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DOUBLE HAPPINESS: ENHANCING THE COUPLED GAINS OF L-LAG COUPLING VIA CONTROL VARIATES

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Abstract: The recently proposed L-lag coupling for unbiased Markov chain Monte Carlo (MCMC) calls for a joint celebration by MCMC practitioners and theoreticians. For practitioners, it circumvents the thorny issue of deciding the burn-in period or when to terminate an MCMC sampling process, and opens the door for safe parallel implementation. For theoreticians, it provides a powerful tool to establish elegant and easily estimable bounds on the exact error of an MCMC approximation at any finite number of iterates. A serendipitous observation about the bias-correcting term leads us to introduce naturally available control variates into the L-lag coupling estimators. In turn, this extension enhances the gains of L-lag coupling, because it results in more efficient unbiased estimators, as well as a better bound on the total variation error of any MCMC iteration, albeit the gains diminish as L increases. Specifically, the new upper bound is theoretically guaranteed to never exceed the one given previously. We also argue that L-lag coupling represents a coupling for the future, breaking from the coupling-from-the-past type of perfect sampling, by reducing the generally unachievable requirement of being *perfect* to one of being *unbiased*, a worthwhile trade-off for ease of implementation in most practical situations. The theoretical analysis is supported by numerical experiments that show tighter bounds and a gain in efficiency when control variates are introduced.

Key words and phrases: Coupling from the Past, maximum coupling, median absolute deviation, parallel implementation, total variation distance, unbiased MCMC.

1. If Being Perfect is Impossible, Let's Try Being Unbiased

1.1. Perfect coupling – too much to hope for?

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