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Multiply robust nonparametric multiple imputation for the treatment of missing data

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Abstract

Imputation is an effective way for handling missing values. We propose a nonparametric multiple imputation procedure that makes use of multiple outcome regression models and/or multiple propensity score models. Our procedure leads to a multiply robust point estimator in the sense that it remains consistent if all the models but one are misspecified. A variance estimate is readily obtained by applying the customary rule advocated by Rubin (1987). The asymptotic properties of the proposed method are established. Results from a simulation study, assessing the proposed method in terms of bias, efficiency and coverage probability, support our findings.

Key words: Double robustness; Missing data; Multiple imputation; Multiple robustness; Variance estimation.

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1 Introduction

Missing data are virtually unavoidable in clinical trials, social research and surveys, among others. Excluding cases with missing values, a technique often referred to as complete case analysis, generally leads to biased estimators unless the data are missing completely at random (Rubin, 1976). A common approach for handling missing values is imputation, whereby a missing value is filled in by one or more replacement values. For example, item nonresponse in surveys conducted by National Statistical Offices is generally treated by some form of single imputation with the objective of producing public use files, allowing secondary analysts to obtain point estimates through complete data estimation procedures.

In this paper, we consider multiple imputation that consists of replacing a missing value by $M > 1$ imputed values, leading to M completed data files. An estimate is obtained from each data file using a complete data estimation procedure and the M estimates are pooled to obtain a final point estimate and a variance estimate; see, e.g., Rubin (1987), Little and Rubin (2002) and van Buuren (2012) for a comprehensive treatment of multiple imputation. An attractive feature of multiple imputation is that variance estimates are readily obtained by using what is often referred to as Rubin's rule. Wang and Robins (1998) discussed proper (type A) and improper (type B) multiple imputation procedures. Despite the fact that Rubin's variance estimator may be biased in the case of improper procedures, it will often be used in practice due to its simplicity. Rubin's variance formula has been shown to work reasonably well in a number of settings with $M = 5$ although

Wang and Robins (1998) have shown that it led to underestimation of the true variance in the case of finite M.

Whether or not one would succeed in reducing the nonresponse bias effectively depends on the availability of powerful auxiliary variables available for all the sample units (respondents and nonrespondents) as well as the validity of the postulated outcome regression model, which is a set of assumptions about the distribution of the variable being imputed. Traditionally, multiple imputation procedures have been based on a single outcome regression model, making the resulting point estimators vulnerable to model misspecification. To overcome this problem, Long et al. (2012) proposed a doubly robust nonparametric multiple imputation procedure for ignorable missing data. Their method makes use of two working models: an outcome regression model and a propensity score model, the latter being a set of assumptions about the unknown nonresponse mechanism. An estimator is doubly robust if it remains consistent for the true parameter when either model is correctly specified; see, e.g., Robins et al. (1994), Scharfstein et al. (1999), Bang and Robins (2005), Haziza and Rao (2006), Tan (2006), Kang and Schafer (2007), Cao et al. (2009) and Kim and Haziza (2014), among others. Although doubly robust methods offer some protection, point and variance estimators may perform poorly in terms of bias and efficiency if both models are misspecified. This point was raised by a number of authors, including Kang and Schafer (2007) and Chen and Haziza (2017).

To provide additional protection, Han and Wang (2013) introduced the concept of multiple robustness; see also Han (2014a, 2014b, 2016a, 2016b),

Chan and Yam (2014), Chen and Haziza (2017) and Duan and Yin (2017). Multiply robust procedures make use of more than one outcome regression models and/or more than one propensity score models. An estimation procedure is said to be multiply robust if it remains consistent if any one of these multiple models is correctly specified. Multiply robust procedures are attractive in many practical situations; see Chen and Haziza (2017) for a discussion. When all the models are misspecified, a number of empirical investigations available in the literature suggest that multiply robust estimation procedures perform much better than doubly robust procedures based on the same working models, in terms of bias and efficiency; see e.g., Han (2014b) and Chen and Haziza (2017). In this paper, we extend the procedure of Long et al. (2012) by proposing a multiply robust nonparametric multiple imputation procedure that can be easily implemented. As argued by Long et al. (2012), the proposed imputation procedure is proper. Therefore, we advocate the use of Rubin's variance formula. The empirical results in Section 5 suggest that Rubin's variance estimator works well, at least in our experiments. Han (2014b) proposed a variance estimator of multiply robust procedures based on Taylor linearization procedures. Rubin's variance formula is somehow simpler to implement in practice.

The paper is organized as follows. In Section 2, we present the basic setup. The proposed procedure is presented in Section 3. We establish some asymptotic properties in Section 4. The results of a simulation study, assessing the performance of the proposed method in terms of bias, efficiency and coverage probability, are reported in Section 5. Technical details and proofs are relegated to the Appendix.

2 Basic setup

We are interested in estimating the population mean, $\mu_0 = E(Y)$, of a study variable Y subject to missingness. We consider a random sample of size n selected at random from the target population. Let X be a p-vector of fully observed variables and let R be a response indicator such that $R = 1$ if Y is observed and $R = 0$, otherwise. The observed data are the independent and identically distributed triplets $(Y_i R_i, X_i, R_i), i = 1, 2, \ldots, n$. Let s_r and s_m denote respectively the sets of responding units and nonresponding units to the study variable Y.

The relationship between Y and X can be described by the following outcome regression model:

$$Y_i = m(X_i; \beta_0) + \epsilon_i, \quad i = 1, 2 \ldots, n, \tag{1}$$

where $m(X_i; \cdot)$ is a given function with parameter β evaluated at β_0 such that $E(\epsilon_i \mid X_i) = 0$, $var(\epsilon_i \mid X_i) = \sigma^2$ and $E(\epsilon_i \epsilon_j \mid X_i, X_j) = 0$ for $i \neq j$. We assume that Y is missing at random (Rubin, 1976); i.e., the true propensity score $p(X_i)$ satisfies

$$\Pr(R_i = 1 | X_i, Y_i) = \Pr(R_i = 1 | X_i) \equiv p(X_i; \alpha_0), \tag{2}$$

where $p(X_i; \cdot)$ is a given function with parameter α evaluated at α_0.

