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A NOTE ON FIRST PASSAGE TIMES OF STATIONARY SEQUENCES

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Abstract: Let $G(\cdot)$ be a Borel function applied to a stationary, possibly long-memory, sequence of standard Gaussian random variables $\{X_i\}$. Define the first passage time $T(c) = \inf\{n \geq 1, S_n \geq c\}, c > 0$, for partial sums $S_n = \sum_{i=1}^n G(X_i)$. Suppose $G(X_i)$ has finite positive mean μ. When $G(X_i)$ itself is positive or its negative part is under some moment conditions, it is proved that $E(T(c)/c)^\gamma \to \mu^{-\gamma}$ for $\gamma > 0$ as c tends to infinity.

Key words and phrases: Elementary renewal theorem, first passage time, Gaussian sequence, long-memory, long-range dependence, self-similar.

1. Introduction

Let $\{Y_i, i \in Z\}$ be an i.i.d. sequence with finite $EY_n = \mu > 0$. For partial sums $S_n = \sum_{i=1}^n Y_i$, define the first passage time $T(c) = \inf\{n \geq 1, S_n \geq c\}, c > 0$. An important property regarding the asymptotic behavior of $T(c)$ is the so-called elementary renewal theorem (cf. Bhattacharya and Waymire (1990), p.217) which asserts that

$$\lim_{c \to \infty} E(T(c)/c) = 1/\mu. \tag{1.1}$$

In view of a wide-range of applications that have been generated by (1.1), particularly in the context of sequential analysis, it is necessary to extend (1.1) from the i.i.d. case to other dependent time series. Lai (1977, Theorem 3) established (1.1) for certain types of stationary strong mixing sequences. For more restrictive classes of weakly dependent stationary sequences such as absolutely regular (or weak Bernoulli) sequences and functionals of Markov chains and of moving averages having C^1 spectral density, (1.1) can be strengthened to a Blackwell type renewal theorem (Berbee (1979) and Lalley (1986)). This note aims to extend Lai's result in another direction to show that (1.1) still holds for a class of stationary sequences which may not be strongly mixing. The model we consider is defined by $Y_i = G(X_i)$, where $G(\cdot)$ is a Borel function and $\{X_i, i \in Z\}$ is a stationary sequence of standard Gaussian random variables whose covariance function $r(n) = EX_iX_{i+n}$ satisfies (1.2) below. The stationary sequences $\{X_i\}$ with

$$r(n) = |n|^{-\theta} L(n) \tag{1.2}$$

for some $0 < \theta < 1$ and slowly varying $L(n)$ have increasingly gained attention in recent years from statisticians and probabilists. The assumption of (1.2) has proved to be a useful and parsimonious way to describe the covariances of many empirical time series that have been documented in such diverse applied areas as river flows in hydrology (Mandelbrot and Taqqu (1979)), security prices and asset returns in finance (Cutland, Kopp and Willinger (1993)) and network traffic in telecommunication (Willinger, Taqqu, Leland and Wilson (1993)). The basic asymptotic property (1.1) of $T(c)$ with regard to the aggregations of those series mentioned above would naturally be worthy of investigation. A sequence $\{X_i\}$ which satisfies (1.2) is known to be not strongly mixing (cf. Rosenblatt (1961)) and is nowadays often referred to as a long-memory sequence (or a sequence with long-range dependence) to reflect the fact that $\sum_n r(n) = \infty$, or, assuming some regularity conditions on $r(n)$, that the spectral density of $\{X_i\}$ behaves like $x^{\theta-1}$ (modulo a slowly varying factor) as $x \to 0$ (see e.g. Cox (1984), Beran (1992), Künsch (1986) and Robinson (1994), for reviews and more references on long memory sequences). An immediate consequence of (1.2) is the $\mathrm{Var}\,(\sum_{i=1}^n X_i) = n^{2-\theta}L(n)$, which is the main feature often used to distinguish long-memory sequences from traditional short memory dependent series such as Markov and *ARMA* processes. Long-memory sequences are closely related to the self-similar processes introduced by Mandelbrot and van Ness (1968). One well-known example of a long-memory sequence is fractional Gaussian noise, a stationary Gaussian sequence $\{Z_i, i = 1, 2, \ldots\}$ with $EZ_i = 0$, $EZ_i^2 = \sigma_x^2$ and

