KOO APPROACH FOR SCALABLE VARIABLE SELECTION PROBLEM IN LARGE-DIMENSIONAL REGRESSION

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Supplementary Material

This supplementary material includes additional simulation studies, additional real data analysis and proofs of the main theorems for general error distributions using random matrix theory.

S1 Additional simulation results

The simulation results for Settings I and II, and six cases of distribution of ${\bf E}$ are tabulated in Tables 1–6.

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				$\alpha =$	0.2, <i>c</i> =	= 0.2					
		1	n = 100				1	n = 500			
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	
U-S	0	79	0	15	0	0	589	0	0	0	
T-S	198	921	228	983	966	570	411	655	999	954	
O-S	802	0	772	2	34	430	0	345	1	46	
A-S	2.05	-	1.97	1	1	1.37	-	1.27	1	1.04	
		r_{i}	n = 1000				r	n = 2000			
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^{C}_{*}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	$\hat{\mathbf{j}}^A_*$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{j}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$	
U-S	0	993	0	0	0	0	1000	0	0	0	
T-S	956	7	972	1000	958	1000	0	1000	999	951	
O-S	44	0	28	0	42	0	0	0	1	49	
A-S	1.02	-	1	-	1.02	-	-	-	1	1.04	
				$\alpha =$	0.2, c =	= 0.4					
		1	n = 100				1	n = 500			
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	
U-S	0	938	0	640	19	0	1000	0	0	0	
T-S	35	62	0	360	940	2	0	0	1000	953	
O-S	965	0	1000	0	41	998	0	1000	0	47	
A-S	3.69	_	7.06	-	1.05	6.86	_	46.30	_	1.04	
		r_{i}	n = 1000				r	n = 2000			
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(5)}$					
U-S	0	1000	0	0	0	0	1000	0	0	0	
T-S	23	0	0	998	957	505	0	0	1000	961	
O-S	977	0	1000	2	43	495	0	1000	0	39	
ΔS	3 0 2	_	95.28	1	1.05	1.47	_	194.82	_	1	
-D	0.34		00.20	-	1.00	1.11		101101		- 1	
<u></u>	5.52		00.20	$\alpha =$	0.4, c =	= 0.2		101102			
M -0	0.02	1	n = 100	$\alpha =$	0.4, <i>c</i> =	= 0.2	1	n = 500			
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\frac{n = 100}{\hat{\mathbf{j}}_{*}^{C}}$	$\hat{\boldsymbol{\alpha}} =$ $\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{0.4}, c =$ $\hat{\mathbf{j}}_{*}^{(5)}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\frac{n = 500}{\hat{\mathbf{j}}_{*}^{C}}$	$\hat{\mathbf{j}}_{*}^{(0)}$		
U-S	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}_{*}^{B}$ 42	$n = 100$ $\hat{\mathbf{j}}_{*}^{C}$ 0	$\frac{\alpha}{\hat{\mathbf{j}}_{*}^{(0)}}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0	$\hat{\mathbf{j}}^B_*$ 129	$\frac{n = 500}{\hat{\mathbf{j}}_{*}^{C}}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	
U-S T-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 $	\hat{j}_{*}^{B} 42 923	$n = 100$ $\hat{\mathbf{j}}_{*}^{C}$ 0 3	$\frac{\alpha}{\hat{\mathbf{j}}_{*}^{(0)}}$ $\frac{\hat{\mathbf{j}}_{*}^{(0)}}{828}$ 172	0.4, c = $\hat{j}_{*}^{(5)}$ 41 919	$ \hat{\mathbf{j}}_{*}^{A} $	\hat{j}_{*}^{B} 129 871	$n = 500$ $\hat{\mathbf{j}}_{*}^{C}$ 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998	$\hat{j}_{*}^{(5)}$ 0 965	
U-S T-S O-S	\hat{j}_{*}^{A} 0 0 1000	\hat{j}_{*}^{B} 42 923 35	$n = 100$ $\hat{\mathbf{j}}_{*}^{C}$ 0 3 997	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 828 172 0	$ \begin{array}{r} 100 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ \end{array} $		\hat{j}_{*}^{B} 129 871 0	$n = 500$ $\hat{\mathbf{j}}_{*}^{C}$ 0 1000	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2	$\hat{j}_{*}^{(5)}$ 0 965 35	
U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 16.50	$\hat{\mathbf{j}}_{*}^{B}$ 42 923 35 1.09		$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 828 172 0 -	$ \begin{array}{r} \hline 100 \\ \hline 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ 1.12 \end{array} $		$\hat{\mathbf{j}}_{*}^{B}$ 129 871 0 -		$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2 1	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1	
U-S T-S O-S A-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 16.50 \end{array} $	\hat{j}_{*}^{B} 42 923 35 1.09 n	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 828 172 0 -	$ \begin{array}{r} 1.00\\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)}\\ 41\\ 919\\ 40\\ 1.12 \end{array} $	$ \begin{array}{r} \hline 1.41 \\ = 0.2 \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 100.87 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - 7	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 n = 2000	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2 1	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1	
