

ROBUSTNESS AND TRACTABILITY FOR NONCONVEX M-ESTIMATORS

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Abstract: We investigate two important properties of M-estimators, namely, robustness and tractability, in a linear regression setting, when the observations are contaminated by some arbitrary outliers. Specifically, robustness is the statistical property that the estimator should always be close to the true underlying parameters, *regardless of the distribution of the outliers*, and tractability refers to the computational property that the estimator can be computed efficiently, even if the objective function of the M-estimator is *nonconvex*. In this article, by examining the empirical risk, we show that under some sufficient conditions, many M-estimators enjoy nice robustness and tractability properties simultaneously when the percentage of outliers is small. We extend our analysis to the high-dimensional setting, where the number of parameters is greater than the number of samples, $p \gg n$, and prove that when the proportion of outliers is small, the penalized M-estimators with the L_1 penalty enjoy robustness and tractability simultaneously. Our research provides an analytic approach to determine the effects of outliers and tuning parameters on the robustness and tractability of some families of M-estimators. Simulations and case studies are presented to illustrate the usefulness of our theoretical results for M-estimators under Welsch's exponential squared loss and Tukey's bisquare loss.

Key words and phrases: Computational tractability, gross error, high-dimensionality, nonconvexity, robust regression, sparsity.

1. Introduction

M-estimation plays an essential role in linear regression, owing to its robustness and flexibility. From a statistical viewpoint, it has been shown that many M-estimators enjoy desirable robustness properties in the presence of outliers, and asymptotic normality when the data are normally distributed without outliers. Some general theoretical properties and reviews of robust M-estimators can be found in Bai, Rao and Wu (1992), Huber and Ronchetti (2009), Cheng and Huang (2010), Hampel et al. (2011), and El Karoui et al. (2013). In the high-dimensional setting, where the dimensionality is greater than the number of

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