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# USE OF LINEAR TRANSFER FUNCTION ANALYSIS IN ECONOMETRIC TIME SERIES MODELLING

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*Abstract:* In the application of large scale econometric models for forecasting, the single-equation ordinary least squares (OLS) method is often used to estimate parameters in each model equation. This paper investigates the properties of the parameter estimates under single-equation estimation methods. Since the disturbance of a time series regression model is seldom a white noise process, it is found that bias is almost inevitable as long as contemporaneous endogenous variables are present in a model equation. This paper proposes a model identification method based on reduced form linear transfer function (LTF) models that can avoid or reduce bias of transfer function weight estimates under rather practical assumptions. It is found that forecasts can be greatly improved if appropriate models are identified and employed.

*Key words and phrases:* Econometric models, reduced form models, structural form models, simultaneous transfer function models, linear transfer function models, serial correlations, consistent estimates, LTF analysis.

## 1. Introduction

The development and application of time series analysis in econometric forecasting has occurred rapidly during the past two decades. In recent years, the focus in this area has shifted from univariate or single equation to multivariate and simultaneous equation models. In particular, there has been a great deal of study on dynamic equation systems (Zellner and Palm (1974)), rational structural form models (Wall (1976)), and vector autoregressive-moving average models (Tiao and Box (1981), Jenkins and Alavi (1981)).

Despite vast advancements in the development of econometric time series modelling, "classical" econometric models are still one of the major tools used by many commercial economic forecasting firms to provide national economic forecasts. Here we refer to the "classical" econometric models as the simultaneous equation systems originally proposed by Tinbergen (1949, 1951) and Klein (1950), and studied extensively by a number of econometricians. In typical applications of classical econometric models, a simultaneous equation system often consists of a set of linear lag regression equations with white noise disturbances.

For typical national economic forecasting systems, the number of variables and equations included in the systems is often large (e.g., ranging from 350 to 44,000 equations, see Chen (1987)). Therefore it is impossible to perform a *joint* parameter estimation of the full system as recommended in modern time series econometric models. Even though the classical econometric model is often referred to as a system of equations, it is important to note that in typical applications of large scale econometric models, the use of "system" or "joint model" comes in at the forecasting stage, rather than at the model estimation stage. In terms of model estimation, the ordinary least squares (OLS) method is usually applied to each equation in the system individually. In such a case, a large system of equations is merely a collection of single-equation linear regression or lag regression models as far as model estimation is concerned. Since the single-equation OLS estimates may have serious bias, the accuracy of forecasts based on such biased estimates is dubious. A number of bias-reduction methods, such as 2SLS, LIML and FIML, have been studied extensively. However, such methods in practice can only be applied to a small system of equations with white noise disturbances.

The usefulness of large scale econometric models has been subject to many criticisms, particularly, the validity of the models and their forecasting performance. In this paper we consider an extension of classical econometric models that may avoid some pitfalls in modelling and improve the forecasting performance of the models. In addition, we investigate the issue of tentative model identification in econometric time series modelling. In almost any statistical modelling, one major issue is the tentative identification (specification) of the model. Since the properties of parameter estimates and the accuracy of forecasts hinge on the assumption that the form of the model is correctly specified, the importance of model identification is apparent. In this paper, we propose the use of linear transfer function (LTF) analysis in the identification of a reduced form econometric model (Liu and Hanssens (1982), Liu (1987)). In the next section, we provide an overview of classical econometric models and their extensions. The LTF method is also briefly described in this section. The theoretical properties of the reduced form LTF estimates are presented in Section 3. In Section 4, the LTF analysis is applied to a set of simulated time series and a set of actual macroeconomic time series.

## 2. Classical Econometric Models and their Extensions

In this section, we briefly outline the framework of classical econometric models and their extensions in light of time series models. For notational simplicity, we will consider a system of equations that consists of two variables, $Y_t$ and $X_t$, where $Y_t$ and $X_t$ may be inter-related and both can be *endogenous* variables in the system. A general form of a classical econometric model for such a system can be expressed as

$$Y_t = C_1 + \beta_1(\mathrm{B})X_t + u_{1t}, \tag{2.1}$$
$$X_t = C_2 + \beta_2(\mathrm{B})Y_t + u_{2t}, \tag{2.2}$$

where B is the backshift operator with $\mathrm{B}Y_t = Y_{t-1}$, and the transfer functions $\beta_1(\mathrm{B})$ and $\beta_2(\mathrm{B})$ are linear polynomials of B with finite orders. In classical econometric models, it is often assumed that $u_{1t}$ and $u_{2t}$ may be contemporaneously cross correlated, but both disturbance processes individually are *serially independent*, i.e., both $u_{1t}$ and $u_{2t}$ follow white noise processes. Occasionally, a first or second order adjustment is employed in parameter estimation of the model. In such situations, it implies that the disturbance process follows a first or second order autoregressive (AR) model. Since the disturbance processes are seldom white and the transfer functions (TF) may be in a rational rather than linear form, the model in (2.1) and (2.2) needs to be generalized to (2.3) and (2.4) to accommodate more realistic situations,

$$Y_t = C_1 + \frac{\omega_1(\mathrm{B})}{\delta_1(\mathrm{B})}X_t + u_{1t}, \quad u_{1t} = \frac{\theta_1(\mathrm{B})}{\phi_1(\mathrm{B})}a_{1t} \tag{2.3}$$
$$X_t = C_1 + \frac{\omega_2(\mathrm{B})}{\delta_2(\mathrm{B})}Y_t + u_{2t}, \quad u_{2t} = \frac{\theta_2(\mathrm{B})}{\phi_2(\mathrm{B})}a_{2t} \tag{2.4}$$

where $[a_{1t} \quad a_{2t}]'$ are independently and identically distributed as multivariate normal $N(\mathbf{0}, \boldsymbol{\Sigma})$, $\theta_i(\mathrm{B})/\phi_i(\mathrm{B})$'s are the autoregressive-moving average (ARMA) operators of the disturbance term, and $\omega_i(\mathrm{B})/\delta_i(\mathrm{B})$'s are the transfer functions (Box and Jenkins (1976)), where

$$\phi(\mathrm{B}) = 1 - \phi_1\mathrm{B} - \phi_2\mathrm{B}^2 - \cdots - \phi_p\mathrm{B}^p, \tag{2.5}$$
$$\theta(\mathrm{B}) = 1 - \theta_1\mathrm{B} - \theta_2\mathrm{B}^2 - \cdots - \theta_q\mathrm{B}^q, \tag{2.6}$$
$$\omega(\mathrm{B}) = \omega_0 + \omega_1\mathrm{B} + \omega_2\mathrm{B}^2 + \cdots + \omega_g\mathrm{B}^g, \tag{2.7}$$
$$\delta(\mathrm{B}) = 1 - \delta_1\mathrm{B} - \delta_2\mathrm{B}^2 - \cdots - \delta_r\mathrm{B}^r. \tag{2.8}$$

Here the subscript $i$'s are omitted for notational simplicity. The model in (2.3) and (2.4) is also referred to as a rational structural form (RSF) model (Wall (1976)), or a simultaneous transfer function (STF) model (Liu and Hudak (1984), Liu et al. (1986)). In general, a $k$-equation STF model can be written in a compact matrix form as shown in the literature mentioned above.

In typical applications of econometric models, a large number of equations may be considered. Ideally, the parameters in a system of equations should be estimated by using a joint estimation method such as the full information maximum likelihood (FIML) method. However, since the number of equations is large, the parameters are usually estimated by using the single-equation OLS method. Depending on the form of the model and the extent of serial correlation, the single-equation OLS estimates of the parameters can be rather biased. More details are shown in Section 3.

As will be discussed in the next section, the bias of the single-equation parameter estimates can be greatly reduced if the contemporaneous endogenous variables are not included in an equation, i.e., the model equation is in a *reduced form*. Even though such a reduced form model may not be immediately useful in the structural analysis of a system, it can be directly employed in forecasting. When only reduced form models are considered, the $\omega_0$ parameter in the $\omega_i(\mathrm{B})$ polynomial in (2.7) is restricted to zero.