U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$	\hat{j}_{*}^{B} 42 923 35 1.09 n \hat{j}_{*}^{B}	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C}	$ \begin{array}{r} 1 \\ \alpha = \\ \hline \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hline \hat{\mathbf{j}}_{*}^{(0)} \\ \end{array} $	$ \begin{array}{r} 1.00 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ 1.12 \\ \hat{\mathbf{j}}_{*}^{(5)} \end{array} $		\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B}	$n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 8 $n = 2000$ $\hat{\mathbf{j}}_{c}^{C}$	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2 1 $\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$	
U-S T-S O-S A-S U-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0	\hat{j}_{*}^{B} 42 923 35 1.09 \hat{j}_{*}^{B} 729	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0	$ \begin{array}{c} $	$ \frac{\hat{\mathbf{j}}_{*}^{(5)}}{\hat{\mathbf{j}}_{*}^{(5)}} \\ \frac{\hat{\mathbf{j}}_{*}^{(5)}}{41} \\ \frac{1}{919} \\ \frac{40}{1.12} \\ \frac{\hat{\mathbf{j}}_{*}^{(5)}}{\hat{\mathbf{j}}_{*}} \\ 0 $	$ \begin{array}{c} \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 1000 \\ 100.87 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999	$ \begin{array}{r} n = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 8 \\ n = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \end{array} $		$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0	
U-S T-S O-S A-S U-S T-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 0	$ \begin{array}{r} \hat{j}_{*}^{3} \\ 42 \\ 923 \\ 35 \\ 1.09 \\ \hline \pi \\ \hat{j}_{*}^{8} \\ 729 \\ 271 \\ \end{array} $	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41	$\begin{array}{c} \mathbf{a} = \\ \mathbf{\hat{j}}_{*}^{(0)} \\ \mathbf{\hat{s}}_{*} \\ 828 \\ 172 \\ 0 \\ - \\ \mathbf{\hat{j}}_{*}^{(0)} \\ \mathbf{\hat{j}}_{*} \\ 0 \\ 1000 \end{array}$	$ \begin{array}{r} \hline \mathbf{j}_{*}^{(5)} \\ \hline \mathbf{j}_{*}^{(5)} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \mathbf{j}_{*}^{(5)} \\ \hline \mathbf{j}_{*}^{(5)} \\ \hline 0 \\ 954 \\ \end{array} $	$ \begin{array}{c} \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 1000 \\ 100.87 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $n = 2000$ \hat{j}_{*}^{C} 0 748	$ \begin{array}{r} \hat{j}_{*}^{(0)} \\ 0 \\ 998 \\ 2 \\ 1 \\ $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940	
U-S T-S O-S A-S U-S T-S O-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{B} \\ \hat{\mathbf{j}}_{*}^{B} \\ 42 \\ 923 \\ 35 \\ 1.09 \\ \hat{\mathbf{j}}_{*}^{B} \\ 729 \\ 271 \\ 0 \\ \end{array} $	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959	$ \begin{array}{c} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \end{array} $	$ \begin{array}{r} \hline \mathbf{j}_{*}^{(5)} \\ \hline \mathbf{j}_{*}^{(5)} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \mathbf{j}_{*}^{(5)} \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ \hline 46 \\ \end{array} $	$ \begin{array}{r} \hline \\ \\ \\ $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $n = 2000$ \hat{j}_{*}^{C} 0 748 252		$\begin{array}{c} \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 965 \\ 35 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 940 \\ 60 \end{array}$	
U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52	\hat{j}_{*}^{B} 42 923 35 1.09 \hat{j}_{*}^{B} 729 271 0 $-$	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28	$\begin{array}{c} \alpha = \\ \hat{\mathbf{j}}^{(0)}_{*} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}^{(0)}_{*} \\ 0 \\ 1000 \\ 0 \\ - \end{array}$	$\begin{array}{c} 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \end{array}$	$ \begin{array}{c} \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 1000 \\ 100.87 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 450.55 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0 -	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $n = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16		$\begin{array}{c} \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 965 \\ 35 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 940 \\ 60 \\ 1 \\ \end{array}$	
