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<b>Title</b>	Discrete Choice Models for Nonmonotone Nonignorable Missing Data: Identification and Inference
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<b>Complete List of Authors</b>	Eric J. Tchetgen Tchetgen Linbo Wang and BaoLuo Sun
<b>Corresponding Author</b>	Eric J. Tchetgen Tchetgen
<b>E-mail</b>	etchetgen@gmail.com
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CDF of  $(R, L)$ .

Lemma 1 clarifies what the identification task entails, because under assumption (1),  $f(L|R=1)$  is just-identified, and therefore  $f(R, L)$  is nonparametrically just-identified only if one can just-identify  $\text{Odds}_r(L)$  for all  $r$ . Below we describe a sufficient condition for identification under the discrete choice model of the nonresponse process.

### 3 Identification

#### 3.1 The discrete choice nonresponse model

The DCM associates with each realized nonresponse pattern  $l$  an underlying utility function  $U_r = \mu_r(L) + \varepsilon_r$ , where  $\{\varepsilon_r : r\}$  are i.i.d. with cumulative distribution function  $F_\varepsilon$ , and  $\mu_r(L)$  encodes the dependence of a person's utility on  $L$  (McFadden and Train, 2009). Some common choices of  $F_\varepsilon$  include the extreme value distribution (further discussed below) and the normal distribution, although in principle any CDF could be specified. It is then assumed that a person's observed response pattern maximizes her utility, that is  $R = \arg \max_r \{U_r : r\}$ . Together, these assumptions imply that for each

$$\Pi_r = \pi_r(l) = \pi_r(L=l, R=r) = \int \prod_{s \neq r} F_\varepsilon(\Delta\mu_{rs}(L) + \varepsilon) dF_\varepsilon(\varepsilon), \quad (2)$$

where  $\Delta\mu_{rs}(L) = \mu_r(L) - \mu_s(L)$  captures the dependence on  $L$  of a difference in utility in comparing a person's choice between nonresponse patterns  $r$  and  $s$ , see Train (2009). The integral in (2) is generally not available in closed form for most choices of  $F_\varepsilon$  (with the notable exception of the extreme value distribution, see Section 2.2), but can easily be evaluated by numerical integration using say, Gaussian quadrature. Two interesting observations about equation (2) are worth noting. Although not immediately apparent from the expression in the display, equation (2) gives rise to a proper probability mass function, that is  $\sum_r \pi_r(l) = 1$  for all values of  $l$  and for any choice of  $F_\varepsilon$ . This remarkable result is a direct consequence of utility maximization as a formal principle for generating multinomial probabilities  $\{\pi_r : r\}$ . A second interesting observation is that only differ-













































