Statistica Sinica **33** (2023), 1-30
doi:https://doi.org/10.5705/ss.202020.0340

FREQUENTIST MODEL AVERAGING FOR THE NONPARAMETRIC ADDITIVE MODEL

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Abstract: This study develops an optimal frequentist model averaging approach for estimating the unknown conditional mean function in the nonparametric additive model when the covariates and the degree of smoothing are subject to uncertainty. Our weight choice criterion selects model weights by minimizing a plug-in estimator of the risk of the model average estimator under a squared error loss function. We derive the convergence rate of the model weights obtained from our proposed method to the infeasible optimal weights, and prove that the resultant model average estimators are asymptotically optimal. An extension to the additive autoregressive model for time series data is also considered. Our simulation analysis shows that the proposed model average estimators significantly outperform several commonly used model selection estimators and their model averaging counterparts in terms of the mean squared error in a large part of the parameter space. We further illustrate our methods using two real data studies.

Key words and phrases: Additive model, asymptotic optimality, autoregressive model, consistency, model averaging.

1. Introduction

The nonparametric additive model (AM) (Stone (1985); Hastie and Tibshirani (1990)) is a well-known statistical modeling approach. The AM belongs to a class of regression models in which the usual linear relationship between the response and the covariates is replaced by a sum of univariate smooth functions. AMs thus avoid much of the curse of dimensionality that afflicts fully nonparametric regression and afford more flexibility than traditional linear models with respect to the covariate effects. The smooth functions in AMs are commonly estimated by backfitting (Buja, Hastie and Tibshirani (1989); Mammen, Linton and Nielsen (1999); Opsomer (2000); Nielsen and Sperlich (2005); Ravikumar et al. (2009)), smoothing splines (Stone (1985); Doksum and Koo (2000); Huang and Yang (2004); Chen, Fan and Li (2018)), or marginal integration methods

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