U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52	\hat{j}_{*}^{B} 42 923 35 1.09 π \hat{j}_{*}^{B} 729 2711 0 $-$	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28	$ \begin{array}{r} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hline 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ $	$\begin{array}{c} 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline 0.4, c = \end{array}$	$ \begin{array}{c} \mathbf{j}_{*}^{A} \\ \mathbf{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 100.87 \\ \hline \mathbf{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 450.55 \\ = 0.4 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $n = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16	$ \hat{\mathbf{j}}_{*}^{(0)} \\ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{998} \\ 2 \\ 1 \\ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{995} \\ 5 \\ 1 \\ $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1	
U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52	\hat{j}_{*}^{B} 42 923 35 1.09 π \hat{j}_{*}^{B} 729 271 0 $-$	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$		$ \begin{array}{r} \hline 1.00 \\ \hline 0.4, c = \\ \hline \mathbf{\hat{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \mathbf{\hat{j}}_{*}^{(5)} \\ \hline \mathbf{\hat{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline 0.4, c = \\ \end{array} $	$ \begin{array}{r} 1.41 \\ = 0.2 \\ \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 100.87 \\ \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 450.55 \\ = 0.4 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $n = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$	$ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 998 \\ 2 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 995 \\ 5 \\ 1 $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1	
U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52	$ \begin{array}{r} \hat{j}_{*}^{B} \\ 42 \\ 923 \\ 35 \\ 1.09 \\ \hat{j}_{*}^{B} \\ 729 \\ 2711 \\ 0 \\ - \\ \hat{j}_{*}^{B} \\ \hat{j}$	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C}	$ \begin{array}{c} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \end{array} $	$ \begin{array}{r} \hline \mathbf{\hat{j}}_{*}^{(5)} \\ \hline 0 \\ \hline 0.4, c = \\ \hline \mathbf{\hat{j}}_{*}^{(5)} \end{array} $	$ \begin{array}{r} 1.41 \\ = 0.2 \\ \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 1000 \\ 1000 \\ 450.55 \\ = 0.4 \\ \hat{j}_{*}^{A} \\ \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 450.55 \\ = 0.4 \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0 - \hat{j}_{*}^{B} \hat{j}_{*}^{B} \hat{j}_{*}^{B}	$ \begin{array}{r} n = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 1000 \\ 8 \\ n = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 748 \\ 252 \\ 1.16 \\ n = 500 \\ \hat{\mathbf{j}}_{*}^{C} \end{array} $	$ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{\hat{\mathbf{j}}_{*}} = \frac{\hat{\mathbf{j}}_{*}^{(0)}}{2} $ $ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{\hat{\mathbf{j}}_{*}} = \frac{\hat{\mathbf{j}}_{*}^{(0)}}{2} $ $ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{\hat{\mathbf{j}}_{*}} = \frac{\hat{\mathbf{j}}_{*}^{(0)}}{2} $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$	
U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52 $\hat{\mathbf{j}}_{*}^{A}$ 0	$ \begin{array}{r} \hat{j}_{*}^{B} \\ 42 \\ 923 \\ 35 \\ 1.09 \\ \hat{j}_{*}^{B} \\ 729 \\ 271 \\ 0 \\ - \\ \hat{j}_{*}^{B} \\ 623 \\ \end{array} $	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0	$ \begin{array}{r} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*} \\ 828 \\ 172 \\ 0 \\ - \\ $	$\begin{array}{c} \hline 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline 0.4, c = \\ \hline \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 889 \\ \end{array}$	$ \begin{array}{r} 1.41 \\ = 0.2 \\ \hat{j}_{*}^{A} \\ 0 \\ 0 \\ $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0 - \hat{j}_{*}^{B} 999 1 0 - \hat{j}_{*}^{B} 1000	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $a = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0	$ \hat{\mathbf{j}}_{*}^{(0)} \\ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{998} \\ 2 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 995 \\ 5 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 1 \\ 1 \\ 10 $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 0 0 1 0 1 0 0 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 0 0 0 0 0 0	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{B} \\ 42 \\ 923 \\ 35 \\ 1.09 \\ \hline \\ \hat{\mathbf{j}}_{*}^{B} \\ 729 \\ 271 \\ 0 \\ - \\ 0 \\ \hline \\ \hat{\mathbf{j}}_{*}^{B} \\ 623 \\ 294 \\ \end{array} $	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0 0	$ \begin{array}{r} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*} \\ 828 \\ 172 \\ 0 \\ - \\ $	$\begin{array}{c} \hline 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline 0.4, c = \\ \hline \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 889 \\ 103 \\ \end{array}$	$ \begin{array}{r} \hline \\ \\ \\ $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 9909 1 0 - \hat{j}_{*}^{B} 1000 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $n = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0 0	$ \hat{\mathbf{j}}_{*}^{(0)} \\ \frac{\hat{\mathbf{j}}_{*}^{(0)}}{998} \\ 2 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 9995 \\ 5 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 1 \\ 0 \\ 990 \\ 0 \\ 990 \\ $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1	
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U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52	\hat{j}_{*}^{B} 42 923 35 1.09 π \hat{j}_{*}^{B} 729 271 0 $ \hat{j}_{*}^{R}$ 623 294 83 1.51	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0 0 1000 29.35	$\begin{array}{c} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 999 \\ 1 \\ 0 \\ - \\ \end{array}$	$\begin{array}{c} \hline 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 889 \\ 103 \\ 8 \\ 1.25 \\ \end{array}$	$ \begin{array}{r} 1.41 \\ \hline 1.41 \\ \hline 0.2 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 100.87 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 450.55 \\ = 0.4 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.59 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 9999 1 0 \hat{j}_{*}^{B} 1000 0 0 -	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $a = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0 0 1000 193.17	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 0 995 5 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 1 0 990 0 -	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05	
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52	\hat{j}_{*}^{B} 42 923 35 1.09 π \hat{j}_{*}^{B} 729 271 0 $ \hat{j}_{*}^{R}$ 623 294 83 1.51 π	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0 1000 29.35 $n = 1000$	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{*}}$ 828 172 0 $\hat{j}_{*}^{(0)}$ 0 1000 0 $\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0	$\begin{array}{c} \hline 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 889 \\ 103 \\ 8 \\ 1.25 \\ \hline \end{array}$	$ \begin{array}{r} 1.41 \\ \hline 1.41 \\ \hline 0.2 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 100.87 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 450.55 \\ = 0.4 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.59 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 9999 1 0 \hat{j}_{*}^{B} 1000 0 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $a = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0 0 1000 193.17 $a = 2000$	$\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05	
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U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 16.50 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 213.52 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 31.05 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 31.05	\hat{j}_{*}^{B} 42 923 35 1.09 π \hat{j}_{*}^{B} 729 271 0 $ \hat{j}_{*}^{B}$ 623 294 83 1.51 π \hat{j}_{*}^{B} 1000	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0 1000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 0 10000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 0 10000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 0 0	$\begin{array}{c} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 