*Identification of a transfer function model: the* LTF *method*

Determining the appropriate form of the models in (2.3) and (2.4) is crucially important. In classical econometric modelling, the specification of a model is usually based on economic theory. However, in most practical applications of econometric time series modelling, theory or a priori knowledge of an equation system may not be sufficient to completely specify an appropriate model, in particular, the lag structure in a model. In such cases, empirical model identification is important. For reduced form STF models, we find that the linear transfer function (LTF) method proposed in Liu et al. (1986) and Liu (1987) can be used for tentative identification of a model. The LTF method and its rationale are outlined here. For convenience of illustration, we shall consider only the reduced form equation described in (2.3).

There are two basic concerns in the identification of a transfer function model: (1) the form of the rational polynomials $\omega_i(\mathrm{B})/\delta_i(\mathrm{B})$; and (2) the form of the disturbance process, $\{\theta_i(\mathrm{B})/\phi_i(\mathrm{B})\}a_t$. The main spirit of the LTF method is to express the transfer function $\omega(\mathrm{B})/\delta(\mathrm{B})$ for each input variable in an approximate linear form as $V(\mathrm{B}) = v_1\mathrm{B} + v_2\mathrm{B}^2 + \cdots + v_s\mathrm{B}^s$ during the model identification stage. The linear transfer function $V(\mathrm{B})$ has a finite number of terms and $V(\mathrm{B}) = \omega(\mathrm{B})$ when $\delta(\mathrm{B}) = 1$. Conversely, $V(\mathrm{B})$ theoretically has an infinite number of terms when $\delta(\mathrm{B}) \neq 1$. The values $v_1, v_2, \ldots$ are referred to as the transfer function weights for the related input series. Thus the reduced form equation for (2.3) can be written in an approximate linear form as

$$Y_t = C_1 + V_1(\mathrm{B})X_t + u_{1t}. \tag{2.9}$$

Using the model in (2.9), the transfer function weights can be estimated using the ordinary least squares method, where the length of lag $s$ for each input variable can be determined by a stepwise vector autoregression analysis (Tiao and Box

(1981), Tsay (1985)) or chosen judiciously. However, since the disturbance, $u_{1t}$, is seldom a white noise process, the least squares estimates of transfer function weights may be inefficient. In general, the disturbance should not be assumed to be white noise. In the case of nonseasonal series, a useful first approximation to $u_{1t}$ is that of a simple autoregressive process, such as an AR(1) or AR(2) model. For seasonal time series, an initial approximation of the disturbance term may be a first-order multiplicative model. It is important to note that these low order AR models are only used during the estimation of intermediate models. Typically the estimates of transfer function weights are not sensitive to the choice of AR models. After the linear transfer functions $V_i(\mathrm{B})$ are estimated, we can easily obtain the corresponding estimated disturbance series $\widehat{u}_{1t}$, and then identify an ARMA model for $\widehat{u}_{1t}$. Furthermore, the form of transfer functions $\omega_i(\mathrm{B})/\delta_i(\mathrm{B})$ can be determined using the corner method based on the estimated transfer function weights if the weights have a die-out pattern (Beguin, Gourieroux and Monfort (1980), Liu and Hanssens (1982), Tsay (1985)). However, if the transfer function weights have a cut-off pattern, it is not required to employ the corner method.

## 3. Properties of the Transfer Function Weight Estimates under the LTF Method

In this section, we study the properties of the transfer function weight estimates under the LTF method. We are particularly interested in the cause of bias in the single-equation OLS estimation and how it can be avoided or reduced. Without loss of generality, we assume $C_1 = 0$ and $C_2 = 0$ in models (2.3) and (2.4). Since all zeros of $\delta_1(\mathrm{B})$ and $\delta_2(\mathrm{B})$ are restricted to lie outside the unit circle, the reduced form model of (2.3) and (2.4) can be written approximately as

$$Y_t = V_1(\mathrm{B})X_t + u_{1t}, \tag{3.1}$$

$$X_t = V_2(\mathrm{B})Y_t + u_{2t}, \quad t = 1, 2, \ldots, n, \tag{3.2}$$

where

$$V_1(\mathrm{B}) = v_1\mathrm{B} + v_2\mathrm{B}^2 + \cdots + v_s\mathrm{B}^s, \tag{3.3}$$

$$V_2(\mathrm{B}) = w_1\mathrm{B} + w_2\mathrm{B}^2 + \cdots + w_\ell\mathrm{B}^\ell, \tag{3.4}$$

where $s$ and $\ell$ are sufficiently large integers. Note that the disturbances $u_{1t}$ and $u_{2t}$ may be serially and cross correlated. Using $\boldsymbol{Z}_t = [Y_t\ X_t]'$ and $\boldsymbol{u}_t = [u_{1t}\ u_{2t}]'$, the reduced form model in (3.1) and (3.2) can be written in vector autoregressive form as

$$(\boldsymbol{I} - \boldsymbol{\Phi}_1\mathrm{B} - \boldsymbol{\Phi}_2\mathrm{B}^2 - \cdots - \boldsymbol{\Phi}_p\mathrm{B}^p)\boldsymbol{Z}_t = \boldsymbol{u}_t, \quad p = \max\{s, \ell\} \tag{3.5}$$

where the diagonals of the $\boldsymbol{\Phi}$ matrices are all zeros.

Below, we shall focus on the properties of the TF weight estimates for the first equation of the multivariate model in (3.1) and (3.2). It is well known (see, e.g., Johnston (1984) and Kmenta (1971)) that the asymptotic bias of the OLS estimates of the TF weights in (3.1) is

$$\operatorname{plim}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}) = \boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{ux} \tag{3.6}$$

where

$$\begin{aligned}
&\boldsymbol{\beta} = [v_1\ v_2 \cdots v_s]', \quad \widehat{\boldsymbol{\beta}} = [\widehat{v}_1\ \widehat{v}_2 \cdots \widehat{v}_s]',\\
&\boldsymbol{\Sigma}_{ux} = [\gamma_{ux}(1)\ \gamma_{ux}(2)\ \cdots \gamma_{ux}(s)]', \quad \gamma_{ux}(j) = \operatorname{Cov}(u_{1t}, X_{t-j}),\\
&\boldsymbol{\Sigma}_{xx} = \begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{s-1} \\ \gamma_1 & \gamma_0 & \cdots & \gamma_{s-2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{s-1} & \gamma_{s-2} & \cdots & \gamma_0 \end{bmatrix}, \quad \gamma_j = \operatorname{Cov}(X_t, X_{t-j}).
\end{aligned} \tag{3.7}$$

Note that in model (3.1), we assumed that it did not include contemporaneous $X_t$ as an input variable. If the contemporaneous $X_t$ is included in the model, the linear transfer function $V_1(\mathrm{B})$ in (3.1) can be expressed as

$$V_1(\mathrm{B}) = v_0 + v_1\mathrm{B} + v_2\mathrm{B}^2 + \cdots + v_s\mathrm{B}^s. \tag{3.8}$$

The bias of the single-equation OLS estimates for $\boldsymbol{\beta}$ in (3.8) is similar to that expressed in (3.6), where

$$\begin{aligned}
&\boldsymbol{\beta} = [v_0\ v_1\ v_2 \cdots v_s]', \quad \widehat{\boldsymbol{\beta}} = [\widehat{v}_0\ \widehat{v}_1\ \widehat{v}_2 \cdots \widehat{v}_s]',\\
&\boldsymbol{\Sigma}_{ux} = [\gamma_{ux}(0)\ \gamma_{ux}(1) \cdots \gamma_{ux}(s)]', \quad \gamma_{ux}(j) = \operatorname{Cov}(u_{1t}, X_{t-j}),
\end{aligned} \tag{3.9}$$

and $\boldsymbol{\Sigma}_{xx}$ is an $(s+1) \times (s+1)$ matrix similar to that in (3.6) except that an additional $\gamma_j$ element is added at the end of each row and column.

From the result in (3.6) and (3.7), it is obvious that if the cross correlations between $u_{1t}$ and lagged $X_t$ can be avoided, then the bias of the TF weight estimates can be eliminated if a reduced form model is employed. Also, if $\boldsymbol{\Sigma}_{xx}$ is in the form of a diagonal matrix (or close to diagonal matrix), then the bias can be limited to a few isolated lags with cross correlations rather than propagating themselves to the entire vector of estimates. Below we study the cross covariances between $u_{1t}$ and $X_{t-j}$'s.