If complete reliance is placed on the outcome regression model (1), a natural estimator of μ_0 is the imputed estimator

$$\hat{\mu}_{RI} = \frac{1}{n} \sum_{i=1}^{n} \left\{ R_i Y_i + (1 - R_i) m(X_i; \hat{\beta}) \right\}, \tag{3}$$

where $\hat{\beta}$ is a consistent estimator of β_0 that may be obtained by solving the following estimating equations:

$$S_m(\beta) = \frac{1}{n}\sum_{i=1}^{n} \phi_i R_i \left\{Y_i - m(X_i;\beta)\right\} \frac{\partial m(X_i;\beta)}{\partial \beta} = 0, \tag{4}$$

where ϕ_i is a coefficient attached to unit i. Often, the coefficient ϕ_i is set to 1.

If complete reliance is placed on the propensity score model (2), a natural estimator of μ_0 is the inverse probability weighted estimator

$$\hat{\mu}_{IPW} = \frac{1}{n}\sum_{i=1}^{n} \frac{R_i}{p(X_i;\hat{\alpha})} Y_i, \tag{5}$$

where $\hat{\alpha}$ is a consistent estimator of α_0 that may be obtained by solving the following estimating equations:

$$S_p(\alpha) = \frac{1}{n}\sum_{i=1}^{n} \frac{R_i - p(X_i;\alpha)}{p(X_i;\alpha)\left\{1 - p(X_i;\alpha)\right\}} \frac{\partial p(X_i;\alpha)}{\partial \alpha} = 0. \tag{6}$$

Both (3) and (5) are based on a single working model. Estimators of the form

$$\hat{\mu}_{DR} = \frac{1}{n}\sum_{i=1}^{n} \frac{R_i}{p(X_i;\hat{\alpha})} Y_i + \frac{1}{n}\sum_{i=1}^{n} \left(1 - \frac{R_i}{p(X_i;\hat{\alpha})}\right) m(X_i;\hat{\beta}) \tag{7}$$

are doubly robust in the sense that they remain consistent for μ_0 if either (1) or (2) is correctly specified. Unlike (3) and (5), the estimator (7) makes use of two working models: an outcome regression model and a propensity score model. However, when both models are incorrectly specified, doubly robust estimators are inconsistent and tend to have poor numerical properties; see Han (2014b) and Chen and Haziza (2017).

Unlike doubly robust procedures, multiply robust procedures provide additional protection against model misspecification because they rely on multiple propensity score and/or multiple outcome regression models see, e.g., Han and Wang (2013) Han (2014, 2016a, 2016b), Cham and Yam (2014) and Chen and Haziza (2017). Let $\mathcal{C}_1 = \{p^j(X_i;\alpha^j); j = 1,\ldots,J\}$ be a class consisting of J propensity score models, where $p^j(\cdot;\alpha^j)$ is a known function associated with the jth propensity score model and let $\mathcal{C}_2 = \{m^k(x_i;\beta^k); k = 1,\ldots,K\}$ be a class consisting of K outcome regression working models, where $m^k(\cdot;\beta^k)$ is a known function associated with the kth outcome regression model. The corresponding estimators $\hat{\alpha}^j$ and $\hat{\beta}^k$ may be obtained by solving the following estimating equations:

$$S_p^j(\alpha^j) = \frac{1}{n}\sum_{i=1}^{n} s_p^j(X_i, R_i;\alpha^j) = 0, \quad j = 1,\cdots J, \tag{8}$$

and

$$S_m^k(\beta^k) = \frac{1}{n}\sum_{i=1}^{n} s_m^k(X_i, R_i;\beta^k) = 0, \quad k = 1,\cdots K, \tag{9}$$

respectively, where

$$s_p^j(X_i, R_i;\alpha^j) = \frac{R_i - p^j(X_i;\alpha^j)}{p^j(X_i;\alpha^j)\left\{1 - p^j(X_i;\alpha^j)\right\}}\frac{\partial p^j(X_i;\alpha^j)}{\partial \alpha^j}$$

and

$$s_m^k(X_i, R_i;\beta^k) = R_i\left\{Y_i - m^k(X_i;\beta^k)\right\}\frac{\partial m^k(X_i;\beta^k)}{\partial \beta}.$$

A multiply robust estimator of μ_0 is

$$\hat{\mu}_{MR} = \sum_{i=1}^{n} w_i R_i Y_i, \tag{10}$$

where w_i is a weight attached to unit i satisfying the calibration constraints

$$\sum_{i=1}^{n} w_i R_i = 1,$$

$$\sum_{i=1}^{n} w_i R_i L\left\{1/p^j(X_i;\widehat{\alpha}^j)\right\} = n^{-1}\sum_{i=1}^{n} L\left\{1/p^j(X_i;\widehat{\alpha}^j)\right\},$$

$$\sum_{i=1}^{n} w_i R_i m^k(X_i;\widehat{\beta}^k) = n^{-1}\sum_{i=1}^{n} m^k(X_i;\widehat{\beta}^k),$$

where the function $L(t)$ depends on the calibration method; see Chen and Haziza (2017) for discussion about the choice of the calibration method. The estimator (10) is multiply robust in the sense that it is consistent if all models in $\mathcal{C}_1$ or $\mathcal{C}_2$ but one are misspecified.