999 \\ 1 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 0 \\ 0 \\ - \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	$\begin{array}{c} 1.00 \\ \hline 0.4, c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 889 \\ 103 \\ 8 \\ 1.25 \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ \hline \\ \end{array}$		\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 1000 0 0 - \hat{j}_{*}^{B} 1000 0 0 \hat{j}_{*}^{B}	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $a = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0 1000 193.17 $a = 2000$ \hat{j}_{*}^{C} 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 0 995 5 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 5 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 0 0 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 0 0 0 0 0 0	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05 $\hat{\mathbf{j}}_{*}^{(5)}$ 0	
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S	$\begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 16.50 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 213.52 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 31.05 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c} \hat{\mathbf{j}}_{*}^{B} \\ 42 \\ 923 \\ 35 \\ 1.09 \\ \hline \\ \hat{\mathbf{j}}_{*}^{B} \\ 729 \\ 271 \\ 0 \\ \hline \\ 0 \\ \hline \\ \hat{\mathbf{j}}_{*}^{B} \\ 623 \\ 294 \\ 83 \\ 1.51 \\ \hline \\ \hat{\mathbf{j}}_{*}^{B} \\ 1000 \\ 0 \\ \end{array}$	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0 1000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 0 10000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 0 0 0	$\begin{array}{c} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 999 \\ 1 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 998 \end{array}$	$\begin{array}{c} \hline \mathbf{0.4, c} = \\ \hline \mathbf{0.4, c} = \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \mathbf{0.4, c} = \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 889 \\ 103 \\ 8 \\ 1.25 \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 0 \\ 952 \\ \end{array}$		\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 9999 1 0 - \hat{j}_{*}^{B} 1000 0 - \hat{j}_{*}^{B} 1000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $a = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0 1000 193.17 $a = 2000$ \hat{j}_{*}^{C} 0 0 1000 193.17 $a = 2000$ \hat{j}_{*}^{C} 0 0 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ 0 998 2 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 0 995 5 1 $\hat{\mathbf{j}}_{*}^{(0)}$ 10 990 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 - 0 0 - 0 0 0 - 0 0 0 0 0 0 0 0	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 930	
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 16.50 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 213.52 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 31.05 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 31.05 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ \end{array}$	$\begin{array}{c} & & \\ \hat{\mathbf{j}}_{*}^{B} & \\ 42 \\ 923 \\ 35 \\ 1.09 \\ & \\ \hat{\mathbf{j}}_{*}^{B} \\ 729 \\ 271 \\ 0 \\ - \\ 271 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{B} \\ 623 \\ 294 \\ 83 \\ 1.51 \\ & \\ \hat{\mathbf{j}}_{*}^{B} \\ 1000 \\ 0 \\ 0 \\ 0 \\ \end{array}$	$n = 100$ \hat{j}_{*}^{C} 0 3 997 6.67 $n = 1000$ \hat{j}_{*}^{C} 0 41 959 3.28 $n = 100$ \hat{j}_{*}^{C} 0 1000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 1000 29.35 $n = 1000$ \hat{j}_{*}^{C} 0 0 1000 200 3	$\begin{array}{c} \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 828 \\ 172 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \alpha = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 999 \\ 1 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 999 \\ 2 \\ \end{array}$	$\begin{array}{c} \hline \mathbf{0.4, c} = \\ \hline \mathbf{0.4, c} = \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ \hline 41 \\ 919 \\ 40 \\ 1.12 \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \mathbf{0.4, c} = \\ \hline \mathbf{0.4, c} = \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 889 \\ 103 \\ 8 \\ 1.25 \\ \hline \mathbf{\hat{j}_{*}^{(5)}} \\ 0 \\ 952 \\ 48 \\ \end{array}$	$ \begin{array}{c} \mathbf{\hat{j}}_{*}^{A} \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 10000 \\ 10000 \\ 10000 \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10000 \\ 450.55 \\ \mathbf{=} 0.4 \\ \hline \\ \begin{array}{c} \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 10000 \\ 194.59 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 10000 \\ 10000 \\ 10000 \\ \end{array} $	\hat{j}_{*}^{B} 129 871 0 - \hat{j}_{*}^{B} 999 1 0 - \hat{j}_{*}^{B} 999 1 0 - \hat{j}_{*}^{B} 1000 0 0 - \hat{j}_{*}^{B} 1000 0 0 0 0 0	$n = 500$ \hat{j}_{*}^{C} 0 1000 8 $a = 2000$ \hat{j}_{*}^{C} 0 748 252 1.16 $n = 500$ \hat{j}_{*}^{C} 0 1000 193.17 $a = 2000$ \hat{j}_{*}^{C} 0 0 1000 193.17 $a = 2000$ \hat{j}_{*}^{C} 0 0 1000	$ \hat{\mathbf{j}}_{*}^{(0)} \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 2 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 9995 \\ 5 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 10 \\ 9990 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 990 \\ 0 \\ 0 \\ - \\ 996 \\ 4 $	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 965 35 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 940 60 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 963 37 1.05 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 930 70	

Table 1: Selection times of the KOO methods with AIC, BIC, C_p thresholds and bootstrap methods under Settings (I) and (i) based on 1,000 replications.

S1. ADDITIONAL SIMULATION RESULTS

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$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{r} 39 \\ 1.05 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \end{array} $
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	1.05 $\hat{\mathbf{j}}_{*}^{(5)}$ 0
$\begin{tabular}{ c c c c c c c c c c c c c c c c } \hline & n = 1000 & n = 2000 \\ \hline $\hat{\mathbf{j}}_{*}^{A}$ & $\hat{\mathbf{j}}_{*}^{B}$ & $\hat{\mathbf{j}}_{*}^{C}$ & $\hat{\mathbf{j}}_{*}^{(5)}$ & $\hat{\mathbf{j}}_{*}^{A}$ & $\hat{\mathbf{j}}_{*}^{B}$ & $\hat{\mathbf{j}}_{*}^{C}$ & $\hat{\mathbf{j}}_{*}^{(0)}$ \\ \hline $U-S$ & 0 & 780 & 0 & 0 & 0 & 1000 & 0 & 0 \\ \hline $T-S$ & 963 & 220 & 978 & 999 & 946 & 1000 & 0 & 1000 & 999 \\ \hline $O-S$ & 37 & 0 & 22 & 1 & 54 & 0 & 0 & 0 & 1 \\ \hline \end{tabular}$	$\hat{\mathbf{j}}_{*}^{(5)}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\frac{\hat{\mathbf{j}}_{*}^{(5)}}{0}$
	0
T-S 963 220 978 999 946 1000 0 1000 999 O-S 37 0 22 1 54 0 0 0 1	
O-S 37 0 22 1 54 0 0 0 1	953
	47
A-S 1.03 - 1 1 1.02 1	1.04
$\alpha = 0.2, c = 0.4$	
n = 100 $n = 500$	
$\hat{\mathbf{j}}_{*}^{A} \hat{\mathbf{j}}_{*}^{B} \hat{\mathbf{j}}_{*}^{C} \hat{\mathbf{j}}_{*}^{(0)} \hat{\mathbf{j}}_{*}^{(5)} \hat{\mathbf{j}}_{*}^{A} \hat{\mathbf{j}}_{*}^{B} \hat{\mathbf{j}}_{*}^{C} \hat{\mathbf{j}}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$
U-S 2 734 0 487 41 0 1000 0 0	0
T-S 40 266 0 513 937 1 0 0 1000	945
O-S 958 0 1000 0 22 999 0 1000 0	55
A-S $3.63 - 6.99 - 1.05 - 6.59 - 45.85 - 6.59 - 45.85$	1.02
n = 1000 $n = 2000$	
$\hat{\mathbf{j}}_{*}^{A} \hat{\mathbf{j}}_{*}^{B} \hat{\mathbf{j}}_{*}^{C} \hat{\mathbf{j}}_{*}^{(0)} \hat{\mathbf{j}}_{*}^{(5)} \hat{\mathbf{j}}_{*}^{A} \hat{\mathbf{j}}_{*}^{B} \hat{\mathbf{j}}_{*}^{C} \hat{\mathbf{j}}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$
U-S 0 1000 0 0 0 0 1000 0 0	0
T-S 26 0 0 1000 962 490 0 0 999	943
O-S 974 0 1000 0 38 510 0 1000 1	57
A-S 3.95 - 95.28 - 1 1.46 - 194.80 1	1.02
$\alpha = 0.4, c = 0.2$	
n = 100 $n = 500$	
$\hat{\mathbf{i}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$
U-S 0 56 2 245 53 0 113 0 0	0
T-S 0 917 2 755 913 0 887 0 999	958
O-S 1000 27 996 0 34 1000 0 1000 1	42
A-S 16.56 1.04 6.79 - 1.03 102.01 - 8.19 1	1
n = 1000 $n = 2000$	
$\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S 0 465 0 0 0 0 902 0 0	0
T-S 0 535 54 1000 940 0 98 780 999	959
O-S 1000 0 946 0 60 1000 0 220 1	41
A-S 213.34 - 3.27 - 1.02 449.91 - 1.11 1	1.05
$\alpha = 0.4, c = 0.4$	
n = 100 $n = 500$	
	$\hat{j}_{*}^{(5)}$
\hat{j}_{*}^{A} \hat{j}_{*}^{B} \hat{j}_{*}^{C} $\hat{j}_{*}^{(0)}$ $\hat{i}_{*}^{(5)}$ \hat{i}_{*}^{A} \hat{j}_{*}^{B} \hat{i}_{*}^{C} $\hat{i}_{*}^{(0)}$	U ·
$\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$ U-S 0 465 1 984 744 0 996 0 17	3
$\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{(0)}$ U-S 0 465 1 984 744 0 996 0 17 T-S 0 420 0 16 240 0 4 0 983	3 937
