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ESTIMATION FOR FUNCTIONAL SINGLE INDEX MODELS WITH UNKNOWN LINK FUNCTIONS

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Abstract: This study examines an estimating problem in single index models with functional predictors. An estimating approach is developed to estimate the slope function in the single-index and the nonparametric link function. Optimal convergence rates for the estimator of the slope function are established in a minimax sense, under mild conditions, using a functional principal component analysis and the estimating equation technique. For the estimator of the nonparametric link function, both the uniform and mean squared convergence rates are obtained. An error variance estimator is also defined and is proved to be asymptotically normal. The finite-sample performance of the proposed estimators is illustrated by simulations and a real-data application.

Key words and phrases: Functional data analysis, functional principal components analysis, kernel smoother, local linear smoothing, nonparametric models, semiparametric models.

1. Introduction

Functional data analysis has received substantial interest in recent decades. There have been extensive studies on functional linear models with a scalar response (Cai and Hall (2006); Hall and Horowitz (2007); Yuan and Cai (2010); Cai and Yuan (2012); Delaigle and Hall (2012); Hilgert, Mas and Verzelen (2013)) and nonlinear models with known link functions (James (2002); Dou, Pollard and Zhou (2012)). For nonparametric models with functional predictors, Ferraty and Vieu (2006) and Goia and Vieu (2016) provide comprehensive discussions on this topic. However, because functional data are inherently infinite dimensional, the statistical performance of full nonparametric methods is unfavorable owing to the so-called curse of dimensionality.

To avoid the curse, we model scalar responses with functional covariates as single-index models, which include a simple linear term and a flexible nonparametric link function. Early works assumed that either the link function is monotonic (Müller and Stadtmüller (2005)) or that the slope function lies in a

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