Statistica Sinica **33** (2023), 1-22
doi:https://doi.org/10.5705/ss.202020.0415

TESTS OF UNIT ROOT HYPOTHESIS WITH HEAVY-TAILED HETEROSCEDASTIC NOISES

Rui She

Southwestern University of Finance and Economics

Abstract: This study examines unit-root testing with unspecified and heavy-tailed heteroscedastic noise. A new weighted least squares estimation (WLSE) is designed for the Dickey–Fuller (DF) test, the asymptotic normality of which is verified. However, the performance of the DF test relies strongly on the estimation accuracy of the asymptotic variance, which is not stable for dependent time series. To overcome this issue, we develop two novel unit-root tests by applying the empirical likelihood technique to the WLSE score equation. We show that both empirical likelihood-based tests converge weakly to a chi-squared distribution with one degree of freedom. Furthermore, the limiting theory is extended to the weighted M-estimation score equation. In contrast to existing unit-root tests for heavy-tailed time series, empirical likelihood tests do not involve any estimators of the unknown parameters or any restrictions on the tail index of the noise. This makes them appealing in practice, with wide applications in finance and econometrics. Extensive simulation studies are conducted to examine the effectiveness of the proposed methods.

Key words and phrases: Empirical likelihood, GARCH type noise, heavy-tailed, unit-root.

1. Introduction

Consider the following AR(1) model:

$$y_t = \phi y_{t-1} + \varepsilon_t, \tag{1.1}$$

where the noise $\{\varepsilon_t\}$ is a sequence of stationary random variables. We are interested in detecting a possible unit root in model (1.1); that is, we test the null hypothesis $H_0 : \phi = 1$ versus the alternative $H_1 : |\phi| < 1$. There is an extensive and relatively complete body of literature on unit-root estimation and testing when $E\varepsilon_t^2$ is finite. When the noise $\{\varepsilon_t\}$ is an independent and identically distributed (i.i.d.) random variable, Dickey and Fuller (1979) and Evans and Savin (1981) proposed the classical Dickey–Fuller (DF) test and Student's t test, respectively, based on the ordinary least squares estimator (LSE) of the

Corresponding author: Rui She, Center of Statistical Research, School of Statistics, Southwestern University of Finance and Economics, Chengdu, China. E-mail: rshe@swufe.edu.cn.