$$\begin{aligned}\mathrm{Cov}\,(Z_i, Z_{i+n}) &= \sigma_x^2(|n+1|^{2H} - 2|n|^{2H} + |n-1|^{2H})/2 \\ &\sim \sigma_x^2 H(2H-1)|n|^{2H-2},\end{aligned}$$

where $1/2 < H < 1$, and "$\sim$" means the ratio of the right side and left side is asymptotically one. Here H is called the self-similarity parameter (cf. Samorodnitsky and Taqqu (1994), Ch. 7).

We shall prove (1.1) with $S_n = \sum_{i=1}^n G(X_i)$ under some moment conditions on $G(X_i)$. The Gaussian sequences $\{X_i\}$ under consideration can be long-memory as well as short-memory. The paper's main results, Theorems 1 and 2, are stated and proved in Section 2. The key ingredient of the proof is a proposition, possibly be of independent interest. That is established in the Appendix.

2. Main Results

For the rest of the paper, we focus on $Y_i = G(X_i)$ for a Borel function $G(\cdot)$ such that $EY_i = \mu$ is finite, and assume that the covariance function $r(n) = EX_iX_{i+n}$ of the stationary Gaussian sequence $\{X_i\}$ satisfies

$$|r(n)| \le |n|^{-\lambda}L(n), \quad \forall \ \ |n| \ge 1, \tag{2.1}$$

for some $\lambda > 0$ and for slowly varying function $L(n)$. The bound Condition (2.1) contains (1.2) and covers most of the common seen Gaussian models in time series analysis, including the *ARMA* and the fractional *ARIMA* processes (cf. Brockwell and Davis (1987)). Let

$$S_n = \sum_{i=1}^{n} Y_i \text{ with } S_0 = 0, \qquad S_n^* = \sum_{i=1}^{n}(Y_i - \mu),$$

$$T(c) = \inf\{n \geq 1, S_n \geq c\}. \tag{2.2}$$

For any random variable X, let $X^+ = X \vee 0$ and $X^- = -(X \wedge 0)$. Statements of the main theorems follow.

Theorem 1. *Assume* (2.1)*,* $Y_i \geq 0$ *a.s. and* $0 < EY_i = \mu < \infty$*. Then for all* $\gamma > 0$

$$\lim_{c \to \infty} E\Big(\frac{T(c)}{c}\Big)^{\gamma} = \frac{1}{\mu^{\gamma}}. \tag{2.3}$$

Theorem 2. *Assume* (2.1)*,* $EY_i = \mu > 0$ *and* $E(Y_i^-)^q < \infty$ *for* $q > \max\{2, 12/\lambda\}$*. Then* (2.3) *holds for* γ *satisfying* $0 < \gamma < (q\lambda/4 - 3)/(3 + \lambda)$.

The proofs have two main steps. First we establish an upper bound for $P\{\max_{1 \leq j \leq n} |S_j^*| \geq \epsilon n\}, \;\; \epsilon > 0$. To achieve this, we reduce the problem to the i.i.d. case by partitioning $\{Y_1, \ldots, Y_n\}$ into an appropriate number of blocks so that elements of each block can be treated as if they were independent. The next step is to use the bound to construct an γ-integrable "last time", $\sup\{n \geq 1, |\sum_{i=1}^{n} Y_i I(Y_i \leq b) - nEY_1 I(Y_1 \leq b)| \geq n\epsilon\}$ for some $b > 0$, to dominate $T(c)/c$. Then (2.3) is derived by using the ergodicity of $\{Y_i\}$.

