

## A Skewed Version of the Robbins-Monro-Joseph Procedure for Binary Response

Dianpeng Wang<sup>1</sup>, Yubin Tian<sup>1</sup> and C. F. JEFF WU<sup>2</sup>

<sup>1</sup>*Beijing Institute of Technology* and <sup>2</sup>*Georgia Institute of Technology*

### Supplementary Material

## S1 The results of simulation for large skewness coefficient $\lambda$

Table 1: Mean square error of estimate for  $p$  between 0.001 and 0.25, true model=normal distribution

	$p = 0.001$	$p = 0.01$	$p = 0.05$	$p = 0.1$	$p = 0.15$	$p = 0.20$	$p = 0.25$
RMJ	0.6436	0.3787	0.2091	0.1518	0.1231	0.1103	0.0948
$\lambda = 1000$	0.2110	0.1242	0.0882	0.0857	0.0850	0.0865	0.0899
$\lambda = 5000$	0.1744	0.1109	0.0894	0.0881	0.0894	0.0971	0.1068
$\lambda = 50000$	0.1442	0.1030	0.0917	0.0966	0.1089	0.1339	0.1683
$\lambda = 500000$	0.1314	0.0978	0.1018	0.1244	0.1760	0.2687	0.3863

RMJ: Robbins-Monro-Joseph procedure.

Table 2: Mean square error of estimate for  $p$  between 0.75 and 0.999, true model=normal distribution

	$p = 0.75$	$p = 0.80$	$p = 0.85$	$p = 0.90$	$p = 0.95$	$p = 0.99$	$p = 0.999$
RMJ	0.0895	0.0992	0.1160	0.1401	0.1919	0.3587	0.6040
$\lambda^{-1} = 1000$	0.0963	0.0936	0.0852	0.0871	0.0917	0.1311	0.1976
$\lambda^{-1} = 5000$	0.1179	0.1048	0.0962	0.0858	0.0897	0.1083	0.1695
$\lambda^{-1} = 50000$	0.1940	0.1537	0.1257	0.0996	0.0928	0.1053	0.1522
$\lambda^{-1} = 500000$	0.3915	0.2814	0.1998	0.1457	0.1069	0.0947	0.1281

RMJ: Robbins-Monro-Joseph procedure.

Table 3: Mean square error of estimate for  $p$  between 0.001 and 0.25, true model=logistic distribution

	$p = 0.001$	$p = 0.01$	$p = 0.05$	$p = 0.1$	$p = 0.15$	$p = 0.20$	$p = 0.25$
RMJ	0.8336	0.5052	0.2555	0.1559	0.1268	0.1080	0.0910
$\lambda = 1000$	0.4397	0.2501	0.1231	0.0937	0.0842	0.0871	0.0886
$\lambda = 5000$	0.4357	0.2626	0.1319	0.0980	0.0935	0.1008	0.1182
$\lambda = 50000$	0.4903	0.3031	0.1518	0.1146	0.1258	0.1603	0.2277
$\lambda = 500000$	0.5977	0.3448	0.1748	0.1543	0.2119	0.3401	0.4882

RMJ: Robbins-Monro-Joseph procedure.

Table 4: Mean square error of estimate for  $p$  between 0.75 and 0.999, true model=logistic distribution

	$p = 0.75$	$p = 0.80$	$p = 0.85$	$p = 0.90$	$p = 0.95$	$p = 0.99$	$p = 0.999$
RMJ	0.0844	0.0968	0.1206	0.1516	0.2256	0.4791	0.8203
$\lambda^{-1} = 1000$	0.0925	0.0824	0.0824	0.0891	0.1186	0.2219	0.4029
$\lambda^{-1} = 5000$	0.1227	0.1036	0.0928	0.0957	0.1220	0.2329	0.3891
$\lambda^{-1} = 50000$	0.2362	0.1613	0.1256	0.1195	0.1444	0.2681	0.4570
$\lambda^{-1} = 500000$	0.5393	0.3619	0.2217	0.1580	0.1717	0.3125	0.5373

RMJ: Robbins-Monro-Joseph procedure.

Table 5: Mean square error of estimate for  $p$  between 0.001 and 0.25, true model=skewed logistic distribution

	$p = 0.001$	$p = 0.01$	$p = 0.05$	$p = 0.1$	$p = 0.15$	$p = 0.20$	$p = 0.25$
RMJ	0.7800	0.4925	0.2918	0.2065	0.1672	0.1464	0.1324
$\lambda = 1000$	0.3909	0.2050	0.1399	0.1332	0.1391	0.1549	0.1747
$\lambda = 5000$	0.3675	0.1993	0.1334	0.1404	0.1566	0.1874	0.2085
$\lambda = 50000$	0.3479	0.1845	0.1376	0.1664	0.1939	0.2401	0.2938
$\lambda = 500000$	0.3671	0.1855	0.1488	0.2042	0.2613	0.3647	0.4316

RMJ: Robbins-Monro-Joseph procedure.