$ \begin{array}{ c c c c c c c c c } \hline & \hat{\mathbf{j}}_{*}^{A} & \hat{\mathbf{j}}_{*}^{B} & \hat{\mathbf{j}}_{*}^{C} & \hat{\mathbf{j}}_{*}^{(0)} & \hat{\mathbf{j}}_{*}^{(5)} & \hat{\mathbf{j}}_{*}^{A} & \hat{\mathbf{j}}_{*}^{B} & \hat{\mathbf{j}}_{*}^{C} & \hat{\mathbf{j}}_{*}^{(0)} \\ \hline & \text{U-S} & 0 & 465 & 1 & 984 & 744 & 0 & 996 & 0 & 17 \\ \hline & \text{T-S} & 0 & 420 & 0 & 16 & 240 & 0 & 4 & 0 & 983 \\ \hline & \text{O-S} & 1000 & 115 & 999 & 0 & 16 & 1000 & 0 & 1000 & 0 \\ \hline \end{array} $	3 937 60
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c } \hline \hat{\mathbf{j}}_{*}^{A} & \hat{\mathbf{j}}_{*}^{B} & \hat{\mathbf{j}}_{*}^{C} & \hat{\mathbf{j}}_{*}^{(0)} & \hat{\mathbf{j}}_{*}^{(5)} & \hat{\mathbf{j}}_{*}^{A} & \hat{\mathbf{j}}_{*}^{B} & \hat{\mathbf{j}}_{*}^{C} & \hat{\mathbf{j}}_{*}^{(0)} \\ \hline \mathbf{U}\text{-S} & 0 & 465 & 1 & 984 & 744 & 0 & 996 & 0 & 17 \\ \hline \mathbf{T}\text{-S} & 0 & 420 & 0 & 16 & 240 & 0 & 4 & 0 & 983 \\ \hline \mathbf{O}\text{-S} & 1000 & 115 & 999 & 0 & 16 & 1000 & 0 & 1000 & 0 \\ \hline \mathbf{A}\text{-S} & 30.98 & 1.39 & 29.18 & - & 1.06 & 194.59 & - & 193.12 & - \\ \hline & & & & & & & n = 1000 & & & & n = 2000 \\ \hline \end{array} $	3 937 60 1
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\frac{3}{937}$ 60 1 $\hat{j}_{*}^{(5)}$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\frac{3}{937}$ 60 1 $\hat{j}_{*}^{(5)}$ 0
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{r} 3 \\ 937 \\ 60 \\ 1 \\ \\ \hat{j}_{*}^{(5)} \\ 0 \\ 950 \\ \end{array} $
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{r} 3 \\ 937 \\ 60 \\ 1 \\ \hat{j}_{*}^{(5)} \\ 0 \\ 950 \\ 50 \\ \end{array} $

Table 2: Selection times of the KOO methods with AIC, BIC, C_p thresholds and bootstrap methods under Settings (I) and (ii) based on 1,000 replications.

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$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$					$\alpha =$	0.2, c =	= 0.2				
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			1	n = 100				1	n = 500		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{j}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{j}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	120	0	25	0	0	543	0	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	182	880	204	974	969	572	457	663	999	955
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	O-S	818	0	796	1	31	428	0	337	1	45
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	A-S	2.10	_	2.02	1	1.10	1.35	_	1.23	1	1.04
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			r	n = 1000				r	n = 2000		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$\hat{\mathbf{i}}^A$	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}^C$	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$	$\hat{\mathbf{i}}^A$	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}^C$	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	J *	975				J *	J* 1000			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	963	25	981	1000	963	999	0	1000	999	953
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	O-S	37	0	19	0	37	1	0	0	1	47
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	1	_	1	_	1	1	_	_	1	1.02
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	11-5	1		1		0.0	- 0.4			1	1.02
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$				n = 100	$\alpha =$	0.2, c =	= 0.4		n = 500		
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		î A	ĵ₿	îC	î (0)	<u>;</u> (5)	ĵA	îВ	îC	<u>;</u> (0)	<u>;</u> (5)
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	US	J *	3%	<u>J*</u>	J* 210	J* 94	J *	J* 1000	<u>J*</u>	<u>]*</u>	<u>]*</u>
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0-5 TS	37	105	1	219	032	0	1000	0	000	041
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	09	063	100	000	2	44	4 006	0	1000	1	50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0-5 A S	368	0	999 7 1 1	1	44	990 6.60	0	46.14	1	1.02
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	л-ы	5.00	- ~	$\frac{7.11}{2} = 1000$	1	1.07	0.00	- ~	$\frac{40.14}{2}$	1	1.02
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		î A	îB	$\hat{i} = 1000$ $\hat{i}C$	<u>-</u> (0)	î (5)	î.A	îB	$\frac{1 - 2000}{C}$	<u>-</u> (0)	<u>;</u> (5)