3 Proposed method

The multiply robust procedure described above relied on an optimization problem with $K + J + 1$ constraints. To reduce the computational burden, we adopt the approach of Duan and Yin (2017), who proposed to compress the working models through weighted averages. Let

$$\widehat{U}_{pi} = (p^1(X_i;\widehat{\alpha}^1),\ldots,p^J(X_i;\widehat{\alpha}^J))^\top, \quad \widehat{U}_{mi} = (m^1(X_i;\widehat{\beta}^1),\ldots,m^K(X_i;\widehat{\beta}^K))^\top.$$

To summarize the working models information, we regress R_i on $\widehat{U}_{pi}$ and Y_i on $\widehat{U}_{mi}$, which leads to the least square regression coefficients

$$\widehat{\eta}_p = \left(\sum_{i=1}^{n}\widehat{U}_{pi}\widehat{U}_{pi}^\top\right)^{-1}\left(\sum_{i=1}^{n}\widehat{U}_{pi}R_i\right) \tag{11}$$

and

$$\widehat{\eta}_m = \left(\sum_{i=1}^{n}R_i\widehat{U}_{mi}\widehat{U}_{mi}^\top\right)^{-1}\left(\sum_{i=1}^{n}R_i\widehat{U}_{mi}Y_i\right). \tag{12}$$

The prediction $\widehat{U}_{pi}^{\top}\widehat{\eta}_p$ converges to $p(X_i;\alpha_0)$ if one of the propensity score models is correctly specified, whereas the prediction $\widehat{U}_{mi}^{\top}\widehat{\eta}_m$ converges to $m(X_i;\beta_0)$ if one of the outcome regression models is correctly specified. Define

$$\widehat{p}_i = \widehat{U}_{pi} \times \frac{\widehat{\eta}_p^2}{\widehat{\eta}_p^{\top}\widehat{\eta}_p} \tag{13}$$

and

$$\widehat{m}_i = \widehat{U}_{mi} \times \frac{\widehat{\eta}_m^2}{\widehat{\eta}_m^{\top}\widehat{\eta}_m} \tag{14}$$

with a^2 denoting the column vector $(a_1^2,\ldots,a_q^2)^{\top}$ for a given vector $a = (a_1,\ldots,a_q)^{\top}$. The scores $\widehat{p}_i$ and $\widehat{m}_i$ are standardized version of $\widehat{U}_{pi}^{\top}\widehat{\eta}_p$ and $\widehat{U}_{mi}^{\top}\widehat{\eta}_m$ and, as such, enjoy the same statistical properties. The prediction $\widehat{p}_i$ compresses the J propensity score models into a single number, whereas the prediction $\widehat{m}_i$ compresses the K outcome regression models.

Let $Z_i = (Z_{1i}, Z_{2i}), i = 1,\ldots,n$, with $Z_{1i} = \widehat{m}_i$, $Z_{2i} = \widehat{p}_i$ and let $S_i = (S_{1i}, S_{2i})$ be the vector of standardized scores, where $S_{1i} = \widehat{\sigma}_m^{-1}(\widehat{m}_i - \bar{\widehat{m}}_n)$ and $S_{2i} = \widehat{\sigma}_p^{-1}(\widehat{p}_i - \bar{\widehat{p}}_n)$, with $\bar{\widehat{m}}_n$ and $\bar{\widehat{p}}_n$ denoting respectively the usual sample means of the $\widehat{p}_i$'s and $\widehat{m}_i$'s and $\widehat{\sigma}_m$ and $\widehat{\sigma}_p$ denoting the corresponding sample standard deviations. Our proposed multiply robust nonparametric multiple imputation (MRM) procedure can be implemented along these steps:

(Step1). Calculate $S_i, i = 1,\ldots,n$ from the original sample.

(Step2). To obtain the lth imputation, draw a bootstrap sample of size n with replacement from the original sample, fit the working models in $\mathcal{C}_1$ and $\mathcal{C}_2$ using the bootstrap sample, and calculate $S_i^{(l)}$, the version of S_i corresponding to the lth imputation.

(Step3). Calculate the distance between each missing subject $i \in s_m$ and every response subject $j \in s_r^{(l)}$ as

$$d^{(l)}(i,j) = \left\{ \lambda \left(S_{1i} - S_{1j}^{(l)} \right)^2 + (1-\lambda) \left(S_{2i} - S_{2j}^{(l)} \right)^2 \right\}^{1/2}, \qquad (15)$$

where $0 \leq \lambda \leq 1$ and $s_r^{(l)}$ denotes the set of respondents corresponding to the lth imputation. The value of λ reflects one's confidence about the different working models. A small value of λ places more weight on the outcome regression models, whereas a large value of λ places more weight on the propensity score models.

(Step4). Define the nearest-H neighborhood $R_H^{(l)}(i)$ for each missing unit $i \in s_m$ as H unit in $s_r^{(l)}$ who have the smallest H distances (d) from unit i.

(Step5). Randomly select one unit from $R_H^{(l)}(i)$ and use its value $Y_i^{*(l)}$ as the imputed value for unit i.

(Step6). Repeat (Step2) to (Step5) L times to obtain L multiply imputed data sets with $\tilde{Y}_i^{(l)} = \left\{ R_i Y_i + (1-R_i) Y_i^{*(l)} \right\}, i = 1, \ldots, n,\ l = 1, \ldots, L.$

A point estimator is obtained by pooling the L multiply imputed data sets, which leads to

$$\widehat{\mu}_{MRM} = \frac{1}{L} \sum_{l=1}^{L} \widehat{\mu}^{(l)}, \qquad (16)$$

where $\widehat{\mu}^{(l)} = n^{-1} \sum_{i=1}^{n} \tilde{Y}_i^{(l)}$. Using Rubin's rule, the variance of (16) is readily estimated by

$$\widehat{V}_{MRM} = \bar{U}_L + \left(1 + \frac{1}{L} \right) B_L, \qquad (17)$$

where $\bar{U}_L$ and B_L denote the within and between components given respectively by

$$\bar{U}_L = \frac{1}{L}\sum_{l=1}^{L} U^{(l)}, \tag{18}$$

and

$$B_L = \frac{1}{L-1}\sum_{l=1}^{L} \left(\widehat{\mu}^{(l)} - \widehat{\mu}_{MRM}\right)^2, \tag{19}$$

with $U^{(l)}$ denoting the sampling variance of $\widehat{\mu}_{MRM}$ based on the lth imputed data set.

An $100(1-\gamma)$-th confidence interval of μ_0 can be constructed as follows:

$$\left(\widehat{\mu}_{MRM} - z_{1-\gamma/2}\widehat{V}_{MRM}^{1/2}, \quad \widehat{\mu}_{MRM} + z_{1-\gamma/2}\widehat{V}_{MRM}^{1/2}\right),$$

where $z_{1-\gamma/2}$ is the $100(1-\gamma/2)$-th percentile from standard normal distribution.

