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Complete List of Authors	Xin Wang Min Yang and Wei Zheng
Corresponding Author	Min Yang
E-mail	minyang.stat@gmail.com
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3 Complete Class of Designs

In this section, we try to find the complete class, that is a subclass of designs containing the optimal designs under various design criteria simultaneously. Meanwhile, the designs in the derived complete class have very few (mostly minimum) number of supporting points, which tremendously facilitates the numerical search of specific optimal designs. As compared to existing results on complete class, Model (2.5) imposes additional challenges here. There are two layers of approximate designs as represented by (2.4) and (2.6). Moreover, the information matrix in (2.4) does not possess the desirable additivity property as in most studies. We shall establish complete classes separately for the two layers.

3.1 Complete Class of Between-Group Designs

By (2.4), the within-group information matrix under a design, say $\boldsymbol{\xi}$, can be represented by

$$M(\boldsymbol{\xi}) = c_1 L(\boldsymbol{\xi}) - c_2 G(\boldsymbol{\xi}) G(\boldsymbol{\xi})^T \quad (3.1)$$

$$L(\boldsymbol{\xi}) = \int g(x) g(x)^T \boldsymbol{\xi}(dx) \quad (3.2)$$

$$G(\boldsymbol{\xi}) = \int g(x) \boldsymbol{\xi}(dx) \quad (3.3)$$

The concavity of $M(\boldsymbol{\xi})$ as shown by Lemma 3.1 is substantial for the proofs of two main results of the paper below, i.e. Theorem 3.2 and 4.2.

Lemma 3.1. *$M(\boldsymbol{\xi})$ is concave in $\boldsymbol{\xi}$ by Lowner's ordering.*

Proof. Since $L(\boldsymbol{\xi})$ is linear in $\boldsymbol{\xi}$, it is sufficient to show that $G(\boldsymbol{\xi})G(\boldsymbol{\xi})^T$ is convex in $\boldsymbol{\xi}$ in view of $c_2 > 0$. For a constant $0 < \alpha < 1$ and two measures $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, we have

$$\begin{aligned} & \alpha G(\boldsymbol{\xi}_1) G(\boldsymbol{\xi}_1)^T + (1-\alpha) G(\boldsymbol{\xi}_2) G(\boldsymbol{\xi}_2)^T - G(\alpha\boldsymbol{\xi}_1 + (1-\alpha)\boldsymbol{\xi}_2) G(\alpha\boldsymbol{\xi}_1 + (1-\alpha)\boldsymbol{\xi}_2)^T \\ = & \alpha(1-\alpha)[G(\boldsymbol{\xi}_1) - G(\boldsymbol{\xi}_2)][G(\boldsymbol{\xi}_1) - G(\boldsymbol{\xi}_2)]^T \geq 0 \end{aligned}$$

Hence, the proof is completed. □