Expressing the model in (3.5) in its inverted form, we have

$$\boldsymbol{Z}_t = \boldsymbol{u}_t + \boldsymbol{\Psi}_1\boldsymbol{u}_{t-1} + \boldsymbol{\Psi}_2\boldsymbol{u}_{t-2} + \cdots \tag{3.10}$$

where

$$\boldsymbol{\Psi}_1 = \boldsymbol{\Phi}_1, \quad \boldsymbol{\Psi}_2 = \boldsymbol{\Phi}_1\boldsymbol{\Psi}_1 + \boldsymbol{\Phi}_2, \quad \ldots, \quad \text{and}$$
$$\boldsymbol{\Psi}_j = \boldsymbol{\Phi}_1\boldsymbol{\Psi}_{j-1} + \boldsymbol{\Phi}_2\boldsymbol{\Psi}_{j-2} + \cdots + \boldsymbol{\Phi}_p\boldsymbol{\Psi}_{j-p}, \quad \text{if} \quad j \geq p \quad (\boldsymbol{\Psi}_0 = \boldsymbol{I}).$$

Using the above results, we can conveniently derive the covariance matrix $E(\boldsymbol{u}_t\boldsymbol{Z}'_{t-j})$ for $j = 1, 2, \ldots, s$. Note that the cross covariances between $u_{1t}$ and $X_{t-j}$'s are contained in the matrix $E(\boldsymbol{u}_t\boldsymbol{Z}'_{t-j})$ where $\gamma_{ux}(j)$ is the (1,2)th element of the matrix $E(\boldsymbol{u}_t\boldsymbol{Z}'_{t-j})$. If a lagged dependent variable is included in (3.1), then the cross covariance between $u_{1t}$ and $Y_{t-j}$ is the (1,1)th element of the matrix $E(\boldsymbol{u}_t\boldsymbol{Z}'_{t-j})$.

Now we examine the consistency property of $\widehat{\boldsymbol{\beta}}$ under certain joint relationships between $u_{1t}$ and $u_{2t}$. First we consider a simple case. Assuming that $\boldsymbol{u}_t$ follows a white noise process with mean $\mathbf{0}$ and covariance $\boldsymbol{\Sigma}$, we have

$$E(\boldsymbol{u}_t\boldsymbol{Z}'_t) = \boldsymbol{\Sigma} \quad \text{and} \quad E(\boldsymbol{u}_t\boldsymbol{Z}'_{t-j}) = \mathbf{0} \quad \text{if} \quad j \geq 1. \tag{3.11}$$

By examining (3.7) and (3.9), it is useful to note that (i) $\widehat{\boldsymbol{\beta}}$ is always consistent as long as the contemporaneous $X_t$ is not included in (3.1); and (ii) if the contemporaneous $X_t$ is included in (3.1), the OLS estimates of the TF weights are all biased unless $\sigma_{12} = 0$.

*Reduced form* STF *models*

Under the general formulation of the STF models, it is typically assumed that $u_{1t}$ and $u_{2t}$ may be serially correlated, but there is no cross-equation relationships between $u_{1t}$ and $u_{2t}$. Assuming that $u_{1t}$ and $u_{2t}$ follow ARMA processes described in (2.3) and (2.4), we can express $\boldsymbol{u}_t$ in a special vector MA model as

$$\boldsymbol{u}_t = (\boldsymbol{I} - \boldsymbol{\Lambda}_1\mathrm{B} - \boldsymbol{\Lambda}_2\mathrm{B}^2 - \cdots - \boldsymbol{\Lambda}_q\mathrm{B}^q)\boldsymbol{a}_t \tag{3.12}$$

for sufficiently large $q$, where the $\boldsymbol{\Lambda}_i$'s are diagonal matrices containing the psi-weights of $u_{1t}$ and $u_{2t}$ as their diagonal elements. It can be shown that

$$E(\boldsymbol{u}_t\boldsymbol{Z}'_{t-j}) = \boldsymbol{\Upsilon}_j + \boldsymbol{\Upsilon}_{j+1}\boldsymbol{\Psi}'_1 + \boldsymbol{\Upsilon}_{j+2}\boldsymbol{\Psi}'_2 + \cdots, \quad j = 0, 1, 2, \ldots, \tag{3.13}$$

where

$$\boldsymbol{\Upsilon}_j = E(\boldsymbol{u}_t\boldsymbol{u}'_{t-j}) = \begin{cases} \sum_{i=0}^{q-j} \boldsymbol{\Lambda}_{i+j}\boldsymbol{\Sigma}\boldsymbol{\Lambda}'_i, & j = 0, \ldots, q \quad (\boldsymbol{\Lambda}_0 = -\boldsymbol{I}) \\ \mathbf{0}, & j > q. \end{cases} \tag{3.14}$$

From the above result we see that serious bias will occur as long as $u_{1t}$ is autocorrelated. However *if $u_{1t}$ follows a white noise process* (regardless of the process for $u_{2t}$), then the $\boldsymbol{A}_j$ matrices in (3.12) have the following form

$$\boldsymbol{A}_j = \begin{bmatrix} 0 & 0 \\ 0 & * \end{bmatrix}, \quad j = 1,2,\ldots,q, \tag{3.15}$$

where "$*$" represents a nonzero value. Under this special case, the $\boldsymbol{\Upsilon}_j$ matrices ($j \geq 1$) in (3.14) also have the same form as $\boldsymbol{A}_j$'s. Therefore the first row of the covariance matrix between $\boldsymbol{u}_t$ and $\boldsymbol{Z}_{t-j}$ ($j \geq 1$) is a zero vector. Since $\gamma_{ux}(j)$ is either the (1,2)th or the (1,1)th element of the matrix $E(\boldsymbol{u}_t \boldsymbol{Z}_{t-j}')$, it is obvious that $\gamma_{ux}(j) = 0$ for all $j \geq 1$ in this special case. Therefore, the estimate $\widehat{\boldsymbol{\beta}}$ is consistent as long as $u_{1t}$ is white noise and the contemporaneous $X_t$ is not included in the linear transfer function model. It is also worth noting that if $u_{1t}$ is not a white noise process, the seriousness of the bias is dependent upon the extent to which $u_{1t}$ is autocorrelated.

In general, $u_{1t}$ may not be a white noise process. However, we may filter both $Y_t$ and $X_t$ by the disturbance model of the first equation, that is $\phi_1(\mathrm{B})/\theta_1(\mathrm{B})$. Thus the models in (3.1) and (3.2) become

$$y_t = V_1(\mathrm{B})x_t + a_{1t}, \tag{3.16}$$

$$x_t = V_2(\mathrm{B})y_t + \frac{\phi_1(\mathrm{B})\theta_2(\mathrm{B})}{\theta_1(\mathrm{B})\phi_2(\mathrm{B})}a_{2t}, \tag{3.17}$$

where $y_t = \{\phi_1(\mathrm{B})/\theta_1(\mathrm{B})\}Y_t$ and $x_t = \{\phi_1(\mathrm{B})/\theta_1(\mathrm{B})\}X_t$. With this reformulation, the single equation TF weight estimates of (3.16) are unbiased.

From the above results, we demonstrate that filtering all variables in an equation by the same ARMA model of the disturbance term will result in consistent estimates if no cross-equation MA terms are present or will at least reduce the bias of the estimates if cross-equation MA terms do exist. In applying the above result, we need only include an appropriate ARMA model for the disturbance in each equation instead of actually prefiltering the series. However since the true disturbance model for an equation is usually unknown, it is necessary to begin the analysis with an approximation. As shown in the above derivation, the LTF method outlined in Section 2 can be employed in the identification of reduced form STF models.

In a more complex situation, $\boldsymbol{u}_t$ in (3.5) may follow a vector MA($q$) process to allow for cross-equation MA relationships. Following similar derivations to those shown above, it is easy to see that bias may occur in such a situation. However, it can be shown that bias may be reduced if an appropriate ARMA model is employed in each equation during the estimation of transfer function weights.

In using the LTF method, it is recommended that simple AR models be used to approximate the model of the disturbance term. This is partially due to the fact that most real-life economic time series are highly autocorrelated; thus AR filters will improve the diagonality condition of the $\Sigma_{xx}$ matrix, while MA filters, in general, will degrade such a condition of this matrix. From the result in (3.6), we see that $\Sigma_{xx}$ is another contributing factor in the bias. If $\Sigma_{xx}$ is diagonal (i.e., $X_t$ is white noise), then the bias can be limited to only a few isolated lags and more desirable estimates will be yielded.