Remark 3.1 *The size of the neighborhood H in Step 4 of the proposed procedure is assumed to satisfy $H/n = o(1)$ and $\log(n)/H = o(1)$; see Condition C4 in Appendix A. Long et al. (2012) evaluated empirically different choices of H for their doubly robust imputation procedure. They found that the bias of point estimators increased as H increased, while their standard errors decreased slightly. In their empirical experiments, the point estimators exhibited a slightly lower mean square errors with $H = 3$. In practice, choosing the optimal value of H is challenging. We suggest applying the proposed procedure with different values of H (say between 2 and 6) and then choosing the value of H corresponding to the smallest standard error computed through (17).*

4 Asymptotic properties

We show that the estimator (16) is multiply robust and establish its asymptotic normality. The regularity conditions are described in Appendix A.

Theorem 1 establishes the multiply robustness property of (16).

Theorem 1 *Assume that the regularity conditions (C1)-(C5) in Appendix A hold. If* $0 < \lambda < 1$ *and at least one of the outcome regression models or the propensity score models is correctly specified, then* $\widehat{\mu}_{MRM}$ *is consistent for* μ_0.

A sketch of the proof of Theorem 1 is presented in Appendix B.

Let $\theta = (\alpha^1, \ldots, \alpha^J, \beta^1, \ldots, \beta^K)^\top$, $\widehat{\theta} = (\widehat{\alpha}^1, \ldots, \widehat{\alpha}^J, \widehat{\beta}^1, \ldots, \widehat{\beta}^K)^\top$ and let $\theta^* = (\alpha^{*1}, \ldots, \alpha^{*J}, \beta^{*1}, \ldots, \beta^{*K})^\top$ denote the probability limit of $\widehat{\theta}$. Define

$$S_{mp}(\theta) = \left(S_p^1(\alpha^1), \ldots, S_p^J(\alpha^J), S_m^1(\beta^1), \ldots, S_m^K(\beta^K)\right)$$

and

$$s(X_i, R_i; \theta) = \left(s_p^1(X_i, R_i; \alpha^1), \ldots, s_p^J(X_i, R_i; \alpha^J), s_m^1(X_i, R_i; \beta^1), \ldots, s_m^K(X_i, R_i; \beta^K)\right).$$

Theorem 2 establishes the root-n consistency and asymptotic normality of (16).

Theorem 2 *Assume that the regularity conditions (C1)-(C5) in Appendix A hold. If* $0 < \lambda < 1$ *and at least one of the outcome regression models or the propensity score models is correctly specified, then*

$$\widehat{\mu}_{MRM} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{R_i}{\pi(Z_i^*)}Y_i + \left\{1 - \frac{R_i}{\pi(Z_i^*)}\right\}\mu(Z_i^*) + As(X_i, R_i; \theta^*)\right] + o_p(n^{-1/2}), \tag{20}$$

as $n \to \infty$ *and* $L \to \infty$, *where* Z_i^* *denotes* Z_i *evaluated at* θ^*, $\pi(Z_i^*) = E(R_i = 1|Z_i^*)$, $\mu(Z_i^*) = E(Y_i|Z_i^*)$ *and*

$$A = -E\left\{(1-R_i)\frac{\partial \mu(Z_i^*)}{\partial \theta}\right\}\left\{E\left(\frac{\partial S_{mp}(\theta^*)}{\partial \theta}\right)\right\}^{-1}.$$

Furthermore, we have

$$n^{1/2}(\widehat{\mu}_{MRM} - \mu_0) \to^d N(0, \sigma^2_{MRM}),$$

where

$$\sigma^2_{MRM} = V\left\{\frac{R_i}{\pi(Z_i^*)}Y_i + \left\{1 - \frac{R_i}{\pi(Z_i^*)}\right\}\mu(Z_i^*) + As(X_i, R_i; \theta^*)\right\}. \quad (21)$$

A sketch of the proof of Theorem 2 is presented in Appendix C.

Remark 4.1 *Alternatively, the estimator* $\widehat{\mu}_{MRM}$ *in Theorem 2 can be written as*

$$\widehat{\mu}_{MRM} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{R_i}{\pi(Z_i)}Y_i + \left\{1 - \frac{R_i}{\pi(Z_i)}\right\}\mu(Z_i)\right] + o_p(n^{-1/2}), \quad (22)$$

where $\pi(Z_i) = E(R_i = 1|Z_i)$ *and* $\mu(Z_i) = E(Y_i|Z_i)$. *If one of propensity score models and one of outcome regression models are correctly specified, then according to (22) it can be shown that*

$$\begin{aligned}
\widehat{\mu}_{MRM} &= \frac{1}{n}\sum_{i=1}^{n} m(X_i;\beta_0) + \frac{1}{n}\sum_{i=1}^{n}\frac{R_i}{p(X_i;\alpha_0)}\{Y_i - m(X_i;\beta_0)\} \\
&+ \frac{1}{n}\sum_{i=1}^{n} R_i\left\{\frac{1}{\pi(Z_i)} - \frac{1}{p(X_i;\alpha_0)}\right\}\{Y_i - m(X_i;\beta_0)\} \\
&+ \frac{1}{n}\sum_{i=1}^{n}\left\{1 - \frac{R_i}{\pi(Z_i)}\right\}\{\mu(Z_i) - m(X_i;\beta_0)\} \\
&= \frac{1}{n}\sum_{i=1}^{n}\left[\frac{R_i}{p(X_i;\alpha_0)}Y_i + \left\{1 - \frac{R_i}{p(X_i;\alpha_0)}\right\}m(X_i;\beta_0)\right] \\
&+ o_p(n^{-1/2}). \quad (23)
\end{aligned}$$

Therefore, $\widehat{\mu}_{MRM}$ achieves semi-parametric lower bound in this scenario.

Using Theorem 2, we can construct a normal approximation-based confidence interval of μ_0. However, estimating the variance of $\widehat{\mu}_{MRM}$ given by (21) is generally tedious and would require a non-parametric method such as kernel smoothing for estimating the density. To overcome this problem, we recommend using Rubin's formula for variance estimation. According to Long et al. (2012), the proposed imputation procedure is proper. The addition of a bootstrap step allows the estimation of the variance of estimators in other settings as well (Heitjan and Little, 1991, Rubin and Schenker, 1991). In our numerical experiments, $L = 5$ imputations led to good performances in finite samples; see also Rubin (1987).

