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ASYMPTOTIC OPTIMALITY OF C_P-TYPE CRITERIA IN HIGH-DIMENSIONAL MULTIVARIATE LINEAR REGRESSION MODELS

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Abstract: We study the asymptotic optimality of C_p-type criteria from the perspective of prediction in high-dimensional multivariate linear regression models, where the dimension of a response matrix is large, but does not exceed the sample size. We derive conditions in order that the generalized C_p (GC_p) exhibits asymptotic loss efficiency (ALE) and asymptotic mean efficiency (AME) in such high-dimensional data. Moreover, we clarify that one of the conditions is necessary for GC_p to exhibit both ALE and AME. As a result, we show that the modified C_p can claim both ALE and AME, but the original C_p cannot in high-dimensional data. The finite-sample performance of the GC_p with several tuning parameters is compared by means of a simulation study.

Key words and phrases: Asymptotic theory, high-dimensional statistical inference, model selection/variable selection.

1. Introduction

Variable selection problems are crucial in statistical fields for improving the prediction accuracy and/or interpretability of a resultant model. There is a burgeoning body of literature that has attempted to solve the variable selection problem, and many selection procedures and their theoretical properties have been studied.

For example, Mallows' C_p criterion (Mallows (1973)) and the Akaike information criterion (AIC) (Akaike (1974)) are useful selection methods from a predictive point of view, because these procedures are optimal in some predictive sense (see Shibata (1981, 1983); Li (1987); Shao (1997)). On the other hand, the Bayesian information criterion (BIC) proposed by Schwarz (1978) is consistent (Nishii (1984)) under appropriate conditions; that is, the probability that a model selected by the BIC coincides with the true model converges to one as the sample size n tends to infinity. In this sense, the BIC is a feasible method from the

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