## 4. Application of the LTF Method on Simulated and Actual Examples

In this section we illustrate the application of the LTF identification method using simulated and actual data sets. For the simulated data set we intentionally use a simpler system of equations for the sake of clarity. The actual data set is a collection of macroeconomic time series of Taiwan, Republic of China (R.O.C.). We use this set of time series to illustrate the practical use of the LTF method and the importance of model identification in forecasting.

### 4.1. Simulated example

In this simulated example, we consider three time series. The first two time series $Y_{1t}$ and $Y_{2t}$ follow the STF model listed bellow:

$$Y_{1t} = 0.8Y_{2(t-3)} + (1 - 0.6\mathrm{B})a_{1t} \tag{4.1}$$

$$Y_{2t} = 0.7Y_{1(t-1)} + (1 - 0.7\mathrm{B}^4)a_{2t} \tag{4.2}$$

$$\begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} 1.00 & 0.60 \\ 0.60 & 1.44 \end{bmatrix}\right), \quad t = 1,2,\ldots,n \quad \text{and} \quad n = 100.$$

In the above model, both $Y_{1t}$ and $Y_{2t}$ are endogenous, and $Y_{2t}$ possesses slight quarterly seasonality. In addition to $Y_{1t}$ and $Y_{2t}$, an unrelated series $Y_{3t}$ is simulated according to the following ARMA(1,4) model

$$(1 - 0.7\mathrm{B})Y_{3t} = (1 - 0.8\mathrm{B}^4)a_{3t}, \quad a_{3t} \sim N(0, 1.00). \tag{4.3}$$

We are interested in (i) whether the LTF identification method can reveal that $Y_{3t}$ is an unrelated variable with respect to $Y_{1t}$ and $Y_{2t}$ in both directions; and (ii) whether the lag structure in each equation can be correctly identified.

Before employing the LTF analysis, it is helpful to know an appropriate maximum lag order ($s$) to use in the LTF models. We may employ vector AR models and the stepwise autoregression analysis discussed in Tiao and Box (1981), Tsay (1985) and Liu et al. (1986) to obtain such information. The M($\ell$) statistics

(Tiao and Box (1981)) for vector AR(1) to AR(8) models are listed below:

| Model | AR(1) | AR(2) | AR(3) | AR(4) | AR(5) | AR(6) | AR(7) | AR(8) |
|---|---|---|---|---|---|---|---|---|
| $M(\ell)$ | 66.66 | 26.50 | 69.65 | 51.66 | 30.57 | 9.07 | 20.11 | 10.81 |

Critical values for $\chi^2_9$ : 16.9 for 5%, 21.7 for 1% (upper tail)

The above $M(\ell)$ statistics suggest that a vector AR model of order 5 to 7 may be an appropriate approximation of the STF model to be identified. When AR(5), AR(6) and AR(7) models are fitted, the estimated models clearly indicated the first two time series are inter-related. However, the estimated models fail to indicate that $Y_{3t}$ is an unrelated variable in the system since there are non-zero off-diagonal elements in the third rows and the third columns of the AR matrices. In general, the usefulness of stepwise autoregression analysis decreases as the number of series included in the system increases. Extensive discussion of this topic can be found in Tiao and Tsay (1989) and Tsay (1989).

*The* LTF *Method*

Based on the results in stepwise autoregression, we may employ the maximum lag order of 7 in the LTF analysis. However, since the TF weights at lags 6 and 7 are all insignificant, we shall present the results based on the maximum lag order of 5. In the first step of the LTF estimation, an AR(1) disturbance model is used for all three equations. In examining the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) of the disturbances of the above three equations, it is found that the disturbance model can be better approximated by $(1-\phi_1 B-\phi_2 B^2)^{-1}a_t$ for the $Y_{1t}$ equation, and $(1-\phi_1 B)^{-1}(1-\phi_2 B^4)^{-1}a_t$ for the $Y_{2t}$ and $Y_{3t}$ equations. The estimates of the TF weights under the revised disturbance models are listed in Step 1 of Table 1. From the estimates and their $t$-values in this table, it is clear that $Y_{2t}$ is not influenced by $Y_{3t}$, and $Y_{3t}$ is not influenced by $Y_{1t}$. After eliminating unrelated variables and insignificant lags from each equation, we obtain the estimates of the TF weights in Step 2 of Table 1. From the results in Step 2, it is clear that $Y_{1t}$ is not influenced by $Y_{3t}$, and $Y_{3t}$ is not influenced by $Y_{2t}$, also all lag structures in the $Y_{1t}$ and $Y_{2t}$ equations are correctly identified. Examining the sample ACF, PACF, and extended autocorrelation function (Tsay and Tiao (1984)) of the disturbance term for each equation, we find that the disturbance models of $Y_{1t}$, $Y_{2t}$ and $Y_{3t}$ equations are the same as those in the simulation models (4.1)–(4.3).

In this simulation example, it is found that we have correctly concluded that $Y_{1t}$ and $Y_{2t}$ are not influenced by $Y_{3t}$, and $Y_{3t}$ is not influenced by $Y_{1t}$ and $Y_{2t}$. Furthermore, the lag structure and disturbance model in each equation are correctly identified.

Table 1. Estimates and $t$-values of the transfer function weights using the LTF method: the simulated data

*Step 1:* Estimation using all lags between 1 and 5

| Variables | | Transfer function weights and their $t$-values (in parentheses) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Output | Input | $C$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $\phi_1$ | $\phi_2$ |
| $Y_1$ | $Y_2$ | −.03 | .09 | −.19* | .80* | .18* | −.11 | −.60* | −.46* |
| | | (.63) | (1.34) | (2.62) | (12.42) | (2.49) | (1.70) | (6.28) | (4.31) |
| | $Y_3$ | | .11 | −.32* | .36* | −.17 | −.01 | | |
| | | | (1.39) | (2.65) | (3.00) | (1.51) | (.08) | | |
| $Y_2$ | $Y_1$ | .03 | .53* | −.11 | −.02 | −.13 | .08 | .06 | −.47* |
| | | (.36) | (5.64) | (1.61) | (.32) | (1.87) | (.95) | (.12) | (3.57) |
| | $Y_3$ | | .01 | −.20 | .13 | .01 | −.11 | | |
| | | | (.12) | (1.74) | (1.07) | (.07) | (1.12) | | |
| $Y_3$ | $Y_1$ | −.26 | .11 | −.16 | −.06 | −.18 | −.01 | .55* | −.45* |
| | | (1.51) | (.99) | (1.05) | (.50) | (1.57) | (.15) | (6.32) | (4.80) |
| | $Y_2$ | | .22* | .14 | .29* | −.02 | −.00 | | |
| | | | (2.01) | (1.18) | (2.29) | (.11) | (.02) | | |

*Step 2:* Estimation using possibly significant lags only

| Variables | | Transfer function weights and their $t$-values (in parentheses) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Output | Input | $C$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $\phi_1$ | $\phi_2$ |
| $Y_1$ | $Y_2$ | −.02 | −− | −.13 | .84* | .10 | −− | −.63* | −.34* |
| | | (.34) | | (1.97) | (11.69) | (1.51) | | (6.44) | (3.37) |
| | $Y_3$ | | −− | −.07 | .07 | −− | −− | | |
| | | | | (1.27) | (1.19) | | | | |
| $Y_2$ | $Y_1$ | .05 | .65* | −− | −− | −− | −− | .06 | −.51* |
| | | (.57) | (11.81) | | | | | (.60) | (5.58) |
| $Y_3$ | $Y_2$ | −.22 | .03 | .10 | .04 | −− | −− | .55* | −.42* |
| | | (1.15) | (.51) | (1.40) | (.66) | | | (6.37) | (4.51) |

(*) The asterisk indicates that the estimate is significant at 5% level

$$\text{Correlation matrix of the residual series:} \quad \Sigma = \begin{bmatrix} 1.00 & & \\ 0.32 & 1.00 & \\ 0.14 & 0.06 & 1.00 \end{bmatrix}.$$

## 4.2. National economic time series of Taiwan, R.O.C.

In this section, we study a macroeconometric forecasting system employed by the Directorate General of Budget, Accounting and Statistics (DGBAS), R.O.C. We shall use this study to illustrate the problems with the inclusion of contemporaneous input variables in the identification of a transfer function model and their effects on the forecasting performance of the model.