5 Simulation study

We performed a simulation study to evaluate the performance of our proposed estimators. We used the simulation setup of Kang and Schafer (2007), also considered by Chan and Yam (2014), Han (2014a) and Chen and Haziza (2017). The Monte Carlo size was set to $B = 1,000$ and the sample size, to $n = 400$. A vector of covariates $X = (X_1, X_2, X_3, X_4)^\top$ was first generated from a standard multivariate normal distribution. Then, a study variable Y was generated according to the linear regression model $Y = 210 + 27.4X_1 + 13.7(X_2 + X_3 + X_4) + \epsilon$, where the errors ϵ were generated from a standard normal distribution. Given X and Y, the response indicators R were generated from a Bernoulli distribution with probability

$$p = \frac{\exp\left(\alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4\right)}{1 + \exp\left(\alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4\right)},$$

where $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (0, -1, 0.5, -0.25, -0.1)$. The corresponding response rate was about 50%.

To evaluate the performance of the proposed procedure when some models are misspecified, we considered the following transformations of the X-variables: $V_1 = X_2 \{1 + \exp(X_1)\}^{-1} + 10$ and $V_2 = (X_1 X_3/25 + 0.6)^3$; see also Kang and Schafer (2007). The correct models were fitted using $X = (X_1, X_2, X_3, X_4)$ as predictors in both the outcome regression and the propensity score models. The incorrect models were fitted by using $V = (V_1, V_2)$ as predictors in both models.

We were interested in estimating the population mean $\mu_0 = E(Y) = 210$. Because the proposed imputation procedure may be based on different combinations of the models, we use four digits between parentheses to distinguish between estimators constructed using different models. The first two digits correspond to the correct and incorrect propensity score models, respectively, whereas the last two digits correspond to the correct and incorrect outcome regression models, respectively. For example, the estimator $\widehat{\mu}(1010)$ is based on the correct nonresponse and outcome regression models, whereas $\widehat{\mu}(1111)$ denotes the estimator based on the four models.

We computed the following estimators:

1. The complete data estimator $\widehat{\mu}_{COM} = n^{-1} \sum_{i=1}^{n} Y_i$, which assumes no missing values.

2. The unadjusted estimator $\widehat{\mu}_{RES} = n_r^{-1} \sum_{i=1}^{n} R_i Y_i$ based on the complete cases, where n_r denotes the number of responding units.

3. The customary multiply imputed estimators (MI) based on a single outcome regression model: $\widehat{\mu}_{MI}(0010)$ and $\widehat{\mu}_{MI}(0001)$, where the two estimators were obtained under the correct and incorrect outcome regression models with normal errors. The imputations were performed using the R package 'mice'; see van Buuren (2014).

4. The doubly robust nonparametric multiple imputation estimator (DRM) of Long et al. (2012) based on a single propensity score model and a single outcome regression model: $\widehat{\mu}_{DRM}(1010)$, $\widehat{\mu}_{DRM}(1001)$, $\widehat{\mu}_{DRM}(0110)$ and $\widehat{\mu}_{DRM}(0101)$.

5. The proposed multiply robust nonparametric multiple imputation estimator (MRM) given by (16): $\widehat{\mu}_{MRM}(0111)$, $\widehat{\mu}_{MRM}(1011)$, $\widehat{\mu}_{MRM}(1101)$, $\widehat{\mu}_{MRM}(1110)$ and $\widehat{\mu}_{MRM}(1111)$.

For the DRM and the MRM estimators, we used $H = 3$ units in each neighborhood and $L = 5$ imputed values. We also evaluated the effect of λ in (15) by considering $\lambda = 0.2$, 0.5 and 0.8. For each estimator, we computed the Monte Carlo bias (Bias), standard error (SE) and root mean square error (RMSE). The results, presented in Tables 1-3, correspond to values of λ equal to $0.2, 0.5$ and 0.8, respectively.

From Table 1–Table 3, the complete data estimator, $\widehat{\mu}_{COM}$, performed the best in terms of RB, RSE and RRMSE, as expected. On the other hand, the unadjusted estimator, $\widehat{\mu}_{RES}$, was biased in all the scenarios, as expected. When either the propensity score model or the outcome regression model was correctly specified, the DRM estimators showed small values

of RB suggesting that they are doubly robust. However, when both models were misspecified, the estimator $\widehat{\mu}_{DRM}(0101)$ exhibited a large bias. For example, for $\lambda = 0.5$, the relative bias was approximately equal to -5.1%; see Table 2. Turning to the MRM estimators, they showed negligible biases in all the scenarios. The estimators $\widehat{\mu}_{DRM}(1111)$ based on the four models were almost as efficient as the estimator $\widehat{\mu}_{DRM}(1010)$ based on two correctly specified models. This suggests that incorporating an additional propensity score model and an additional outcome regression model does not seem to affect much the efficiency of the resulting estimator.

We also computed the coverage rate (CR) and relative average length (RAL) of confidence intervals based on DRM and MRM estimators for $\lambda = 0.5$; see Table 4. The results for $\lambda = 0.2$ and $\lambda = 0.8$ were very similar and are not presented here. According to Table 4, the coverage rate of the MRM estimators were close to the nominal rate 95% and were slightly better than those obtained for the DRM estimators. In most of the cases, the relative average length of the MRM estimators were smaller than those corresponding to the DRM estimators. When both models were incorrect, the DRM method showed a very low coverage rate with a value of 11.7%.

