Supplementary File for "Censored Quantile Regression via Box-Cox Transformation under Conditional Independence"

Chenlei Leng and Xingwei Tong

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1 Justification of the Bootstrap Procedure

To show the validity of the bootstrap estimator, we briefly discuss the proof to verify the conditions in Theorem B and Theorem 3 in Chen et al. (2003) by following their Example 1. We only prove this result for $p = 1$ with a single continuous covariate.

First we assume that γ_0 is known. Clearly, the conditions in Theorem 3 have been verified by Lemma A.2. To prove the consistency of the bootstrap estimators, it suffices to verify condition (2.6B) in Chen et al. (2003). Following Lemma A.3, one obtains

$$\Gamma_2(\hat{\beta}_n, \hat{F})[\hat{F}^* - \hat{F}] = -E\left[ZI(\hat{F}(Y|Z) \leq \tau)\frac{\hat{F}^*(Y|Z) - \hat{F}(Y|Z)}{1 - \hat{F}(Y|Z)}\middle|\{(Z_i, Y_i, \delta_i)\}_{i=1}^n\right],$$

where $\hat{F}^*(\cdot|z)$ is the bootstrap conditional Kaplan-Meier estimator of $F_0(\cdot|z)$ from the data $\{(Z_i^*, Y_i^*, \delta_i^*)\}_{i=1}^n$, drawn randomly with replacement from $\{(Z_i, Y_i, \delta_i)\}_{i=1}^n$. Following the proof of Lemma A.3 and noting that $U_n(\hat{\beta}_n, \hat{F}) = 0$, one can see that there are i.i.d random vector functions $v_i^*(\beta, F)$ such that $n^{1/2}(U_n^*(\hat{\beta}_n, \hat{F}) - U_n(\hat{\beta}_n, \hat{F}) + \Gamma_2(\hat{\beta}_n, \hat{F})[\hat{F}^* - \hat{F}]) = n^{-1/2}\sum_{i=1}^n v_i^*(\hat{\beta}_n, \hat{F}) + o_{P^*}(n^{-1/2})$, where $U_n^*(\hat{\beta}_n, \hat{F}) = n^{-1}\sum_{i=1}^n Z_i^*\{\delta_i^* I(Y_i^* \leq \exp(Z_i^{*'}\hat{\beta}_n)) - H_\tau(\hat{F}(Y_i^*|Z_i^*))\}$ and v_i^* is similar to v_i in Lemma A.3. Conditioning on the data, $v_1^*, \cdots, v_n^*$ are independent and mean zero random variables and thus Condition (2.6B) in Chen et al. (2003) is satisfied by the triangular array central limit theorem.

References

Chen, X., Linton, O., and van Keilegom, I. (2003). Estimation of semiparametric models when the criterion function is not smooth. *Econometrica* **71**, 1591-1608.