Following the methods developed by Klein (1950) and Wharton Econometric Forecasting Associates (WEFA (1986)), a macroeconometric model consisting of 26 behavioral equations and 42 definitional equations has evolved over a period of 20 years. The sixth revision of this forecasting system (DGBAS (1987)) contains 83 variables, including 68 endogenous variables and 15 exogenous variables. The data used in this study began in the first quarter of 1968 and ended in the fourth quarter of 1988, a total of 84 observations for each variable. Among the 84 observations in each series, the first 72 observations will be used for model identification and parameter estimation. The last 12 observations will be used solely for the comparison of the forecasting performance of the models. One of the most important applications of this econometric model is to forecast the quarterly economic time series of Taiwan.

Among the 26 dependent variables in the behavioral equations of the DGBAS model, we shall only include 12 of them in this study. For the variables excluded in the study, nine of them are price deflation variables, two of them are tax variables, and three other variables are not easily interpretable. Since the wholesale price index (WPI) and consumer price index (CPI) are included in the study, the behavior of these two variables is representative for other price index variables excluded from this study. The tax variables are not included in the study due to the irregular variability of their seasonal patterns, which is mainly caused by changes of tax regulations. For convenience of reference, the abbreviations and definitions of the 12 dependent variables and their relevant explanatory variables are listed below. The time series plots for the 12 dependent variables (original series) are shown in Figure 1. Note that for the variable D, the data were available beginning in the first quarter of 1977 (instead of 1968). Also, the variable CF had a substantial different seasonal pattern prior to the first quarter of 1976; hence, this portion of the data is not included in the analysis. As shown in the time series plots, all the series to be studied are nonstationary and possess strong seasonality (except for WPI and CPI). Since the series CO, IBF, M, X, and MON have greater variability over time, logarithmic transformed data will be employed in this study.

*Dependent variables*

- CF — Food consumption expenditure, in constant million NT$ (NT$: new Taiwan dollar)
- CO — Nonfood consumption expenditure, in constant million NT$
- IBF — Gross fixed capital formation in private sector, in constant million NT$
- D — Provision for domestic fixed capital consumption, in constant million

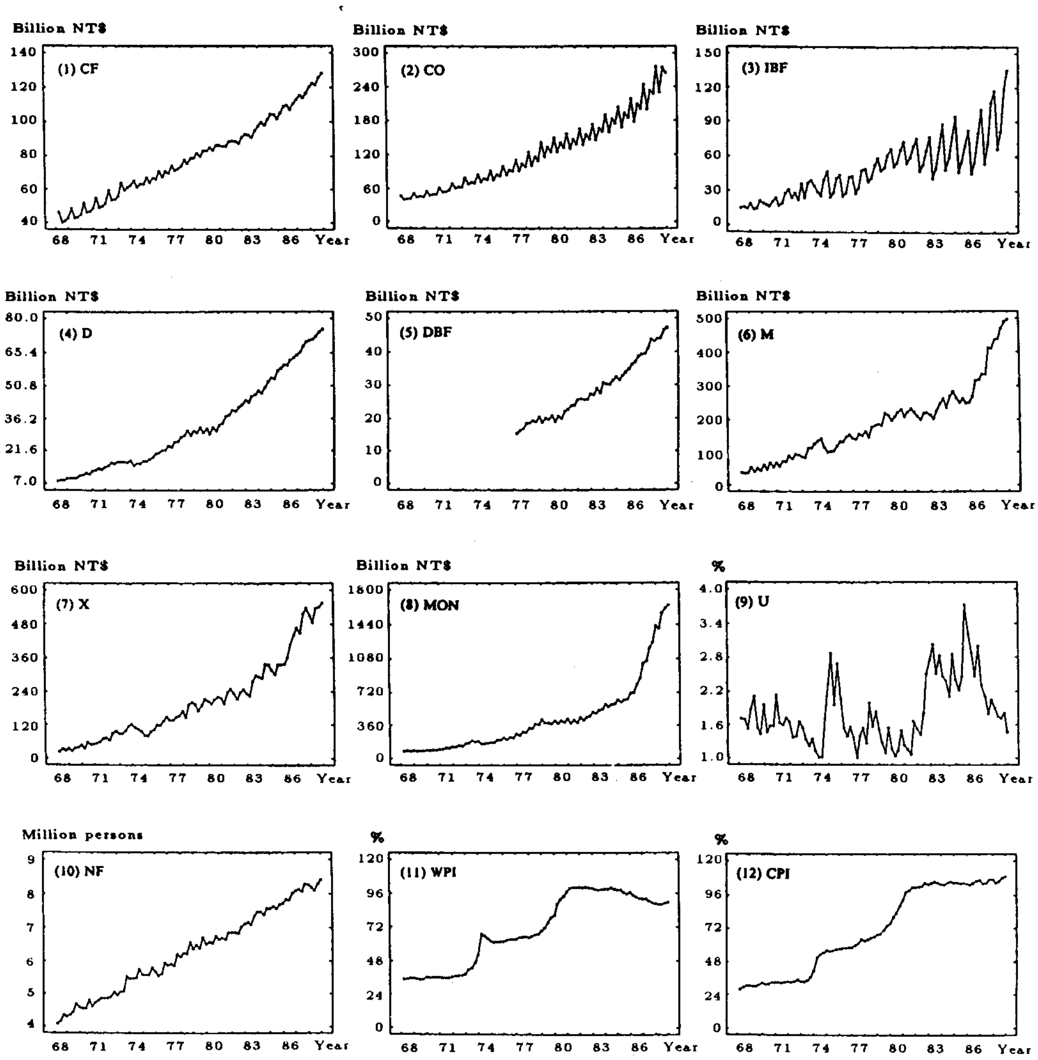


Figure 1. Plots of DGBAS economic time series (1/1968 - 4/1988)

| | |
|---|---|
| | NT$ |
| DBF | Provision for fixed capital consumption in private sector, in constant million NT$ |
| M | Imports of goods and services, in constant million NT$ |
| X | Exports of goods and services, in constant million NT$ |
| MON | Money demand, in constant million NT$, defined as: MON = MON$/PGDP*0.01 |

| | |
|---|---|
| U | Unemployment rate |
| NF | Labor force, in million persons |
| WPI | Wholesale price index in Taiwan area, R.O.C. (1981=100) |
| CPI | Consumer price index in Taiwan area, R.O.C. (1981=100) |

*Relevant explanatory variables*

| | |
|---|---|
| DUM63 | Dummy variable for indication of data after 1974 |
| E | Foreign exchange rate (NT$/US$) |
| GDP | Gross domestic product, in constant million NT$ |
| GNP | Gross national product, in constant million NT$ |
| IR | Rediscount rate of CBC (The Central Bank of China, R.O.C.), average rate per annum |
| KBF | Fixed capital stock on private sector, in constant million NT$ |
| N | Population in Taiwan area, R.O.C., in million persons |
| PDT | Per capita productivity, defined as: PDT = GDP/NE, NE=NF(1 − .01U) |
| PGDP* | Changes in implicit price deflator of GDP, defined as PGDP* = (PGDP − PGDP(−4))/PGDP*100% |
| PM | Implicit price deflator for imports of goods and services |
| PWM | Index of average monthly earnings per manufacturing employee |
| PX | Implicit price deflator for exports |
| Q1 | Dummy variable for the first quarter of each year |
| Q2 | Dummy variable for the second quarter of each year |
| Q3 | Dummy variable for the third quarter of each year |
| QF | Potential GDP estimated by trend-through-peak method |
| TW | World trade quantity index |
| YDD | Disposable income, in constant million NT$ |

The regression models to be considered in this study are based on the sixth revision of the DGBAS quarterly macroeconometric models (DGBAS (1987)). Similar to most macroeconometric models, the variables included in each equation are mainly based on economic theory rather than empirical evidence. The validity of these regression equations remains to be examined; however, this is not the main focus of this study and we shall assume that the variables included in each equation are appropriate. In the following analyses, we shall examine whether the explanatory variables in each equation are useful in improving the forecasting accuracy. The 12 regression models to be studied are listed below. In the original formulation, all these regression models are in simple linear form with white noise disturbance. It is important to note that most of the regression models include a number of contemporaneous explanatory variables which may cause biases in single-equation model estimation, as discussed in Section 3.