Table 1: Relative bias (RB), relative standard error (RSE) and relative root mean squared error (RRMSE) of several estimators with $H = 3$, $L = 5$ and $\lambda = 0.2$

Estimators	RB(%)	RSE(%)	RRMSE(%)
$\widehat{\mu}_{COM}$	0.000	0.871	0.871
$\widehat{\mu}_{RES}$	-4.761	1.220	4.915
$\widehat{\mu}_{MI}(0010)$	-0.002	0.871	0.871
$\widehat{\mu}_{MI}(0001)$	-5.771	1.178	5.890
$\widehat{\mu}_{DRM}(1010)$	-0.207	0.915	0.938
$\widehat{\mu}_{DRM}(1001)$	-0.370	1.093	1.154
$\widehat{\mu}_{DRM}(0110)$	-0.855	0.897	1.239
$\widehat{\mu}_{DRM}(0101)$	-5.096	1.172	5.229
$\widehat{\mu}_{MRM}(0111)$	-0.850	0.904	1.241
$\widehat{\mu}_{MRM}(1011)$	-0.195	0.924	0.944
$\widehat{\mu}_{MRM}(1101)$	-0.363	1.100	1.159
$\widehat{\mu}_{MRM}(1110)$	-0.194	0.924	0.944
$\widehat{\mu}_{MRM}(1111)$	-0.194	0.924	0.944

Appendix

A: Regularity conditions

Let $\widehat{Z}_i = (\widehat{Z}_{1i}, \widehat{Z}_{2i})$, where $\widehat{Z}_1 = \widehat{m}_i$ and $\widehat{Z}_2 = \widehat{p}_i$. Let β^{*K}, α^{*j}, η_m^* and η_p^* be the probability limits of $\widehat{\beta}^k$, $\widehat{\alpha}^j$, $\widehat{\eta}_m$ and $\widehat{\eta}_p$, respectively. Denote $Z_i^* = (Z_{1i}^*, Z_{2i}^*)$ with

$$Z_{1i}^* = \widehat{U}_{mi}^* \times \frac{\eta_m^{*2}}{\eta_p^{*\top}\eta_m^*}, \quad Z_{2i}^* = \widehat{U}_{pi}^* \times \frac{\eta_p^{*2}}{\eta_p^{*\top}\eta_p^*},$$

where

$$\widehat{U}_{mi}^* = \left(m^1(X_i;\beta^{*1}), \ldots, m^K(X_i;\beta^{*K})\right), \quad \widehat{U}_{pi}^* = \left(p^1(X_i;\alpha^{*1}), \ldots, p^J(X_i;\alpha^{*J})\right).$$

Finally, let $f(Z^*)$ be the density function of Z^*.

Table 2: Relative bias (RB), relative standard error (RSE) and relative root mean squared error (RRMSE) of several estimators with $H = 3$, $L = 5$ and $\lambda = 0.5$

Estimators	RB(%)	RSE(%)	RRMSE(%)
$\widehat{\mu}_{COM}$	0.000	0.871	0.871
$\widehat{\mu}_{RES}$	-4.761	1.220	4.915
$\widehat{\mu}_{MI}(0010)$	-0.002	0.871	0.871
$\widehat{\mu}_{MI}(0001)$	-5.781	1.175	5.899
$\widehat{\mu}_{DRM}(1010)$	-0.191	0.907	0.927
$\widehat{\mu}_{DRM}(1001)$	-0.662	1.089	1.274
$\widehat{\mu}_{DRM}(0110)$	-0.506	0.899	1.032
$\widehat{\mu}_{DRM}(0101)$	-5.165	1.205	5.303
$\widehat{\mu}_{MRM}(0111)$	-0.505	0.888	1.022
$\widehat{\mu}_{MRM}(1011)$	-0.182	0.897	0.915
$\widehat{\mu}_{MRM}(1101)$	-0.622	1.077	1.244
$\widehat{\mu}_{MRM}(1110)$	-0.182	0.895	0.913
$\widehat{\mu}_{MRM}(1111)$	-0.182	0.892	0.911

We assume the following regularity conditions needed to prove Theorem 1-Theorem 3. Conditions (C1) and (C2) apply to each propensity score model, $j = 1, \ldots, J$, and each outcome regression model, and $k = 1, \ldots, K$.

(C1). $\widehat{\alpha}^j$ is the unique solution of $S_p^j(\alpha^j) = 0$ and $\widehat{\beta}^k$ is the unique solution of $S_m^k(\beta^k) = 0$, where $S_p^j(\alpha^j)$ and $S_m^k(\beta^k)$ are defined in Section 3.

(C2). $S_p^j(\alpha^j)$ converges almost surely to $S_p^{*j}(\alpha^j) = E\left\{S_p^j(\alpha^j)\right\}$, uniformly in α^j and $S_p^{*j}(\alpha^j) = 0$ has a unique solution α^{*j}. Also, $S_m^k(\beta^k)$ converges almost surely to $S_m^{*k}(\beta^k) = E\left\{S_m^k(\beta^k)\right\}$, uniformly in β^k and $S_m^{*k}(\beta^k) = 0$ has a unique solution β^{*k}.

(C3). $E(Y^2) < \infty$ and $E\left\{\mu^2(Z^*)\right\} < \infty$, where $\mu(Z^*) = E(Y|Z^*)$.

Table 3: Relative bias (RB), relative standard error (RSE) and relative root mean squared error (RRMSE) of several estimators with $H = 3$, $L = 5$ and $\lambda = 0.8$

Estimators	RB(%)	RSE(%)	RRMSE(%)
$\widehat{\mu}_{COM}$	0.000	0.871	0.871
$\widehat{\mu}_{RES}$	-4.761	1.220	4.915
$\widehat{\mu}_{MI}(0010)$	-0.002	0.872	0.872
$\widehat{\mu}_{MI}(0001)$	-5.766	1.180	5.886
$\widehat{\mu}_{DRM}(1010)$	-0.180	0.898	0.916
$\widehat{\mu}_{DRM}(1001)$	-1.049	1.063	1.493
$\widehat{\mu}_{DRM}(0110)$	-0.311	0.897	0.949
$\widehat{\mu}_{DRM}(0101)$	-5.179	1.177	5.311
$\widehat{\mu}_{MRM}(0111)$	-0.305	0.888	0.939
$\widehat{\mu}_{MRM}(1011)$	-0.169	0.894	0.910
$\widehat{\mu}_{MRM}(1101)$	-1.040	1.085	1.503
$\widehat{\mu}_{MRM}(1110)$	-0.170	0.892	0.908
$\widehat{\mu}_{MRM}(1111)$	-0.171	0.892	0.909

Table 4: Coverage rate (CR) and relative average length (RAL) of confidence intervals with $H = 3$, $L = 5$ and $\lambda = 0.5$

Estimators	CR(%)	RAL(%)
$\widehat{\mu}_{MI}(0010)$	95.3	3.38
$\widehat{\mu}_{MI}(0001)$	0.2	4.52
$\widehat{\mu}_{DRM}(1010)$	95.7	3.80
$\widehat{\mu}_{DRM}(1001)$	91.6	4.76
$\widehat{\mu}_{DRM}(0110)$	92.4	3.77
$\widehat{\mu}_{DRM}(0101)$	1.7	4.74
$\widehat{\mu}_{MRM}(0111)$	92.9	3.76
$\widehat{\mu}_{MRM}(1011)$	95.5	3.79
$\widehat{\mu}_{MRM}(1101)$	92.7	4.83
$\widehat{\mu}_{MRM}(1110)$	95.4	3.79
$\widehat{\mu}_{MRM}(1111)$	95.8	3.79

(C4). $H/n = o(1)$ and $\log(n)/H = o(1)$.

