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MODELING NONSTATIONARY AND ASYMMETRIC MULTIVARIATE SPATIAL COVARIANCES VIA DEFORMATIONS

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Abstract: Multivariate spatial-statistical models are often used when modeling environmental and socio-demographic processes. The most commonly used models for multivariate spatial covariances assume both stationarity and symmetry for the cross-covariances, but these assumptions are rarely tenable in practice. In this article, we introduce a new and highly flexible class of nonstationary and asymmetric multivariate spatial covariance models that are constructed by modeling the simpler and more familiar stationary and symmetric multivariate covariances on a warped domain. Inspired by recent developments in the univariate case, we propose modeling the warping function as a composition of a number of simple injective warping functions in a deep-learning framework. Importantly, covariance-model validity is guaranteed by construction. We establish the types of warpings that allow for cross-covariance symmetry and asymmetry, and we use likelihood-based methods for inference that are computationally efficient. The utility of this new class of models is shown through two data illustrations: a simulation study on nonstationary data, and an application to ocean temperatures at two different depths.

Key words and phrases: Cross-covariance, deep learning, Gaussian process, spatial statistics, warping.

1. Introduction

Multivariate spatial-statistical models are used to jointly model two or more variables that are spatially indexed. They find widespread use in several application domains, such as the environmental sciences and the social sciences, where spatial processes interact. The utility of multivariate models lies in the concept of "borrowing strength," where in this setting, information on one process (obtained, for example, through observation) imparts information on the other processes being jointly modeled, but that may or may not be directly observed. Multivariate spatial models need to adequately model both the marginal behavior of the spatial processes and the joint dependence between the processes. Often,

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