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ESTIMATION FOR EXTREME CONDITIONAL QUANTILES OF FUNCTIONAL QUANTILE REGRESSION

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Abstract: Quantile regression as an alternative to modeling the conditional mean function provides a comprehensive picture of the relationship between a response and covariates. It is particularly attractive in applications focused on the upper or lower conditional quantiles of the response. However, conventional quantile regression estimators are often unstable at the extreme tails, owing to data sparsity, especially for heavy-tailed distributions. Assuming that the functional predictor has a linear effect on the upper quantiles of the response, we develop a novel estimator for extreme conditional quantiles using a functional composite quantile regression based on a functional principal component analysis and an extrapolation technique from extreme value theory. We establish the asymptotic normality of the proposed estimator under some regularity conditions, and compare it with other estimation methods using Monte Carlo simulations. Finally, we demonstrate the proposed method by empirically analyzing two real data sets.

Key words and phrases: Extrapolation, extreme quantile, extreme value theory, functional principal component analysis, functional quantile regression, heavy-tailed distribution.

1. Introduction

With modern technology related to data collection and storage, functional data have become increasingly available in many scientific fields, such as meteorology, chemistry, biomedicine, and neuroimaging (Zhu, Fan and Kong (2014); Yu, Kong and Mizera (2016); Miranda, Zhu and Ibrahim (2018)). The most striking feature of functional data is its inherent infinite dimensionality, which poses challenges both for theoretical analysis and statistical computation, and makes traditional multivariate statistical analysis methods no longer applicable. On the other hand, the infinite-dimensional structure of the data is also a rich source of potential useful information, which brings many opportunities for theoretical research and data application. For this very reason, functional data analysis (FDA) has attracted increasing interest in the statistical research community.

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