# Supplementary Material for "Constructing near-Hadamard designs with (almost) $D$-optimality by General Supplementary Difference Sets" 

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#### Abstract

This supplementary material provides a catalog of near-Hadamard designs. Table 1 lists the base blocks of GSDSs for constructing near-Hadamard designs when $N=4 t+2$. The first and second columns are the $N$ and $t$ of the designs. The third column indicates the type of the near-Hadamard design and the classification is referred to Theorem 2 in the main paper. The fourth column lists the GSDS base blocks and the last column is the $D$ efficiencies of the designs. The designs with (*) are conjectured to be $D$-optimal. Although near-Hadamard designs cover cases in which Ehlich's upper bound is not achievable, an exception is $N=58$. In this case neither a near-Hadamard design of Type $I$ exists by exhaustive search nor that of Type $I I$ because of an even $t$. Thus, a near-Hadamard design of Type $G$ of order 58 is listed in Table 1. However, a design of order 58 is listed on the website (http://www.indiana.edu/~maxdet) with a larger determinant than that of a near-Hadamard design of Type $G$. On the other hand, we have confirmed that its determinant is less than the determinant value of a Type $I$ design of order 58 .


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Table 1: Base Blocks of GSDSs for Constructing near-Hadamard designs.

| $N$ | $t$ | Type | GSDS | $D$-efficiency |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 1 | I | $\begin{aligned} & \{1\} \\ & \{1\} \end{aligned}$ | 92.83 |
| 10 | 2 | I | $\begin{aligned} & \{1,3\} \\ & \{1,2\} \end{aligned}$ | 97.67 |
| 14 | 3 | $G$ | $\begin{aligned} & \{1,2,4\} \\ & \{1,2,3\} \end{aligned}$ | 97.79 |
| 18 | 4 | I | $\begin{aligned} & \hline\{1,2,4,6\} \\ & \{1,2,3,5\} \end{aligned}$ | 99.34 |
| 22(*) | 5 | I | $\begin{aligned} & \{1,2,4,6,8\} \\ & \{1,2,3,4,7\} \end{aligned}$ | 99.56 |
| 26 | 6 | $G$ | $\begin{aligned} & \{1,2,3,5,8,9\} \\ & \{1,2,3,4,6,9\} \end{aligned}$ | 99.10 |
| 30 | 7 | $G$ | $\begin{gathered} \{1,2,3,5,8,10,11\} \\ \{1,2,3,4,5,7,11\} \end{gathered}$ | 98.89 |
| 34(*) | 8 | I | $\begin{gathered} \{1,2,3,5,6,9,10,12\} \\ \{1,3,7,9,11,12,13,14\} \end{gathered}$ | 99.82 |
| 38 | 9 | $G$ | $\begin{gathered} \{1,2,3,5,7,9,10,14,15\} \\ \{1,2,3,4,5,6,9,12,15\} \end{gathered}$ | 99.16 |
| 42 | 10 | I | $\begin{gathered} \{1,3,5,8,9,10,11,12,15,17\} \\ \{1,5,8,9,10,11,12,15,17,20\} \end{gathered}$ | 99.88 |
| 46 | 11 | $G$ | $\begin{gathered} \{1,2,4,6,9,10,11,12,13,16,22\} \\ \{1,3,4,5,6,9,10,12,13,18,21\} \end{gathered}$ | 99.36 |
| 50 | 12 | $G$ | $\begin{gathered} \{1,3,6,8,10,11,12,13,14,18,21,24\} \\ \{1,4,5,6,9,10,11,12,13,18,20,24\} \end{gathered}$ | 99.62 |
| 54 | 13 | I | $\begin{gathered} \{1,4,6,11,12,13,14,15,17,20,22,23,26\} \\ \{1,5,6,8,10,12,13,14,16,17,18,20,26\} \end{gathered}$ | 99.93 |
| 58 | 14 | $G$ | $\begin{gathered} \{1,2,3,4,5,10,13,15,18,19,22,23,24,25\} \\ \{1,3,4,8,10,12,13,14,15,16,18,20,25,28\} \end{gathered}$ | 99.50 |
| 62 | 15 | $G$ | $\begin{gathered} \{1,2,4,6,7,11,13,14,15,16,17,19,23,24,30\} \\ \{1,2,4,5,7,8,9,12,13,14,16,18,23,26,28\} \end{gathered}$ | 99.38 |
| 66 | 16 | $G$ | $\begin{gathered} \{1,2,3,4,5,8,10,14,15,16,17,18,22,24,27,32\} \\ \{1,2,3,4,5,8,9,12,15,17,20,24,26,28,29,30\} \end{gathered}$ | 99.43 |
| 70(*) | 17 | I | $\begin{gathered} \\ \{1,2,3,4,7,9,11,15,16,17,20,23,25,26,31,32,33\} \\ \{1,3,5,7,15,16,17,18,20,21,23,24,27,28,30,31,32\} \end{gathered}$ | 99.50 |
| 74 | 18 | $G$ | $\begin{aligned} & \{1,2,3,4,6,7,9,10,15,17,18,19,22,26,27,29,32,36\} \\ & \{1,2,3,4,6,8,10,12,13,16,17,18,19,23,24,28,31,36\} \\ & \hline \end{aligned}$ | 99.50 |
| 78(*) | 19 | II | $\begin{gathered} \{1,2,3,5,9,12,13,16,18,19,21,22,23,24,26,27,31,32,34\} \\ \{1,5,6,7,9,11,12,13,15,18,19,21,23,30,32,33,34,35,37,38\} \end{gathered}$ | 99.91 |
| 82 | 20 | $G$ | $\{1,2,3,4,6,8,10,13,14,17,18,19,20,21,26,27,30,35,38,40\}$ <br> $\{1,2,3,4,5,8,11,14,15,19,20,21,26,29,31,33,34,35,38,40\}$ | 99.73 |
| 86 | 21 | $G$ | $\begin{gathered} \{1,2,6,7,8,9,11,12,14,17,18,21,25,28,30,33,34,35,36,40,42\} \\ \{1,2,7,10,11,12,13,14,18,20,22,24,25,27,28,29,30,31,32,35,40\} \end{gathered}$ | 99.69 |
| 90 | 22 | $G$ | $\{1,2,3,4,6,7,10,11,13,16,21,22,23,26,31,33,34,37,38,40,42,44\}$ $\{1,2,3,4,6,7,8,12,14,16,19,20,21,22,23,25,28,30,33,37,38,44\}$ | 99.61 |
| 94(*) | 23 | II | $\begin{gathered} \{1,2,3,6,10,11,12,16,18,19,21,22,23,24,25,28,30,32,33,35,36,38,39\} \\ \{1,3,4,6,7,10,11,12,14,19,22,24,26,28,30,31,32,34,35,36,37,38,42,43\} \end{gathered}$ | 99.93 |
| 98 | 24 | $G$ | $\begin{gathered} \{1,2,3,4,7,8,12,13,15,17,18,20,21,23,24,25,27,28,30,33,34,36,41,48\} \\ \{1,2,3,4,6,7,8,12,14,15,19,23,24,25,26,29,33,34,36,38,40,42,43,48\} \end{gathered}$ | 99.60 |

