRATIFIED MODELS WITH HIGH-DIMENSIONAL NUISANCE SPECIFICATIONS

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Abstract: Inference about a scalar parameter of interest typically relies on the asymptotic normality of common likelihood pivots, such as the signed likelihood root, the score and the Wald statistics. Nevertheless, the resulting inferential procedures are known to perform poorly when the dimension of the nuisance parameter is large relative to the sample size, and when the information about the parameters is limited. In many such cases, using the asymptotic normality of analytical modifications of the signed likelihood root is known to recover the inferential performance. Here, we prove that the parametric bootstrap of standard likelihood pivots results in inferences as accurate as those of analytical modifications of the signed likelihood root do in stratified models with stratum-specific nuisance parameters. We focus on the challenging case in which the number of strata increases as fast or faster than the size of the stratum samples. We further show that this equivalence holds regardless of whether we use the constrained or the unconstrained bootstrap. In contrast, when the number of strata is fixed or increases more slowly than the stratum sample size, we show that using the constrained bootstrap corrects inference to a higher order than when using the unconstrained bootstrap. Simulation experiments support the theoretical findings and demonstrate the excellent performance of the bootstrap in extreme scenarios.

Key words and phrases: Incidental parameters, location and scale adjustment, modified profile likelihood, profile score bias, two-index asymptotics.

1. Introduction

Standard likelihood inference about a scalar parameter of interest is based on the asymptotic normality of likelihood pivots, such as the signed likelihood root, the score and the Wald statistics. However, this asymptotic approximation can be quite inaccurate in the presence of many nuisance parameters. An alternative, which guarantees higher accuracy, is based on the asymptotic normality of analytical modifications of the signed likelihood root, generally termed

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