Table 6: Mean square error of estimate for  $p$  between 0.75 and 0.999, true model=skewed logistic distribution

	$p = 0.75$	$p = 0.80$	$p = 0.85$	$p = 0.90$	$p = 0.95$	$p = 0.99$	$p = 0.999$
RMJ	0.1883	0.2216	0.2605	0.3046	0.4503	0.7430	0.9148
$\lambda^{-1} = 1000$	0.1579	0.1659	0.1816	0.2054	0.2707	0.4129	0.7389
$\lambda^{-1} = 5000$	0.1759	0.1754	0.1928	0.2202	0.2711	0.4030	0.6876
$\lambda^{-1} = 50000$	0.2197	0.2183	0.2229	0.2564	0.3120	0.4230	0.6286
$\lambda^{-1} = 500000$	0.2773	0.2653	0.2765	0.2970	0.3639	0.4709	0.5851

RMJ: Robbins-Monro-Joseph procedure.

Table 7: Mean square error of estimate for  $p$  between 0.001 and 0.25, true model=cauchy distribution

	$p = 0.001$	$p = 0.01$	$p = 0.05$	$p = 0.1$	$p = 0.15$	$p = 0.20$	$p = 0.25$
RMJ	0.9642	0.9636	0.9434	0.7818	0.5812	0.4362	0.3162
$\lambda = 1000$	0.9642	0.9633	0.8813	0.5795	0.5745	0.4186	0.2802
$\lambda = 5000$	0.9642	0.9632	0.8586	0.6918	0.7650	0.5108	0.3458
$\lambda = 50000$	0.9642	0.9632	0.8125	0.9923	1.0677	0.6869	0.4413
$\lambda = 500000$	0.9642	0.9634	0.7709	1.4280	1.4180	0.8329	0.5269

RMJ: Robbins-Monro-Joseph procedure.

Table 8: Mean square error of estimate for  $p$  between 0.75 and 0.999, true model=cauchy distribution

	$p = 0.75$	$p = 0.80$	$p = 0.85$	$p = 0.90$	$p = 0.95$	$p = 0.99$	$p = 0.999$
RMJ	0.3036	0.3978	0.5302	0.7467	0.9305	0.9637	0.9642
$\lambda^{-1} = 1000$	0.2786	0.4121	0.5685	0.5404	0.8615	0.9636	0.9642
$\lambda^{-1} = 5000$	0.3468	0.5181	0.7631	0.6605	0.8364	0.9636	0.9642
$\lambda^{-1} = 50000$	0.4452	0.6671	1.0729	0.9762	0.7876	0.9634	0.9642
$\lambda^{-1} = 500000$	0.5581	0.8252	1.4003	1.4129	0.7335	0.9633	0.9642

RMJ: Robbins-Monro-Joseph procedure.

Table 9: Mean square error of estimate for  $p$  between 0.001 and 0.25, true model=extreme value distribution

	$p = 0.001$	$p = 0.01$	$p = 0.05$	$p = 0.1$	$p = 0.15$	$p = 0.20$	$p = 0.25$
RMJ	0.3390	0.2184	0.1310	0.1042	0.0924	0.0833	0.0833
$\lambda = 1000$	0.2139	0.1783	0.1811	0.1896	0.2023	0.2225	0.2385
$\lambda = 5000$	0.2920	0.2620	0.2685	0.2885	0.3242	0.3665	0.3825
$\lambda = 50000$	0.5206	0.5043	0.5981	0.6729	0.7335	0.7444	0.8111
$\lambda = 500000$	1.2153	1.3560	1.6071	1.6494	1.5759	1.5789	1.4801

RMJ: Robbins-Monro-Joseph procedure.

Table 10: Mean square error of estimate for  $p$  between 0.75 and 0.999, true model=extreme value distribution

	$p = 0.75$	$p = 0.80$	$p = 0.85$	$p = 0.90$	$p = 0.95$	$p = 0.99$	$p = 0.999$
RMJ	0.1814	0.2051	0.2435	0.3138	0.4383	0.7465	0.9461
$\lambda^{-1} = 1000$	0.1457	0.1610	0.1865	0.1993	0.2710	0.4079	0.7522
$\lambda^{-1} = 5000$	0.1787	0.1840	0.2091	0.2342	0.2828	0.4083	0.6886
$\lambda^{-1} = 50000$	0.2481	0.2426	0.2626	0.2943	0.3331	0.4319	0.6211
$\lambda^{-1} = 500000$	0.3818	0.3413	0.3497	0.3768	0.4198	0.4925	0.5779

RMJ: Robbins-Monro-Joseph procedure.