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TWO-SAMPLE TESTS FOR RELEVANT DIFFERENCES IN THE EIGENFUNCTIONS OF COVARIANCE OPERATORS

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Abstract: This study examines two-sample tests for functional time series data, which have become widely available with the advent of modern complex observation systems. Here, we evaluate whether two sets of functional time series observations share the shape of their primary modes of variation, as encoded by the eigenfunctions of the respective covariance operators. To this end, a novel testing approach is introduced that adds to existing literature in two main ways. First, tests are set up in the relevant testing framework, where interest is not in testing an exact null hypothesis, but rather in detecting deviations deemed sufficiently relevant, with relevance determined by the practitioner and perhaps guided by domain experts. Second, the proposed test statistics rely on a self-normalization principle that helps to avoid the notoriously difficult task of estimating the long-run covariance structure of the underlying functional time series. The main theoretical result of this study is the derivation of the large-sample behavior of the proposed test statistics. Empirical evidence, which indicates that the proposed procedures work well in finite samples and compare favorably with competing methods, is provided through a simulation study and an application to annual temperature data.

Key words and phrases: Functional data, functional time series, relevant tests, self-normalization.

1. Introduction

This study develops testing tools for two independent sets of functional observations, explicitly allowing for temporal dependence within each set. Functional data analysis has become a mainstay for dealing with those complex data sets that may conceptually be viewed as being comprised of curves. Monographs detailing many of the available statistical procedures for functional data are provided by Ramsay and Silverman (2005) and Horváth and Kokoszka (2012). This type of data naturally arises in contexts such as environmental data (Aue, Dubart Norinho and Hörmann (2015)), molecular biophysics (Tavakoli and Panaretos (2016)), climate science (Zhang et al. (2011); Aue, Rice and Sönmez (2018)), and

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