Statistica Sinica **32** (2022), 2381-2403
doi:https://doi.org/10.5705/ss.202020.0312

A MINIMUM DISCREPANCY APPROACH WITH FOURIER TRANSFORM IN SUFFICIENT DIMENSION REDUCTION

Jiaying Weng and Xiangrong Yin

Bentley University and University of Kentucky

Abstract: We propose an optimal family of estimators in sufficient dimension reduction using a Fourier transform based on a quadratic discrepancy function. Our proposed approach has advantages over existing methods in that it avoids the slicing scheme of a response variable and easily handles multivariate responses. We further develop four sub-optimal estimators: degenerated and special estimators for computational efficiency and simplicity, and robust and its degenerated estimators for a less restrictive condition for estimation and inference. Marginal and conditional hypothesis tests for the predictors and dimensions are also obtained. Simulation studies and a real-data analysis illustrate the efficacy of our proposed methods.

Key words and phrases: Fourier transform, minimum discrepancy, sufficient dimension reduction.

1. Introduction

With the recent development in data collection and storage techniques, researchers can now use data with huge volume and high dimension to build economic models and create advanced visualization tools. It is easier to achieve these goals if we can obtain a low-dimensional function of the predictor associated with the response variable. This study focuses on sufficient dimension reduction (SDR; Li (1991); Cook (1996)), a model-free approach. It preserves complete regression information, making it attractive to researchers wanting to build a parsimonious model.

SDR considers a regression of $q \times 1$ response $\mathbf{Y}$ given a $p \times 1$ predictor $\mathbf{X}$. It aims to find a dimension reduction matrix $\beta \in \mathbb{R}^{p\times d}(d \leq p)$ such that the reduced variables $\beta^T\mathbf{X}$ retain complete regression information. Matrix β may not be identifiable, but the space spanned by the columns of β, known as the dimension reduction subspace and denoted as $\text{Span}(\beta)$, is identifiable. To achieve the uniqueness and minimum of the subspace, we study the central

Corresponding author: Jiaying Weng, Mathematical Sciences, Bentley University, Waltham, MA 02452, USA. E-mail: jweng@bentley.edu.