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TEST FOR CONDITIONAL VARIANCE OF INTEGER-VALUED TIME SERIES

Yuichi Goto and Kou Fujimori

Kyushu University and Shinshu University

Abstract: We investigate a test for the conditional variance of stationary and ergodic integer-valued time series. This hypothesis testing problem is motivated by the fact that the form of the conditional variance of the process is determined by the conditional distribution and the conditional mean. First, we estimate the unknown parameters of the intensity function using an M-estimator and prove strong consistency and asymptotic normality. Second, we show that the proposed test has asymptotic size α and is consistent. Finally, we discuss the nontrivial power of the proposed test for the local alternative. The proposed test statistic can be applied to various problems, such as specification tests for intensity functions, tests for overdispersion and underdispersion, and goodness-of-fit tests for ergodic and stationary integer-valued time series. A simulation study illustrates the finite-sample performance of the proposed test. Lastly, in a real-data application, we analyze the number of patients with Escherichia coli in Germany.

Key words and phrases: Conditional variance, integer-valued time series, intensity.

1. Introduction

Integer-valued time series are garnering increasing attention in several fields, including analyses of financial data and the number of patients with infectious diseases, among others. One of the most fundamental integer-valued time series is the Poisson process, the conditional distribution of which, given past information, is the Poisson distribution. Based on the Poisson process, we can develop various statistical models, such as the Poisson integer-valued AR model of order p, or, INAR(p), and Poisson integer-valued GARCH model of order p and q, or, INGARCH(p, q). Franke (2010), Neumann (2011), and Doukhan, Fokianos and Tjøstheim (2012) have investigated the stability of these models of Poisson processes. In addition to the Poisson distribution, the negative binomial (NB) distribution is also popular for constructing statistical models of integer-valued time series; see, for examle, Davis and Wu (2009), Zhu and Joe (2010) and Christou and Fokianos (2014).

Corresponding author: Yuichi Goto, Department of Mathematical Sciences, Faculty of Mathematics, Kyushu University, Fukuoka, 819-0395, Japan. E-mail: yuichi.goto@math.kyushu-u.ac.jp.