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# HYPOTHESIS TESTING IN HIGH-DIMENSIONAL LINEAR REGRESSION: A NORMAL-REFERENCE SCALE-INVARIANT TEST

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*Abstract:* Recently, several non-scale-invariant and scale-invariant tests have been proposed for a general linear hypothesis testing problem for high-dimensional data, which include one-way and two-way MANOVA tests as special cases. Many of these tests impose strong assumptions on the underlying covariance matrix to ensure that their test statistics are asymptotically normally distributed. However, a simulation example and some theoretical justifications indicate that these assumptions are rarely satisfied in practice. As a result, these tests may not be able to maintain their nominal size well. To overcome this problem, we propose a normal-reference scale-invariant test. The test has good size control and power, without imposing strong assumptions on the underlying covariance or correlation matrix. A real-data example and several simulation studies demonstrate that the proposed test has much better size control and power than several non-scale-invariant and scale-invariant tests.

*Key words and phrases:* General linear hypothesis testing, high-dimensional linear regression, scale-invariant test.

## 1. Introduction

Modern data collecting and storing technologies mean that many variables are often observed on a few subjects in scientific fields such as biology, medicine, genetics, economics, finance, and so on, resulting in so-called high-dimensional data. Analyzing such data is challenging, because the dimension of the data may be much larger than the sample size. This study is motivated by a corneal surface data set described in Locantore et al. (1999). The data are from a consulting project on a keratoconus disease study with Ms. Nancy Tripoli and Dr. Kenneth L. Cohen of the Department of Ophthalmology, University of North Carolina at Chapel Hill. According to varying degrees of the keratoconus disease, when the corneas are misshaped, 150 corneal surfaces are classified into four groups, and each corneal surface has 6,912 measurements. Of interest is to

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