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Title	Graph Estimation for Matrix-variate Gaussian Data
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This work greatly generalizes the method on FDR control for *i.i.d.* vector-variate GGMs in Liu (2013) and improves over optimization-based approaches. The main contribution and difference between our results and the existing ones for vector-variate GGMs (e.g., Liu (2013)) are summarized as follows,

1. We propose a novel *test statistic* in Section 3.1. By introducing a new construction of the initial regression coefficients (i.e., setting a particular element in each [illegible] Lasso estimator to zero), our testing approach no longer requires a complicated [illegible]-correlation step as in Eq. (6) in Liu (2013). Furthermore, the limiting null distribution of the sample covariance coefficient between residuals (see $\widehat{r}_{ij}$ in (3.[illegible])) [illegible] easily obtained. In fact, this idea can be used to provide simpler testing procedure for ordinary vector-variate high-dimensional graphical [illegible]

2. Instead of relying on the *i.i.d.* assumption in GGM literature, we propose to extract *np correlated* vector-variate "rows samples" (as well nq [illegible] column samples") from matrix-variate observations. By utilizing correlation structure among rows and columns, our approach allows for finite sample size, which is a very attractive property from both theoretical and practical perspectives. More specifically, even in the case that n is a constant and $p \to \infty$ and $q \to \infty$, our method still guarantees the asymptotic normality of the test statistics and FDR control. This is fundamentally different from the case of vector-variate GGMs, which always requires $n \to \infty$ for the support recovery. Therefore, this work greatly generalizes Liu (2013), which only deals with *i.i.d.* vector-variate Gaussian samples, to correlated data.

 In this paper, we developed new techniques and theoretical analysis to address the challenges arising from correlated samples. For example, the proposed *variance correlation technique* can be used as a general technique for high-dimensional inference [illegible] samples. Moreover, the initial estimator is now based on the Lasso with correlated samples. To this end, we establish the consistency result for the Lasso with correlated samples, which itself is of independent interest for high-dimensional linear regression.

3. Theoretically, by utilizing the Kronecker product structure of the covariance matrix of $\mathbf{X}$, the proposed method allows the partial correlation between X_{ij} and X_{kl} (i.e., $\varrho_{ij,kl}$ in (1.1)) to be of the order of $\frac{1}{n-1}\sqrt{\frac{\log p \log q}{pq}}$ so that the corresponding edge can be detected (please see the power analysis in Section 4.3 and Theorem 4 for details.)