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future stages $\pi(A_j, S_j)$, $t \leq j < T$. By this argument, one can show that, for all t, β_t^0 can depend on $\pi(A_j, S_j)$ for all $j = 1, \ldots, T-1$. Consequently, collecting two data sets in the same population, but with different treatment assignment probabilities, can lead to different infinite-sample limits for the E-learning estimators used in the two settings.

We illustrate the meaningful impact that this dependence on treatment mechanism can have on the interpretation of study results through a simple two-stage example. We consider two different data generating mechanisms that are identical in all regards except for their treatment mechanisms. We denote the treatment mechanisms in the two settings by $\pi^{(1)}$ and $\pi^{(2)}$. We will show that the coefficients (removing "value" because it's a special term) in (2.1) vary between the two scenarios. Specifically, we show that $\beta_1^0(\pi^{(1)}) \neq \beta_1^0(\pi^{(2)})$ and $\beta_2^0(\pi^{(1)}) \neq \beta_2^0(\pi^{(2)})$. In both examples, $S_1 = X_1$ is standard normal. $X_2 | A_1 = a_1, X_1 = x_1$ is a normal distribution with mean $a_1 x_1$ and variance 1. We consider a setting where the investigator is only interested in maximizing the final reward, so that $R_1 = 0$ and $R = R_2$. The outcome regression is given by $\mathbb{E}[R|S_2 = s_2, A_2 = a_2] = \mathbb{1}\{a_2 = 1\}[2x_1^2 \mathbb{1}\{a_1 = 1\} + \mathbb{1}\{a_1 = -1\} + 2x_2^2] + \mathbb{1}\{a_2 = -1\}$. We let $\pi_t^{(k)}$ denote $P(A_t = 1|S_t)$ in each scenario k. In the first scenario, we let $\pi_1^{(1)} = \pi_2^{(1)} = 0.5$. In the second scenario, we let $\pi_1^{(2)} = 0.9$ when $X_1 < 0.5$