(1) Food consumption expenditure (CF)
E1: $\mathrm{CF} = f(\mathrm{YDD}, \mathrm{CF}_{-1}, \mathrm{Q1}, \mathrm{Q2}, \mathrm{Q3})$

(2) Nonfood consumption expenditure (CO)
E2: $\ell n(\mathrm{CO}) = f(\ell n(\mathrm{YDD}), \ell n(\mathrm{CO}_{-1}), \mathrm{Q1}, \mathrm{Q2}, \mathrm{Q3})$

(3) Gross fixed capital formation in private sector (IBF)
E3: $\ell n(\mathrm{IBF}) = f(\ell n(\mathrm{X}_{-1}), \ell n(\mathrm{X}_{-2}), \ell n(\mathrm{GDP/QF}), \ell n(\mathrm{IR-PGDP^*}), \mathrm{Q1}, \mathrm{Q2}, \mathrm{Q3})$

(4) Provision for domestic fixed capital consumption (D)
E5: $\mathrm{D} = f(\mathrm{GDP})$

(5) Provision for fixed capital consumption in private sector (DBF)
E6: $\mathrm{DBF} = f([\mathrm{KBF}_{-1} + \mathrm{KBF}_{-2}]/2, \mathrm{Q}_1, \mathrm{Q}_2, \mathrm{Q}_3)$

(6) Imports of goods and services (M)
E7: $\ell n(\mathrm{M}) = f(\ell n(\mathrm{X}), \ell n(\mathrm{X}_{-1}), \ell n(\mathrm{E}), \ell n(\mathrm{E}_{-1}), \ell n(\mathrm{PM/WPI}))$

(7) Exports of goods and services
E8: $\ell n(\mathrm{X}) = f(\ell n(\mathrm{TW}), \ell n(\mathrm{E}), \ell n(\mathrm{PX/WPI}), \ell n(\mathrm{M}_{-1}), \ell n(\mathrm{X}_{-4}))$

(8) Money demand (MON)
E9: $\ell n(\mathrm{MON}) = f(\ell n(\mathrm{GNP}), \ell n(\mathrm{IR}))$

(9) Unemployment rate (U)
E10: $\mathrm{U} = f(\mathrm{GDP/QF}, \mathrm{PWM/PDT}, \mathrm{U}_{-1}, \mathrm{Q1}, \mathrm{Q2}, \mathrm{Q3})$

(10) Labor force
E11: $\mathrm{NF} = f(\mathrm{N}, \mathrm{PWM/PDT})$

(11) Wholesale price index (WPI)
E15: $\mathrm{WPI} = f(\mathrm{PM}, \mathrm{PWM/PDT}, \mathrm{DUM63})$

(12) Consumer price index (CPI)
E16: $\mathrm{CPI} = f(\mathrm{WPI}_{-1}, \mathrm{GDP/QF}, \mathrm{MON}_{-1}/\mathrm{GDP})$

*Criterion of Model Performance*

To evaluate the performance of different models, we shall employ the root mean squared error (RSME) for within-sample and post-sample of each equation. The RMSE is defined as

$$\mathrm{RMSE} = \left\{ \frac{1}{m} \sum_{t=1}^{m} (Y_t - \hat{Y}_t)^2 \right\}^{1/2} \tag{4.4}$$

where $\hat{Y}_t$ is the fitted or predicted value of $Y_t$ based on an estimated model, and $m$ is the number of observations used in the computation. According to the definition of RMSE, the within-sample RMSE is an estimate of the standard deviation of random errors if the parameter estimates of the model are unbiased, and the post-sample RMSE is a measure of forecast performance using the estimated model.

*Results of the analysis*

In this study, we shall examine the model performance of classical regression models, structural form transfer function models, and reduced form transfer function models. We shall also employ univariate autoregressive-integrated moving average (ARIMA) models for the analysis of each dependent variable. The analysis based on ARIMA models is useful since ARIMA models can be regarded as baseline models to which other more complicated models can be compared with. The results of model performance for different classes of models are summarized in Table 2. In this table, the term "relative RMSE" represents the RMSE of a particular model divided by the RMSE of its corresponding ARIMA model for the same sampling period. Therefore, if the relative RMSE is less than 1, it implies that the model has a smaller RMSE than its corresponding ARIMA model, and vice versa. Below we list the ARIMA models that we have developed for the 12 dependent variables (the values within the parentheses are $t$-values, and the operators $\nabla = 1 - \mathrm{B}$ and $\nabla_4 = 1 - \mathrm{B}^4$).

$$(1)\quad \underset{(5.84)}{(1 - .700\mathrm{B})}\nabla_4 \mathrm{CF}_t = \underset{(2.42)}{1214.893} + a_t, \qquad \widehat{\sigma}_a = 1053.90$$

$$(2)\quad \underset{(10.58)}{(1 - .799\mathrm{B})}\nabla_4 \ell n(\mathrm{CO}_t) = \underset{(2.48)}{.0172} + a_t, \qquad \widehat{\sigma}_a = .0168$$

$$(3)\quad \underset{(3.77)}{(1 - .432\mathrm{B})}\nabla_4 \ell n(\mathrm{IBF}_t) = \underset{(3.85)}{.0479} + \underset{(5.58)}{(1 - .586\mathrm{B}^4)}a_t, \qquad \widehat{\sigma}_a = .1437$$

$$(4)\quad \nabla\nabla_4 \mathrm{D}_t = \underset{(3.77)}{(1 - .414\mathrm{B}^4)}a_t, \qquad \widehat{\sigma}_a = 842.65$$

$$(5)\quad \nabla\nabla_4 \mathrm{DBF}_t = \underset{(10.38)}{(1 - .937\mathrm{B}^4)}a_t, \qquad \widehat{\sigma}_a = 827.53$$

$$(6)\quad \underset{(8.56)}{(1 - .742\mathrm{B})}\nabla_4 \ell n(\mathrm{M}_t) = \underset{(2.23)}{.0269} + \underset{(4.44)}{(1 - .444\mathrm{B}^4)}a_t, \qquad \widehat{\sigma}_a = .1036$$

$$(7)\quad \nabla\nabla_4 \ell n(\mathrm{X}_t) = \underset{(2.27)}{(1 - .268\mathrm{B})}\underset{(12.07)}{(1 - .787\mathrm{B}^4)}a_t, \qquad \widehat{\sigma}_a = .1059$$

$$(8)\quad (1 - \underset{(13.02)}{1.372\mathrm{B}} + \underset{(-4.76)}{.520\mathrm{B}^2})\nabla_4 \ell n(\mathrm{MON}_t) = \underset{(2.80)}{.0192} + \underset{(5.14)}{(1 - .587\mathrm{B}^4)}a_t, \quad \widehat{\sigma}_a = .0394$$

$$(9)\quad \underset{(10.15)}{(1 - .813\mathrm{B})}\nabla_4 \mathrm{U}_t = \underset{(7.45)}{(1 - .713\mathrm{B}^4)}a_t, \qquad \widehat{\sigma}_a = .3575$$

$$(10)\ (1 - \underset{(7.96)}{.699}B)\nabla_4 NF_t = \underset{(3.32)}{.0596} + (1 - \underset{(6.54)}{.640}B^4)a_t, \qquad \hat{\sigma}_a = .0712$$

$$(11)\ (1 - \underset{(4.74)}{.494}B)\nabla WPI_t = a_t, \qquad \hat{\sigma}_a = 2.1532$$

$$(12)\ (1 - \underset{(7.09)}{.644}B)\nabla CPI_t = a_t. \qquad \hat{\sigma}_a = 1.5990$$