Introduce an auxiliary i.i.d. sequence $\{Y_i', i \geq 1\}$ with $Y_i' \stackrel{d}{=} Y_i$, and set $S_n' = \sum_{j=1}^{n}(Y_j' - \mu)$. For β with $1 > \beta > 3/(3+\lambda)$, define $d_n = [n^{\beta}]$ ($[x]$ denotes the integer part of x).

Proposition. *Suppose* $E(Y_i - \mu)^2 < \infty$*. Then for all* $\epsilon > 0$,

$$P\{\max_{1 \leq j \leq n} |S_j^*| \geq \epsilon n\} = O(n^{\beta}(P\{|S'_{[n/d_n]}| \geq \epsilon n d_n^{-1}\})^{1/2}), \tag{2.4}$$

as $n \longrightarrow \infty$.

Define the last time

$$N_\epsilon = \sup\{n \geq 1, |S_n^*/n| \geq \epsilon\}, \quad \epsilon > 0,$$

with the convention that $\sup \phi = 0$. To prove the theorems, we use the proposition to show the intergrability of N_ϵ^{γ} for various γ.

Suppose $0 \le Y_i < b$ a.s. and let $\sigma^2 = \mathrm{Var}\,(Y_i)$. Applying Bennett's inequality (cf. Shorack and Wellner (1986), p.851) to the right-hand side of (2.4), we get for all $\gamma > 0$,

$$\sum_{n=1}^{\infty} n^{\gamma-1} P\{\max_{1\le j\le n} |S_j^*| \ge \epsilon n\}$$
$$= O(\sum_{n=1}^{\infty} n^{\gamma-1+\beta} \exp\{-\frac{[n/d_n]\epsilon^2}{4\sigma^2} \cdot \frac{1}{(1+b(3\sigma^2)^{-1})}\}) < \infty. \tag{2.5}$$

Combining (5.11) and (5.12) of Lemma 2 of Chow and Lai (1975) (cf. Lai (1977), Lemma 4, with $\alpha = 1$ and $p\alpha - 1 = \gamma$) shows that

$$E(\sup\{n \ge 1, |S_n^*| \ge 2\epsilon n\})^\gamma \le \gamma \int_0^\infty t^{\gamma-1} P\{\sup_{j\ge[t]} j^{-1}|S_j^*| \ge 2\epsilon\}dt$$
$$\le (2^\gamma - 1)^{-1}\gamma \cdot \int_0^\infty t^{\gamma-1} P\{\max_{j\le[t]} |S_j^*| \ge \frac{\epsilon}{2}t\}dt$$
$$\le O(\sum_{n=1}^{\infty} n^{\gamma-1} P\{\max_{1\le j\le n} |S_j^*| \ge \epsilon n\}). \tag{2.6}$$

With (2.5) this implies

$$EN_\epsilon^\gamma < \infty \text{ for all } \gamma > 0. \tag{2.7}$$

Assume $Y_i < b$ a.s. and the conditions of Theorem 2. It follows that

$$EN_\epsilon^\gamma < \infty \text{ for } 0 < \gamma < (q\lambda/4 - 3)/(3+\lambda). \tag{2.8}$$

To see this, note that since $q > 4(3 + 3\gamma + \gamma\lambda)/\lambda$, we can choose β sufficiently close to $3/(3+\lambda)$ so that $(1+\gamma)/(1-\beta) < 1 + (q/4)$. Chebyshev's inequality in qth power applied to the right-hand side of (2.4) yields

$$\sum_{n=1}^{\infty} n^{\gamma-1} P\{\max_{1\le j\le n} |S_j^*| \ge \epsilon n\} = O\Big(\sum_{n=1}^{\infty} (n^{1-\beta})^{\frac{\gamma+1}{1-\beta} - 2 - \frac{\beta}{1-\beta} - \frac{q}{4}}\Big)$$
$$= O\Big(\sum_{n=1}^{\infty} n^{\frac{1+\gamma}{1-\beta} - 2 - \frac{q}{4}}\Big) < \infty.$$

Here, in the first bound we use the Von Bahr relation $E|S_m'|^q = O(m^{q/2})$ with $q > 2$ (cf. Shorack and Wellner (1986), p.857). From (2.6) we get (2.8).

