Chapter 8 Variance

Why variance ?

Components of variance

$$P = G + E$$

$$\begin{aligned} V_P &= V_G + V_E + 2\ Cov(G,E) \\ &= V_A + V_D + V_I + V_E \qquad \left(if\ \ Cov(G,E) = 0 \right) \end{aligned}$$

Table 8.1 Components of variance.

Variance component	*Symbol*	*Value whose variance is measured*
Phenotypic	V_P	Phenotypic value
Genotypic	V_G	Genotypic value
Additive	V_A	Breeding value
Dominance	V_D	Dominance deviation
Interaction	V_I	Interaction deviation
Environmental	V_E	Environmental deviation

Heritability in the broad sense

$$h_b^2 = V_G / V_P$$

Heritability in the narrow sense

$$h_n^2 = V_A / V_P$$

Heritability measures the relative importance of heredity in determining phenotypic value.

$V_P = V_A + V_D + V_I + V_E$

The partitioning of the variance into its components allows us to estimate the relative importance of various determinants of phenotype, in particular the role of heredity versus environment or nature and nurture.

Estimation of the degree of genetic determination, V_G / V_P

Environmental variance can't be removed because …….

Elimination of genotypic variance can be achieved experimentally.

Inbred lines and their F progeny are genetically uniform populations.

$$\begin{array}{ccc} AA & \times & aa \\ & \downarrow & \\ & Aa & \end{array}$$

Random bred populations are genetically mixed populations.

Example 8.1 : $h_b^2 = 49\%$ under the assumptions (a) the environmental variance is the same in all genotypes, (b) different genotypes show different sensitivities to the environment (genotype by environment interaction).

Population	*Components*	*Observed variance*
Mixed	$V_G + V_E$	0.366
Uniform	V_E	0.186
Difference	V_G	0.180
	$V_G/V_P =$	0.180/0.366 = 49%

Homozygote is more sensitive to environment than heterozygote.

Genetic components of variance

$G = A + D + I$

$V_G = V_A + V_D + V_I$

V_A is the variance of breeding value and is the chief cause of resemblance between relative and therefore the chief determinant of the observable genetic properties of the population and of the response of the population to selection.

V_A / V_P is the heritability of the character.

Additive and dominance variance (under H-W equilibrium)

Note that the means of both breeding value and dominance deviations are zero (pg 116 and pg 119)

$$V_A = 2pq[a + d(q-p)]^2 \cdots\cdots [eqn\ 8.3\ (a)]$$

$$V_D = (2pqd)^2 \cdots\cdots [eqn\ 8.4]$$

$$IF \quad d = 0 \quad , \quad V_A = 2pqa^2 \quad , \quad and \quad V_D = 0$$

$$IF \quad d = a \quad , \quad V_A = 8pq^3a^2$$

$$IF \quad p = q \quad , \quad V_A = \frac{1}{2}a^2 \quad , \quad and \quad V_D = \frac{1}{4}d^2$$

Both V_A and V_D depend on the genotypic values and gene frequencies.

Total genetic variance

$$G = A + D$$

$$V_G = V_A + V_D + 2\ Cov_{AD}$$

Under H-W equilibrium, the covariance between additive and dominance effect is zero (pg 126)

$$V_G = 2pq[a + d(q-p)]^2 + [2pqd]^2$$

Example 8.2 : (pg 127)

	$q = 0.1$	$q = 0.4$
Genotypic, V_G	1.1664	7.1424
Additive, V_A	1.0368	6.2208
Dominance, V_D	0.1296	0.9216

The gene frequency and the degree of dominance influence the magnitude of the genetic components of variance (Fig 8.1)

The total phenotypic variance can then be partitioned into V_A, V_D, V_I, and V_E (Table 8.2)

Table 8.2 Partitioning of the variance of four characters in *Drosophila melanogaster*. Components as percentages of the total phenotypic variance.

		Character			
		(1) *Bristles*	(2) *Thorax*	(3) *Ovary*	(4) *Eggs*
Phenotypic	V_P	100	100	100	100
Additive genetic	V_A	52	43	30	18
Non-additive genetic	$V_D + V_I$	9	6	35	44
Environmental	V_E	39	51	35	38

Characters and sources of data:

(1) Number of bristles on 4th + 5th abdominal segments (Clayton, Morris, and Robertson, 1957; Reeve and Robertson, 1954).
(2) Length of thorax (Robertson, 1957*b*).
(3) Size of ovaries, i.e., number of ovarioles in both ovaries. (Robertson, 1957*a*).
(4) Number of eggs laid in 4 days (4th to 8th after emergence) (Robertson, 1957*b*).

For multiple loci, if no linkage disequilibrium

$$V_G = \sum V_{A_i} + \sum V_{D_i} + \sum V_{I_i}$$

Interaction variance

$$V_I = V_{AA} + V_{AD} + V_{DA} + V_{DD}$$

Interaction involving larger numbers of loci contribute so little variance that they can be ignored,…….

There is no doubt that interaction between loci controlling quantitative characters is a frequent occurrence……(pg 190).

Variance due to linkage disequilibrium

$$D_{AB} = P_{AB} - P_A P_B \qquad \text{(pg 16)}$$

Considering loci A and B,

$$G = G_A + G_B$$

$$V_G = V_A + V_B + 2\ Cov(A,B)$$

Covariance can be positive or negative

$$\left(Cov\ (a,B) > 0 \quad or \quad Cov\ (A,B) < 0 \right)$$

There are two forms of non-random mating that generate disequilibrium

(1) selection

(2) assortative mating

Two sorts of covariance :

$$D_t = D_0 (1-C)^t \qquad \text{(pg 18)}$$

Correlation and interaction between genotype and environment

Correlation

The better genotypes are given better environments.

Example (1) milk yield in cows

(2) human intelligence

$$V_P = V_G + V_E + 2\ Cov(G,E)$$

How to estimate $Cov(G,E)$?

The covariance is best regarded as part of genetic variance because …….

Interaction

Genotype by environment interaction :

$$V_P = V_G + V_E + 2\ Cov(G,E) + V_{GE}$$

The environment variance is a property of the genotype. But the source of variation is environmental and not genetic. It is therefore logical to regard any variance due to G×E as being part of environmental variance included in any estimate of V_E .

Environmental sensitivity

Environmental value :

Environmental sensitivity :

Example 8.3 :

Environmental variance

All variation of non-genetic origin is environmental variance.

Environmental variance is a source of error that reduces precision in genetic studies and the aim of experimenter or breeder is therefore to reduce it as much as possible by careful management or proper design of experiments.

Multiple measurements : repeatability

What is repeatability ?

Temporal repetition and spatial repetition

Spatial environmental variance V_{Es}

General environmental variance V_{Eg}

Gain from multiple measurements :

Increasing the number of measurements reduces the amount of variance due to special environment that appears in phenotypic variance, and this reduction of the phenotypic variance represents the gain in accuracy.

$$V_{P(n)} = V_G + V_{Eg} + \frac{1}{n} V_{Es}$$

Example 8.6 :

The advantage of breeding programmes from the gain in accuracy is the increased proportion of additive genetic variance (two assumptions).

Prediction of future performance

Example 8.7

<u>Summary of variance partitioning</u>

Homework: (1) The variances of leaf number in the F_1 and F_2 generations of a cross of tobacco varieties were calculated in Problem 6.1. The variances were 1.46 in the F_1 and 5.97 in the F_2. Estimate the degree of genetic determination in the F_2 generation. What assumptions have to be made to do this?
(2) Calculate the amounts of the additive genetic and dominance variance arising from genes referred to in the population specified in last homework.