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THAT PRASAD-RAO IS ROBUST: ESTIMATION OF MEAN SQUARED PREDICTION ERROR OF OBSERVED BEST PREDICTOR UNDER POTENTIAL MODEL MISSPECIFICATION

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Abstract: This study examines a measure of uncertainty for robust small area estimation (SAE). We consider the estimation of the mean squared prediction error (MSPE) of the observed best predictor (OBP) in SAE under the Fay-Herriot model with potential model misspecification. Previously, it was thought that the traditional Prasad-Rao (PR) linearization method could not be used, because it is derived under the assumption that the underlying model is correctly specified. However, we show that when it comes to estimating the unconditional MSPE, the PR estimator, derived for estimating the MSPE of the OBP, assuming that the underlying model is correct, remains first-order unbiased, even when the underlying model is misspecified in its mean function. A second-order unbiased estimator of the MSPE is derived by modifying the PR MSPE estimator. The PR and modified PR estimators also have much smaller variation than that of existing MSPE estimators for the OBP. The theoretical findings are supported by empirical results, including simulation studies and real-data applications.

Key words and phrases: Fay-Herriot model, model misspecification, observed best prediction, robustness, second-order unbiasedness, small area estimation

1. Introduction

Robust small area estimation (SAE; e.g., Rao and Molina (2015)) has received considerable attention in recent studies; see, for example, Sinha and Rao (2009), Jiang, Nguyen and Rao (2011), and Jiongo, Haziza and Duchesne (2013). In particular, Jiang, Nguyen and Rao (2011) introduced the observed best prediction (OBP) method which is known to be more robust against model misspecification than is the traditional empirical best linear unbiased prediction (EBLUP) method. See Pfeffermann (2013), Jiang, Nguyen and Rao (2015), Chen, Jiang and Nguyen (2015), and Jiang and Rao (2020) for reviews and extensions. The

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