Using (2.7) and (2.8), we can prove Theorems 1 and 2 simultaneously.

Proof of Theorems 1 and 2. We adopt the approach used in Theorem 3 of Lai (1977). Since $|r(n)| \longrightarrow 0$ as $n \longrightarrow \infty$, by Maruyama's theorem (cf. Rosenblatt (1961)) both $\{X_i\}$ and $\{Y_i\}$ are ergodic. Hence

$$\lim_{n\to\infty} S_n/n = \mu \quad \text{a.s.},$$

which in turn gives, by noting that $S_{T(c)-1}/T(c) < c/T(c) \leq S_{T(c)}/T(c)$,

$$\lim_{c\to\infty}(T(c)/c)^{\gamma} = \mu^{-\gamma} \quad \text{a.s.} \tag{2.9}$$

We now show (2.3). Let $S''_n = \sum_{j=1}^{n} Y_j I(Y_j \leq b)$, where $b > 0$ is sufficiently large so that $EY_j I(Y_j \leq b) \geq \mu/2$, and define $L = \sup\{n \geq 1, S''_n \leq n\mu/3\}$. If $n \geq \max\{L+1, 3\mu^{-1}c\}$, then $S_n \geq S''_n > n\mu/3 \geq c$. Hence $T(c) \leq L + 3\mu^{-1}c$. This implies that, for $c \geq 1$ and $\gamma > 0$, $(T(c)/c)^{\gamma}$ is bounded by the random variable $(L+3\mu^{-1}+1)^{\gamma}$. Note here that $T(c)$ does not depend on the truncation value b although L does. If we can show $EL^{\gamma} < \infty$ then the desired result (2.3) follows by applying the Dominated Convergence Theorem to (2.9). We can choose small $\epsilon > 0$ so that

$$L \leq L^* \equiv \sup\{n \geq 1, |\frac{S''_n}{n} - EY_1 I(Y_1 \leq b)| \geq \epsilon\}.$$

By (2.7) and (2.8), $E(L^*)^{\gamma} < \infty$ holds (1) for all $\gamma > 0$ if $Y_i > 0$ a.s. as assumed in Theorem 1, or (2) for the γ specified in Theorem 2 if the conditions of Theorem 2 are met. The proof is complete.

Appendix

To prove the Proposition, we need two lemmas. The first is a variant of Lévy's Inequality which can be proved by similar argument (cf. Chow and Teicher (1988), pp.70-72).

Lemma 1. *Let $\{Z_i, i \geq 1\}$ be a sequence of independent random variables. Set $V_m = \sum_{i=1}^{m} Z_i$ and $V_{n,j} = \sum_{i=1}^{j} Z_{n-i+1}$, $1 \leq j \leq n$. Assume*

$$\min_{1\leq j\leq n} P\{V_{n,j} \geq 0\} = v_n^+ > 0 \quad \textit{and} \quad \min_{1\leq j\leq n} P\{V_{n,j} \leq 0\} = v_n^- > 0.$$

Then for all $y > 0$

$$P\{\max_{1\leq j\leq n} |V_j| \geq y\} \leq (v_n)^{-1} P\{|V_n| \geq y\}$$

with $v_n = \min\{v_n^+, v_n^-\}$. Suppose the independent sequence $\{Z_i\}$ is identically distributed with mean zero and finite variance. Then the sequence $\{v_n\}$ can be replaced by some positive constant independent of n and y.

Let $H_k(x)$ denote the kth Hermite polynomial with leading coefficient one and let $\phi(x) = (2\pi)^{-1/2} e^{-x^2/2}$ be the standard Gaussian density. The next lemma is due to Taqqu (1977).