(C5). $f(Z^*)$ and $\pi(Z^*)$ are continuous and bounded away from 0 in the compact support of Z^*.

Conditions (C1) and (C2) ensure the consistency of $\widehat{\alpha}^j$ and $\widehat{\beta}^k$. They are satisfied for most regression models such as linear and generalized linear models and some other estimating equations. Condition (C3) is used for deriving the asymptotic expansion of $\widehat{\mu}_{MRM}$ and its asymptotic normality. Condition (C4) is used to control the asymptotic order of H. Condition (C5) is used frequently in nonparametric statistics to avoid extreme values of the density and the propensity score.

B: Sketched proof of Theorem 1

Let $\widehat{\alpha} = (\widehat{\alpha}^1, \ldots, \widehat{\alpha}^J)$, $\widehat{\beta} = (\widehat{\beta}^1, \ldots, \widehat{\beta}^K)$, $\alpha^* = (\alpha^{*1}, \ldots, \alpha^{*J})$ and $\beta^* = (\beta^{*1}, \ldots, \beta^{*K})$. According to (C1), (C2) and Section 5.2 of van der Vaart (1998), it can be shown that $\widehat{\alpha} \to^p \alpha^*$ and $\widehat{\beta} \to^p \beta^*$, even when the working models are incorrect. Next, we show the multiply robustness property of $\widehat{\mu}_{MRM}$. If one of outcome regression models is correct, say $m^1(X_i; \beta^1)$, then we have $\beta^{*1} = \beta_0$ and $\eta_m^* = (1, 0, \ldots, 0)^\top$, which implies that $Z_{1i}^* = m(X_i; \beta_0)$. It follows that

$$\begin{aligned} E(Y|R, Z^*) &= E\{E(Y|R, Z^*, X)|R, Z^*\} \\ &= E\{E(Y|Z^*, X)|R, Z^*\} \\ &= E(Z_1^*|R, Z^*) = Z_1^* = E(Y|Z^*). \end{aligned} \tag{B.1}$$

If one of the propensity score models is correctly specified, say $p^1(X_i;\alpha^1)$, then $\alpha^{*1}=\alpha_0$, $\eta_p^*=(1,0,\ldots,0)^\top$ and $Z_{2i}^*=p(X_i;\alpha_0)$. Therefore,

$$E(Y|R,Z^*) \;=\; E(Y|Z^*), \tag{B.2}$$

due to the fact that Y is independent of R given Z_2^*. According to Devroye and Wagner (1977) and Silverman (1978), it can be shown that

$$\sum_{j\in R_H(i)}\frac{1}{H}Y_j \to^p E(Y_i|Z_i^*,R_i=1), \tag{B.3}$$

$$\frac{1}{H}\sum_{j\in R_H(i)}(Y_j-\bar{Y}_{R_H(i)})^2 \to^p V(Y_i|Z_i^*,R_i=1), \tag{B.4}$$

uniformly for any $i\in S$ as $n\to\infty$, $L\to\infty$ and $H\to\infty$ and conditions (C4) and (C5). In addition, according to Chebyshev's inequality, (B.3) and (B.4), we have

$$\begin{aligned}
&\Pr(|\ \frac{1}{n}\sum_{i=1}^n(1-R_i)\frac{1}{L}\sum_{l=1}^L Y_i^{*(l)}-E\left\{(1-R_i)E(Y_i^{*(l)}|Y,X,R)\right\}\ |>\epsilon)\\
\le\;& \epsilon^{-2}V\left\{\frac{1}{n}\sum_{i=1}^n(1-R_i)\frac{1}{L}\sum_{l=1}^L Y_i^{*(l)}\right\}\\
=\;& \epsilon^{-2}\left[V\left\{\frac{1}{n}\sum_{i=1}^n(1-R_i)E(Y_i^{*(l)}|Y,X,R)\right\}+E\left\{\frac{1}{n^2}\sum_{i=1}^n(1-R_i)\frac{1}{L}V(Y_i^{*(l)}|Y,X,R)\right\}\right]\\
=\;& \epsilon^{-2}\left[V\left\{\frac{1}{n}\sum_{i=1}^n(1-R_i)\sum_{j\in R_H(i)}\frac{1}{H}Y_j\right\}+E\left\{\frac{1}{n^2}\sum_{i=1}^n(1-R_i)\frac{1}{L}\frac{1}{H}\sum_{j\in R_H(i)}(Y_j-\bar{Y}_{R_H(i)})^2\right\}\right]\\
=\;& \epsilon^{-2}\left[V\left\{\frac{1}{n}\sum_{i=1}^n(1-R_i)E(Y_i|Z_i^*,R_i=1)\right\}+E\left\{\frac{1}{n^2}\sum_{i=1}^n(1-R_i)\frac{1}{L}V(Y_i|Z_i^*,R_i=1)\right\}\right]\\
+\;& o(n^{-1})+o(n^{-1}L^{-1})\\
=\;& O(n^{-1})+O(n^{-1}L^{-1}).
\end{aligned} \tag{B.5}$$

