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Title	Constrained Estimation of Causal Invertible VARMA
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$\det(\Phi(z))$ and $\det(\Theta(z))$ do not share any common factor. For what follows it will be convenient to characterize the causal process in terms of the associated monic polynomial $\tilde{\Phi}(z)$ defined as

$$\tilde{\Phi}(z) := z^p \Phi(z^{-1}) = I_m z^p - \Phi_1 z^{p-1} - \cdots - \Phi_p.$$

A VARMA process defined by (1) is causal if $\det(\tilde{\Phi}(z)) \neq 0$, for all $z \in \mathbb{C}$ such that $|z| \geq 1$, or equivalently the process in (1) is causal if all roots of $\det(\tilde{\Phi}(z)) = 0$ lie within the open unit disc $\mathcal{D} = \{z \in \mathbb{C} : |z| < 1\}$. Similarly, define

$$\tilde{\Theta}(z) := z^q \Theta(z^{-1}) = I_m z^q + \Theta_1 z^{q-1} + \cdots + \Theta_q.$$

Invertibility of the process is equivalent to the property that all roots of $\tilde{\Theta}(z)$ lie within $\mathcal{D}$.

We refer to $\tilde{\Phi}(z)$, $\tilde{\Theta}(z)$ and Σ as the parameters of the process defined by (1) and when it is clear we interchangeably refer to the associated coefficient matrices $\Phi = (\Phi_1, \ldots, \Phi_p)$, $\Theta = (\Theta_1, \ldots, \Theta_q)$ and Σ as the parameters as well. Before describing the parameter space of a causal invertible VARMA process, we introduce further notations. Let $\geq_L$ denote the Loewner partial ordering for symmetric matrices. Define

$$\mathscr{S}^m_{++} = \{\Sigma \in \mathscr{S}^m : \Sigma >_L 0\} \tag{4}$$

to be the set of all $m \times m$ symmetric positive definite matrices that constitute the interior of the convex cone, $\mathscr{S}^m_{+}$, of $m \times m$ positive semi-definite matrices in $\mathscr{S}^m$, the set of $m \times m$ symmetric matrices. A matrix monic polynomial $A(z) = z^k I_m - A_1 z^{k-1} - \cdots - A_k$, will be called *Schur-stable* if all roots of $\det(A(z)) = 0$ lie within the unit disc $\mathcal{D}$. Such polynomials are common in the dynamical systems literature (Bhatia 1997; Kaszkurewicz and Bhaya 2000). Let

$$\begin{aligned} \mathfrak{S}^{m,k} \;\; = \;\; & \{A(z) = z^k I_m - A_1 z^{k-1} - \cdots - A_k : A_r \in \mathbb{R}^{m \times m}, \\ & r \geq 1, \text{and } A(z) \text{ is Schur-stable}\} \end{aligned} \tag{5}$$

define the set of all $m-$dimensional Schur-stable matrix monic polynomials of degree k. Let any polynomial $A(z) = z^k I_m - A_1 z^{k-1} - \cdots - A_k$ be associated with the coefficient sequence $A = [A_1, \ldots, A_k]$. We will define a sequence of matrices $A = [A_1, \ldots, A_k]$ to be Schur-stable provided the associated polynomial is Schur-stable. Then, the parameters $(\Phi, -\Theta, \Sigma)$ of an m-dimensional