Lemma 2. *Let $(Z'_1, \ldots, Z'_m)$ be a Gaussian random vector with joint probability density function $f(x_1, \ldots, x_m)$. Assume $EZ'_i = 0, \mathrm{Var}\,(Z'_i) = 1$ and $|EZ'_i Z'_j| <$*

$1/(m-1)$, *for all* $i \neq j$. *Then*

$$f(x_1,\ldots,x_m)=\sum_{q=0}^{\infty}\sum_{\substack{k_1+\cdots+k_m=2q\\ 0\le k_1,\ldots,k_m\le q}}\Big\{E\prod_{i=1}^{m}H_{k_i}(Z_i')\Big\}\prod_{i=1}^{m}\frac{H_{k_i}(x_i)}{k_i!}\cdot\phi(x_i). \quad \text{(A.1)}$$

The series representation (*A*.1) *is uniformly convergent over* $\mathcal{R}^m$.

Proof of Proposition. For fixed positive integers n and k, $1 \le k \le n$, each sum $S_k^* = \sum_{i=1}^{k}(Y_i-\mu)$ is decomposed into d_n sub-sums each of which has $m(k,i,n)+1$ summands,

$$S_k^* = \sum_{i=1}^{d_n}\sum_{j=0}^{m(k,i,n)}(Y_{i+jd_n}-\mu) \equiv \sum_{i=1}^{d_n} S(i,d_n,m(k,i,n)) \quad \text{(A.2)}$$

($\sum_a^b x = 0$ if $a > b$). For notational convenience let $m(n,i,n) = n(i)$. For each i, $1 \le i \le d_n$, the length $m(k,i,n)+1$ of the sub-sum $S(i,d_n,m(k,i,n))$ is given by

$$m(k,i,n) = \begin{cases} [k/d_n] & \text{if } i+[k/d_n]d_n \le k \\ [k/d_n]-1 & \text{if } i+[k/d_n]d_n > k, \end{cases}$$

Fix n. Note that when $k < d_n$ the sum $S(i,d_n,m(k,i,n))$ is zero if $i > k$ and, for fixed i, some elements in $\{S(i,d_n,m(k,i,n))|k=1,\ldots,n\}$ may have duplicates. In other words, with i and n fixed,

$$\{S(i,d_n,m(k,i,n))|k=1,\ldots,n\} - \{S(i,d_n,l)|l=0,1,\ldots,n(i)\} \subseteq \{0\}. \quad \text{(A.3)}$$

Hence

$$\begin{aligned}\max_{1\le k\le n}|S_k^*| &\le \max_{1\le k\le n}\sum_{i=1}^{d_n}|S(i,d_n,m(k,i,n)| \quad \text{(by (A.2))}\\ &\le \sum_{i=1}^{d_n}\max_{1\le k\le n}|S(i,d_n,m(k,i,n)| \qquad \text{(A.4)}\\ &= \sum_{i=0}^{d_n}\max_{0\le l\le n(i)}|S(i,d_n,l)| \quad \text{(by (A.3))}.\end{aligned}$$

Set

$$p_i(\epsilon,n) = P\{\max_{0\le l\le n(i)}|S(i,d_n,l)| \ge \epsilon n d_n^{-1}\}.$$

By (A.4)

$$P\{\max_{1\le k\le n}|S_k^*| \ge \epsilon n\} \le \sum_{i=1}^{d_n} p_i(\epsilon,n). \quad \text{(A.5)}$$

Observe that the indices of the summands in $S(i, d_n, l)$ are at least d_n apart from one another. With this in mind, the next step is to use Lemma 2 to approximate $p_i(\epsilon, n)$. For each $n \in Z$, define

$$r^*(n) = \sup_{|n| \leq j} |r(j)|.$$

Since $\sup_{|n|\leq j} j^{-\lambda} L(j)$ is also regularly varying with the same exponent $-\lambda$ (cf. Seneta (1976), pp.20-21), direct computation shows that for large n,

$$(r^*(d_n))^{1/2}(n/d_n)^{3/2} = o(1). \tag{A.6}$$

Note by (A.6) that for all $j_1 \neq j_2$,

$$n(i)|EX_{i+j_1 d_n} X_{i+j_2 d_n}| = n(i)|r((j_1 - j_2)d_n)| \leq n(i) r^*(d_n) < 1.$$