According to (B.5), we have

$$\frac{1}{n}\sum_{i=1}^{n}(1-R_i)\frac{1}{L}\sum_{l=1}^{L}Y_i^{*(l)} \to^p E\left\{(1-R_i)E(Y_i^{*(l)}|Y,X,R)\right\}, \tag{B.6}$$

as $n \to \infty$, $L \to \infty$ and $H \to \infty$. Therefore, if at least one the models is correctly specified and according to (B.3) and (B.6), we have

$$\begin{aligned}
\widehat{\mu}_{MRM} &= \frac{1}{n}\sum_{i=1}^{n}\left\{R_iY_i + (1-R_i)\frac{1}{L}\sum_{l=1}^{L}Y_i^{*(l)}\right\} \\
&\to^p E(R_iY_i) + E\left\{(1-R_i)E(Y_i^{*(l)}|Y,X,R)\right\} \\
&= E(R_iY_i) + E\left\{(1-R_i)E\left(\sum_{j\in R_H^{(l)}(i)}\frac{1}{H}Y_j^{*(l)}|X,Y,R\right)\right\} \\
&= E(R_iY_i) + E\left\{(1-R_i)\sum_{j\in R_H(i)}\frac{1}{H}Y_j\right\} \\
&\to E(R_iY_i) + E\left\{(1-R_i)E(Y_i|Z_i^*,R_i=1)\right\} \\
&= E(R_iY_i) + E\left\{(1-R_i)Y_i\right\} \\
&= E(Y)
\end{aligned} \tag{B.7}$$

as $n \to \infty$, $L \to \infty$ and $H \to \infty$ where we used the consistent probability weights argument defined in Stone (1977) in the derivations. Hence, Theorem 1 is proved.

C: Sketched proof of Theorem 2

The proposed estimator can be written as $\widehat{\mu}_{MRM} = \widehat{\mu} + \widehat{\mu}_{MRM} - \widehat{\mu}$, where

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\left\{R_iY_i + (1-R_i)\sum_{j\in R_H(i)}\frac{1}{H}Y_j\right\} \tag{C.1}$$

and

$$\widehat{\mu}_{MRM} - \widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n}(1-R_i)\left(\frac{1}{L}\sum_{l=1}^{L}Y_i^{*(l)} - \frac{1}{H}\sum_{j\in R_H(i)}Y_j\right). \tag{C.2}$$

By using a first-order Taylor expansion of $\widehat{\mu}$ at $\theta = \theta^*$, we have

$$\widehat{\mu} = \widehat{\mu}^* + E\left(\frac{\partial\widehat{\mu}^*}{\partial\theta}\right)\left(\widehat{\theta} - \theta^*\right) + o_p(n^{-1/2}), \tag{C.3}$$

where $\partial\widehat{\mu}^*/\partial\theta$ is $\partial\widehat{\mu}/\partial\theta$ evaluated at θ^*. Because $\widehat{\theta}$ is the solution of estimating equation $S_{mp}(\theta) = 0$, it can be shown that

$$\widehat{\theta} - \theta^* = -\left[E\left\{\frac{\partial S_{mp}(\theta^*)}{\partial\theta}\right\}\right]^{-1} S_{mp}(\theta^*) + o_p(n^{-1/2}). \tag{C.4}$$

Plugging (C.4) into (C.3) leads to

$$\widehat{\mu} = \widehat{\mu}^* + AS_{mp}(\theta^*), \tag{C.5}$$

where A is defined in Theorem 2 of Section 4. By using a similar argument to that used by Long et al. (2012), under the regularity conditions (C1)-(C5) in Appendix A, it can be shown that

$$\widehat{\mu}^* - \mu_0 = T_1 + T_2 + T_3 + o_p(n^{-1/2}), \tag{C.6}$$

with

$$T_1 = \frac{1}{n}\sum_{i=1}^{n}\left\{\mu(Z_i^*) - \mu_0\right\}, \quad T_2 = \frac{1}{n}\sum_{i=1}^{n}R_i\left\{Y_i - \mu(Z_i^*)\right\}$$

and

$$T_3 = \frac{1}{n}\sum_{i=1}^{n}\frac{1-\pi(Z_i^*)}{\pi(Z_i^*)}R_i\left\{Y_i - \mu(Z_i^*)\right\}.$$

Because $\widehat{\mu}_{MRM} - \widehat{\mu}$ is asymptotically independent of $\widehat{\mu}$, we have

$$\begin{aligned} V\left(\widehat{\mu}_{MRM} - \widehat{\mu}\right) &= V\left\{E\left(\widehat{\mu}_{MRM} - \widehat{\mu}|X, Y, R\right)\right\} + E\left\{V\left(\widehat{\mu}_{MRM} - \widehat{\mu}|X, Y, R\right)\right\} \\ &= E\left\{V\left(\widehat{\mu}_{MRM} - \widehat{\mu}|X, Y, R\right)\right\} \\ &= \frac{1}{nL} E\left\{\frac{1}{n}\sum_{i=1}^{n}(1 - R_i) V\left(Y_i^{*(l)}|Y, X, R\right)\right\} \\ &= \frac{1}{nL} E\left\{\frac{1}{n}\sum_{i=1}^{n}(1 - R_i)\frac{1}{H}\sum_{j \in R_H(i)}\left(Y_j - \bar{Y}_{R_H(i)}\right)^2\right\} \\ &= O\left(\frac{1}{nL}\right), \end{aligned} \tag{C.7}$$

where $\bar{Y}_{R_H(i)} = H^{-1}\sum_{j \in R_H(i)} Y_j$. According to the asymptotic independence between $\widehat{\mu}_{MRM} - \widehat{\mu}$ and $\widehat{\mu}$, T_1 and T_2, T_1 and T_3, (C.1),(C.2),(C.5)-(C.7), and regularity condition (C3), we have

$$\widehat{\mu}_{MRM} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{R_i}{\pi(Z_i^*)}Y_i + \left\{1 - \frac{R_i}{\pi(Z_i^*)}\right\}\mu(Z_i^*) + As(X_i, R_i; \theta^*)\right] + o_p(n^{-1/2}),$$

as $n \to \infty$ and $L \to \infty$. By the Central Limit Theorem, we have

$$n^{1/2}\left(\widehat{\mu}_{MRM} - \mu_0\right) \to^d N(0, \sigma^2_{MRM}),$$

with σ^2_{MRM} defined in Theorem 2 of Section 4.

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