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Title	Comparison of Extended Empirical Likelihood Methods: Size and Shape of Test Based Confidence Regions
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respectively:

$$ELR_{1n}(\theta) = EL_{1n}(\theta)/\prod_{i=1}^{n} n^{-1}, \quad ELR_{2n}(\theta) = EL_{2n}(\theta)/\prod_{i=1}^{n} n^{-1}. \qquad (3.1)$$

The following theorem shows that results in (2.1) hold for the profile and the plug-in method.

Theorem 2. *Assume conditions under which the bel[illegible]s hold:*

$$n^{1/2}\begin{pmatrix}\hat{\theta}_n - \theta_0\\ \hat{\psi}_n - \psi_0\end{pmatrix} \longrightarrow N_p(0, \Sigma\text{[illegible]}$$

in distribution, where $\Sigma = \{S_{12}S_{11}^{-1}S_{21}\}^{-1}$ *with* $S_{12} = S_{21}^{\top} = E\begin{pmatrix}\partial m/\partial\theta & \text{[illegible]}\\ \partial h/\partial\theta & \text{[illegible]}\end{pmatrix}$,

$S_{11} = E\begin{pmatrix}mm^{\top} & mh^{\top}\\ hm^{\top} & hh^{\top}\end{pmatrix}$ *, and the expectations taken at* $\theta = \theta_0$ *and* $\psi = \psi_0$*. Also assume that for any* $\tilde{\psi}$ *with* $||\tilde{\psi} - \psi_0|| = O_p(n^{-1/2})$, [illegible] $m(Z_i, \theta_0, \tilde{\psi})m^{\top}(Z_i, \theta_0, \tilde{\psi})$ $\to E(mm^{\top})$*. Suppose* $\Sigma_{(jk)}$*,* $j, k = 1, 2$*, denotes th[illegible]s of* Σ *that correspond to* θ *and* ψ*. Then, the results in [illegible]1) ho[illegible] both* $-2\log ELR_{1n}(\theta_0)$ *and* $-2\log ELR_{2n}(\text{[illegible]})$ *with*

$$U \sim N_p(0, \Sigma_{(11)}), \; W = V_1 = \Sigma_{(1\text{[illegible]}} \text{[illegible]} \left[E(\text{[illegible]}/\partial\theta)^{\top}\{E(mm^{\top})\}^{-1}E(\partial m/\partial\theta)\right]^{-1}.$$

Specifically, $-2\log E$[illegible]$_{2n}(\theta$[illegible] $c_j\xi_j$ *in distribution, where* ξ_j *are independent chi-squared dis[illegible]ed [illegible]nd[illegible] variables with degree of freedom 1 and* [illegible]$, c_p$ *are eigen[illegible]es o[illegible] W*[illegible]

The wei[illegible]te[illegible] χ[illegible] as[illegible]tic results stems from the ill-construction of the likelihood: the [illegible]elih[illegible] of $EL_{2n}(\theta)$ takes the same form as that of Owen's likelihood [illegible] of independent data, whereas $m(Z_i, \theta, \hat{\psi}_n)$, $i = 1, \ldots, n$, are not independent [illegible]ymore as they all involve the same $\hat{\psi}_n$.

3.2. C[illegible]ta Analysis

We suppose that $\{Z_i\}_{i=1}^n = \{(T_i, \delta_i)\}_{i=1}^n$ where (T_i, δ_i) denote right censored observations from lifetime variables $\{Y_i\}$ with a common cumulative distribution function F_0 subject to random censoring as follows:

$$T_i = \min(Y_i, C_i) \quad \text{and} \quad \delta_i = I_{[Y_i \le C_i]} \quad \text{for} \;\; i = 1, \ldots, n.$$