Hence Lemma 2 is applicable as $\{\max_{0\leq l \leq n(i)} |S(i, d_n, l)| \geq \epsilon n d_n^{-1}\}$ is an event of the $(n(i)+1)$-dimensional Gaussian vector $(X_i, X_{i+d_n}, \ldots, X_{i+n(i)d_n})$. First, set

$$B_i(\epsilon, n) = \Big\{(x_0, \ldots, x_{n(i)}) \in \mathcal{R}^{n(i)+1} \Big| \max_{0\leq l\leq n(i)} |\sum_{t=0}^{l} (G(x_t) - \mu)| \geq \epsilon n d_n^{-1}\Big\}.$$

By the stationarity of Y_i's and (A.1),

$$p_i(\epsilon, n) = \sum_{q=0}^{\infty} \sum_{\substack{k_0+\cdots+k_{n(i)}=2q \\ 0\leq k_0,\ldots,k_{n(i)}\leq q}} \Big\{E \prod_{j=0}^{n(i)} \frac{H_{k_j}(X_{1+jd_n})}{\sqrt{k_j!}}\Big\}$$
$$\cdot \int_{B_i(\epsilon,n)} \prod_{j=0}^{n(i)} \frac{H_{k_j}(x_j)}{\sqrt{k_j!}} \cdot \phi(x_j) dx_j. \tag{A.7}$$

It follows from the Cauchy-Schwartz inequality and the fact $\int H_k^2(x)\phi(x)dx = k!$ that

$$\begin{aligned} \Big|\int_{B_i(\epsilon,n)} \prod_{j=0}^{n(i)} \frac{H_{k_j}(x_j)}{\sqrt{k_j!}} \cdot \phi(x_j) dx_j\Big| &\leq \Big(\int_{B_i(\epsilon,n)} \prod_{j=0}^{n(i)} \phi(x_j) dx_j\Big)^{1/2} \\ &= (P\{\max_{0\leq l\leq n(i)} |S'_l| \geq \epsilon n d_n^{-1}\})^{1/2} \qquad \text{(A.8)} \\ &\leq B(P\{|S'_{[n/d_n]+1}| \geq \epsilon n d_n^{-1}\})^{1/2} \end{aligned}$$

for some positive constant B independent of n and ϵ. The last inequality is from Lemma 1. It is known (cf. the proof of Proposition 3.1 of Taqqu (1977)) that

$$\sum_{q=0}^{\infty} \sum_{\substack{k_0+\cdots+k_{n(i)}=2q \\ 0\leq k_0,\ldots,k_{n(i)}\leq q}} \Big|E \prod_{j=0}^{n(i)} \frac{H_{k_j}(X_{1+jd_n})}{\sqrt{k_j!}}\Big| \leq \sum_{q=0}^{\infty} \sum_{\substack{k_0+\cdots+k_{n(i)}=2q \\ 0\leq k_0,\ldots,k_{n(i)}\leq q}} \prod_{j=0}^{n(i)} \{r^*(d_n)(n(i)-1)\}^{k_j/2}$$

$$\leq \Big(\sum_{q=0}^{\infty}[r^*(d_n)(n(i)-1)]^{q/2}\Big)^{[n/d_n]-1}$$
$$\leq \max_{1\leq i\leq d_n} \exp\{-([n/d_n]-1)\log(1-\sqrt{r^*(d_n)(n(i)-1)})\} \qquad (A.9)$$
$$= |O(1)| \quad \text{(by (A.8))}.$$

Combining (A.5), (A.7), (A.8) and (A.9), we get (2.4). The proof is complete.

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