Table 2. The within-sample and post-sample RMSE's for ARIMA, white-noise regression and transfer function models

| Dependent variable | RMSE's of ARIMA Models | | Relative RMSE's | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Regression models | | Reduced form TF models | | Structural form TF models | |
| | Within-sample | Post-sample (1/86–4/88) | Within-sample | Post-sample (1/86–4/88) | Within-sample | Post-sample (1/86–4/88) | Within-sample | Post-sample (1/86–4/88) |
| 1.CF | 1053.9004 | 1283.0806 | .99 | 1.98 | 1.00(a) | 1.00 | .77 | 3.02 |
| 2.$\ln$(CO) | .0168 | .0176 | 1.44 | 2.11 | 1.00(a) | 1.00 | 1.00(a) | 1.00 |
| 3.$\ln$(IBF) | .1437 | .0774 | 1.21 | 7.83 | 1.00(a) | 1.00 | .84 | 5.26 |
| 4.D | 842.6536 | 721.1655 | 2.86 | 3.95 | 1.00(a) | 1.00 | 1.00(a) | 1.00 |
| 5.DBF | 827.5281 | 1284.3933 | .94 | 3.77 | 1.00(a) | 1.00 | 1.00(a) | 1.00 |
| 6.$\ln$(M) | .1036 | .0607 | .70 | 3.87 | .83 | 1.26 | .66 | 2.10 |
| 7.$\ln$(X) | .1059 | .0556 | 1.05 | 9.85 | 1.00(a) | 1.00 | .69 | 3.05 |
| 8.$\ln$(MON) | .0394 | .0459 | 1.63 | 4.97 | 1.01 | 1.20 | .78 | 1.21 |
| 9.U | .3575 | .2556 | .90 | .74 | .93 | 1.08 | .93(b) | 1.08 |
| 10.NF | .0712 | .0592 | 1.66 | 3.86 | 1.00(a) | 1.00 | 1.00(a) | 1.00 |
| 11.WPI | 2.1532 | .8713 | .94 | 5.60 | 1.00(a) | 1.00 | .59 | 2.68 |
| 12.CPI | 1.5990 | 1.7323 | 1.79 | 2.68 | .87 | .88 | .69 | .99 |

(a) The model is the same as the ARIMA model for the dependent variable.
(b) The model is the same as the corresponding reduced form transfer function model.

Based on the results in Table 2, we find that classical regression models have rather poor performance for both within-sample and post-sample periods, despite the fact most of their regression coefficients are highly significant. This is not surprising since the disturbances of these models are assumed to be white noise, though in fact they are highly serially correlated (Box and Newbold (1971)). This result confirms the point we showed in Section 3 that the potential bias for the regression coefficients of contemporaneous explanatory variables can be great if the disturbance term is serially correlated and contemporaneous terms are included in the model. To alleviate the serial correlation, we include an AR(1) $\times$ AR$(1)_4$ disturbance model (or an AR(1) model if the variable does not have seasonality) in each regression model. With such a modification, the performance of the regression models is improved, but it is still substantially worse than that of their corresponding ARIMA models, particularly if we compare the RMSE's for the post-sample period.

*Model identification using the* LTF *method*

To obtain more appropriate transfer function models for each dependent variable, we employ the LTF method outlined in Section 2 in the identification of transfer function models. Both the reduced form and structural form models are studied for each equation. In the application of the LTF analysis, we find it is necessary to seasonally difference ($\nabla_4$) most of the variables, except for WPI and CPI. The variables WPI and CPI require regular differencing. If appropriate differencing is not employed during model identification, we find that the final models tend to include spurious explanatory variables, particularly for the structural form models. In the application of the LTF analysis, the maximum order of lags for each explanatory variable is typically set to 5, and reduced to lower orders as the analysis proceeds. Based on the LTF analysis, we find a number of transfer function models degenerate to ARIMA models. Below we list the equations that have either the reduced form or structural form models that are different from ARIMA models. The models listed under (a) are reduced form models, and those under (b) are structural form models. The relative RMSE's for within-sample and post-sample are also listed in Table 2.

(1) Food consumption expenditures (CF)

(a) Same as the ARIMA model

(b) $$\underset{}{\nabla_4 \mathrm{CF}_t} = (\underset{(6.89)}{.107} + \underset{(3.56)}{.056}\mathrm{B})\nabla_4 \mathrm{YDD}_t + a_t \qquad \hat{\sigma}_a = 810.46$$

(3) Gross fixed capital formation on private sector (IBF)

(a) Same as the ARIMA model

(b) $$\nabla_4 \ell n(\mathrm{IBF}_t) = \underset{(8.20)}{.1785} + (\underset{(-4.63)}{-.000006})\nabla_4 \ell n(\mathrm{X}_t) + \underset{(4.80)}{2.517}\nabla_4 \ell n(\mathrm{GDP}_t/\mathrm{QF}_t)$$
$$+(\underset{(2.62)}{.614} - \underset{(-2.41)}{.629}\mathrm{B})\nabla_4 \ell n(\mathrm{IR}_t - \mathrm{PGDP}^*_t) + (1 - \underset{(4.96)}{.549}\mathrm{B}^4)a_t, \qquad \hat{\sigma}_a = .1202$$

(6) Imports of goods and services (M)

(a) $$\nabla_4 \ell n(\mathrm{M}_t) = \underset{(16.98)}{.731}\mathrm{B}^2\nabla_4 \ell n(\mathrm{X}_t) + (1 - \underset{(8.85)}{.762}\mathrm{B}^4)/(1 - \underset{(4.54)}{.502}\mathrm{B})a_t, \ \hat{\sigma}_a = .0857$$

(b) $$\nabla_4 \ell n(\mathrm{M}_t) = \underset{(24.45)}{.724}\nabla_4 \ell n(\mathrm{X}_t) + (\underset{(-5.17)}{-1.704}\mathrm{B})\nabla_4 \ell n(\mathrm{E}_t)$$
$$+(1 - \underset{(6.59)}{.637}\mathrm{B}^4)/(1 - \underset{(2.52)}{.256}\mathrm{B})a_t, \qquad \hat{\sigma}_a = .0681$$

(7) Exports of goods and services (X)

(a) Same as the ARIMA model

(b) $$\nabla_4 \ell n(X_t) = \underset{(2.61)}{.0256} + \underset{(3.44)}{.554}\nabla_4 \ell n(TW_t) + (\underset{(4.19)}{1.872}B)\nabla_4 \ell n(E_t) + (\underset{(2.30)}{.482}B)\nabla_4 \ell n(PX_t/WPI_t) + \underset{(9.33)}{.908}\nabla_4 \ell n(M_t) + (1 - \underset{(7.18)}{.716}B^4)/(1 - \underset{(2.38)}{.244}B)a_t, \qquad \widehat{\sigma}_a = .0735$$

(8) Money supply (MON)

(a) $$\nabla_4 \ell n(MON_t) = \underset{(4.89)}{.1202} + (\underset{(-4.55)}{-.320}B)\nabla_4 \ell n(IR_t) + (1 - \underset{(5.11)}{.610}B^4)/(1 - \underset{(15.03)}{.913}B)a_t, \qquad \widehat{\sigma}_a = .0398$$

(b) $$\nabla_4 \ell n(MON_t) = \underset{(1.28)}{.0407} + \underset{(4.72)}{.857}\nabla_4 \ell n(GNP_t) + (\underset{(-7.40)}{-.414})\nabla_4 \ell n(IR_t) + (1 - \underset{(4.49)}{.547}B^4)/(1 - \underset{(15.93)}{.929}B)a_t, \qquad \widehat{\sigma}_a = .0306$$

(9) Unemployment rate (U)

(a) $$\nabla_4 U_t = (\underset{(-3.65)}{-7.741}B^2)\nabla_4(GDP_t/QF_t) + (1 - \underset{(4.02)}{.518}B^4)/(1 - \underset{(8.05)}{.753}B)a_t, \qquad \widehat{\sigma}_a = .3338$$

(b) Same as the model in (a)

(11) Wholesale price index in Taiwan area (WPI)

(a) Same as the ARIMA model

(b) $$\nabla WPI_t = (\underset{(3.27)}{6350.78} + \underset{(3.07)}{7954.04}B + \underset{(2.03)}{3968.81}B^2)\nabla(PWM_t/PDT_t) + \underset{(10.30)}{11.551}\nabla DUM63_t + 1/(1 - \underset{(6.11)}{.620}B)a_t, \qquad \widehat{\sigma}_a = 1.2669$$

(12) Consumer price index in Taiwan area (CPI)

(a) $$\nabla CPI_t = (\underset{(4.34)}{.336}B + \underset{(1.61)}{.138}B^2 + \underset{(3.75)}{.305}B^3)\nabla WPI_t + (\underset{(3.52)}{7.243}B^2)\nabla(MON_{t-1}/GDP_t) + a_t, \qquad \widehat{\sigma}_a = 1.3759$$

(b) $$\nabla CPI_t = \underset{(8.99)}{.552}\nabla WPI_t + 1/(1 - \underset{(5.09)}{.518}B)a_t. \qquad \widehat{\sigma}_a = 1.1059$$

The analyses shown above reveal several interesting points that are worth further discussion:

(1) When the LTF analysis is employed, a number of transfer function models degenerate to ARIMA models. In particular, this is true when reduced form models are employed. This result indicates that the association between the explanatory variables and the dependent variable is not as strong as the original hypotheses of the models suggested (or what the classical regression models indicated). This is not too surprising if we take the economic environment of Taiwan into consideration. Taiwan has a highly regulated economy. However, due to its tremendous foreign trade and exports, Taiwan has very high but volatile growth in its economy. A number of foreign and domestic events also have had important impacts on Taiwan's economy. All these factors contribute to major disturbances which might weaken the potential relationships between the dependent variables and their explanatory variables. As the free economic environment becomes more mature and the political situation becomes more stable, we may find transfer function models more useful in modelling Taiwan's economic time series.

(2) For a few of the reduced form transfer function models that are different from ARIMA models, we find the post-sample RMSE's of these models are slightly larger than the RMSE's of their corresponding ARIMA models (except for CPI), despite the within-sample RMSE's for these models are smaller than the ARIMA models (see Table 2). This is probably due to the fact that the economic and political conditions changed greatly during the post-sample period (1986–1988) and the historical relationships between a dependent variable and its explanatory variables may not be directly extendable to the evolving environment. Unlike transfer function models, ARIMA models seem to be more adaptive and are able to track the short term behavior of the time series better.

(3) By comparing the within-sample and post-sample RMSE's between the structural form and reduced form TF models shown in Table 2, we find that even though we are able to obtain structural form TF models with smaller within-sample RMSE's, their post-sample RMSE's are much worse than their corresponding reduced form TF models. This phenomenon demonstrates that the bias caused by serially correlations and the inclusion of contemporaneous explanatory variables is probably quite substantial. This result confirms the theoretical proof in Section 3 and demonstrates the usefulness of the reduced form TF models in forecasting practice.

(4) In model building we often focus on the reduction of the within-sample RMSE in model selection. In this study, we find that a model with smaller within-sample RMSE does not always yield smaller post-sample RMSE. As pointed out in (3), bias in the estimates of model parameters may produce an under-estimated standard error for the model. This biased standard error

can be misleading in model selection and forecasting.

## 5. Summary and Discussion

In this paper, we propose the use of the LTF analysis in the identification of a reduced form transfer function model. By using the reduced form LTF models, we demonstrate that consistent estimates of the transfer function weights can be obtained under rather practical assumptions. On the other hand, when contemporaneous endogenous variables are included in the models (i.e., under structural form models), it is rather difficult to avoid bias in parameter estimates using single-equation estimation methods. However, many well-known econometric models are formulated based on structural form models. Estimation of such models using single-equation methods undoubtedly will result in models with rather poor performance. The analysis of the DGBAS macroeconomic time series provides rather insightful evidence for such problems.

There are a number of basic issues that need to be addressed in the application of large scale econometric models. One of the basic issues is the purpose of modelling. Traditionally, an econometric model is expected to serve two purposes: (1) to represent the underlying relationships among the variables in an economic system, and (2) to provide accurate forecasts. It is important to realize that typically it is much more difficult to address the first goal than the second one. In order to address the first goal, it is necessary to employ an equation model that may contain contemporaneous endogenous variables. Since it is difficult to avoid biased estimates as long as contemporaneous endogenous variables are included in a model and single-equation estimation methods are used, it is advisable to separate the goal of structural analysis from forecasting. In terms of forecasting, a reduced form model is appropriate to use. In a reduced form model, we intentionally avoid the use of contemporaneous endogenous variables and thus avoid bias if serial correlations in disturbance terms are appropriately taken care of.

In addition to the bias problem, another common pitfall in classical econometric modelling is the mis-specification of models, for example, the inclusion of an incorrect lag structure or wrong variables. Careful analysis of the reduced form models of the system can help eliminate such problems. Thus, building a reduced form model does not only produce a sound model for forecasting, it can also serve as an intermediate step toward structural form modelling. More details are discussed in Hanssens and Liu (1983) and Liu and Hudak (1984).

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## References

Beguin, J.-M, Gourieroux, C. and Monfort, A. (1980). Identification of a mixed autoregressive-moving average process: the corner method. *Time Series* (Edited by O. D. Anderson), 423-436. North-Holland, Amsterdam.

Box, G. E. P. and Jenkins, G. M. (1976). *Time Series Analysis: Forecasting and Control.* Holden-Day, San Francisco.

Box, G. E. P. and Newbold, P. (1971). Some comments on a paper of Coen, Gomme and Kendall. *J. Roy. Statist. Soc. Ser.A* **134**, 229-240.

Chen, T. (1987). A quick glance at four econometric models. *An Executive's Guide to Econometric Forecasting* (Edited by A. Migliaro and C. L. Jain). Graceway Publishing Company, Flushing, New York.

DGBAS (1987). *Quarterly National Economic Trends, Taiwan Area, The Republic of China.*

Hanssens, D. M. and Liu, L.-M. (1983). Lag specification in rational distributed lag structural models. *J. Bus. & Econom. Statist.* **1**, 316-325.

Jenkins, G. M. and Alavi, A. S. (1981). Some aspects of modelling and forecasting multivariate time series. *J. Time Ser. Anal.* **2**, 1-47.

Johnston, J. (1984). *Econometric Methods.* McGraw-Hill, New York.

Klein, L. R. (1950). Economic fluctuations in the United States, 1921-41. *Cowless Commission Monograph 11.* John Wiley, New York.

Kmenta, J. (1971). *Elements of Econometrics.* Macmillan, New York.

Liu, L.-M. and Hanssens, D. M. (1982). Identification of multiple-input transfer function models. *Comm. Statist. Theory Methods* **11**, 297-314.

Liu, L.-M. and Hudak, G. B. (1984). Unified econometric model building using simultaneous transfer function models. *Time Series Analysis: Theory and Practice 5* (Edited by O. D. Anderson), 291-308. North-Holland, Amsterdam and New York.

Liu, L.-M., Hudak, G. B., Box, G. E. P., Muller, M. E. and Tiao, G. C. (1986). *The SCA Statistical System: The Reference Manual for Forecasting and Time Series Analysis.* Scientific Computing Associates, P. O. Box 625, Dekalb, Illinois 60115.

Liu, L.-M. (1987). Sales forecasting using multi-equation transfer function models. *J. Forecast.* **6**, 223-238.

Tiao, G. C. and Box, G. E. P. (1981). Modeling multiple time series with applications. *J. Amer. Statist. Assoc.* **76**, 802-816.

Tiao, G. C. and Tsay, R. S. (1989). Model specification in multivariate time series. *J. Roy. Statist. Soc. Ser.B* **51**, 151-213.

Tinbergen, J. (1949). Long-term foreign trade elasticities. *Metroeconomica.*

Tinbergen, J. (1951). *Econometrics.* George Allen and Unwin, Ltd, London.

Tsay, R. S. and Tiao, G. C. (1984). Consistent estimates of autoregressive parameters and extended sample autocorrelation function for stationary and nonstationary ARMA models. *J. Amer. Statist. Assoc.* **79**, 84-96.

Tsay, R. S. (1985). Model identification in dynamic regression (distributed lag) models. *J. Bus. & Econom. Statist.* **3**, 228-237.

Tsay, R. S. (1989). Parsimonious parameterization of vector autoregressive moving average models. *J. Bus. & Economic. Statist.* **7**, 327-341.

Wall, K. D. (1976). FIML estimation of rational distributed lag structural form models. *Ann. Econom. and Soc. Measure.* **5**, 53-64.

WEFA (1986). *WEFA User Guide*. Wharton Econometric Forecasting Associates, Inc. Philadelphia, PA.

Zellner, A. and Palm, F. (1974). Time series analysis and simultaneous equation econometric models. *J. Econometrics* **2**, 17-54.

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