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	IL C	J**	J.	<u> </u>	J *´	<u>]*´</u>	J**	J.	<u> </u>	<u>]*</u>	J*
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	1000	0	0	0	0	1000	0	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	35	0	0	1000	937	454	0	0	1000	941
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0-s	965	0	1000	0	63	546	0	1000	0	59
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	3.90	-	95.25	-	1.05	1.46	-	194.82	-	1
$\begin{array}{ c c c c c c } \hline & n = 100 & n = 500 \\ \hline $\hat{j}_{*}^{A} & \hat{j}_{*}^{B} & \hat{j}_{*}^{C} & \hat{j}_{*}^{(0)} & \hat{j}_{*}^{(5)} & \hat{j}_{*}^{A} & \hat{j}_{*}^{B} & \hat{j}_{*}^{C} & \hat{j}_{*}^{(0)} & \hat{j}_{*}^{(5)} \\ \hline $U-S$ & 0 & 61 & 0 & 331 & 43 & 0 & 153 & 0 & 0 & 0 \\ \hline $T-S$ & 0 & 894 & 5 & 666 & 897 & 0 & 847 & 0 & 999 & 969 \\ \hline $O-S$ & 1000 & 45 & 995 & 3 & 60 & 1000 & 0 & 1000 & 1 & 31 \\ \hline $A-S$ & 16.59 & 1.02 & 6.91 & 1 & 1.08 & 101.70 & - & 8.23 & 1 & 1 \\ \hline $n = 1000 & $n = 2000 \\ \hline $\hat{j}_{*}^{A} & \hat{j}_{*}^{B} & \hat{j}_{*}^{C} & \hat{j}_{*}^{(0)} & \hat{j}_{*}^{(5)} & \hat{j}_{*}^{A} & \hat{j}_{*}^{B} & \hat{j}_{*}^{C} & \hat{j}_{*}^{(0)} & \hat{j}_{*}^{(5)} \\ \hline $U-S$ & 0 & 715 & 0 & 0 & 0 & 0 & 1000 & 0 & 0 & 0 \\ \hline $T-S$ & 0 & 285 & 44 & 1000 & 947 & 0 & 0 & 788 & 999 & 963 \\ \hline $O-S$ & 1000 & 0 & 956 & 0 & 53 & 1000 & 0 & 212 & 1 & 37 \\ \hline $A-S$ & 213.18 & - & 3.22 & - & 1.04 & 449.63 & - & 1.13 & 1 & 1 \\ \hline $u-S$ & 0 & 589 & 0 & 994 & 837 & 0 & 1000 & 0 & 2 & 0 \\ \hline $\hat{J}_{*}^{A} & \hat{J}_{*}^{B} & \hat{J}_{*}^{C} & \hat{J}_{*}^{(0)} & \hat{J}_{*}^{(5)} & \hat{J}_{*}^{A} & \hat{J}_{*}^{B} & \hat{J}_{*}^{C} & \hat{J}_{*}^{(0)} & \hat{J}_{*}^{(5)} \\ \hline $u-S$ & 0 & 589 & 0 & 994 & 837 & 0 & 1000 & 0 & 2 & 0 \\ \hline $T-S$ & 0 & 319 & 0 & 6 & 150 & 0 & 0 & 0 & 995 & 950 \\ \hline $O-S$ & 1000 & 92 & 1000 & 0 & 13 & 1000 & 0 & 1000 & 3 & 50 \\ \hline $A-S$ & 30.97 & 1.34 & 29.24 & - & 1.38 & 194.60 & - & 193.15 & 1 & 1.02 \\ \hline $n = 1000 & $n = 1000 & $n = 2000 \\ \hline $\hat{J}_{*}^{A} & \hat{J}_{*}^{B} & \hat{J}_{*}^{C} & \hat{J}_{*}^{(0)} & \hat{J}_{*}^{(5)} \\ \hline $U-S$ & 0 & 1000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $				100	$\alpha =$	0.4, <i>c</i> =	= 0.2		500		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		î A	î B	$\frac{n = 100}{\hat{C}}$	î (0)	î (5)	î A	î B	$\frac{n=500}{\hat{C}}$	î (0)	î (5)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		j *	<u>j</u> *	<u> </u>	j *	J *	j_*	<u>j</u> *	<u> </u>	j *	j *
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0-8	0	61	0	331	43	0	153	0	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	0	894	5	666	897	0	847	0	999	969
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	O-S	1000	45	995	3	60	1000	0	1000	1	31
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	A-S	16.59	1.02	6.91	1	1.08	101.70	-	8.23	1	1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		^ A	<u>^</u>	n = 1000	^(<u>0</u>)	^(5)	^ 4	<u>^</u>	n = 2000	^(<u>0</u>)	^(5)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		\mathbf{j}_*^A	\mathbf{j}^B_*	\mathbf{j}^{C}_{*}	$\mathbf{j}_{*}^{(0)}$	$\mathbf{j}_{*}^{(0)}$	\mathbf{j}_{*}^{A}	\mathbf{j}^B_*	\mathbf{j}^{C}_{*}	$\mathbf{j}_{*}^{(0)}$	j *
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	715	0	0	0	0	1000	0	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	0	285	44	1000	947	0	0	788	999	963
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	O-S	1000	0	956	0	53	1000	0	212	1	37
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	213.18	_	3.22	_	1.04	449.63	_	1.13	1	1
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		1			$\alpha =$	0.4, <i>c</i> =	= 0.4				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				n = 100	¢(0)	A(E)		1	n = 500	¢(0)	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	j *	$\mathbf{j}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	j *	j *
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	589	0	994	837	0	1000	0	2	0
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	0	319	0	6	150	0	0	0	995	950
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	O-S	1000	92	1000	0	13	1000	0	1000	3	50
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	30.97	1.34	29.24		1.38	194.60	_	193.15	1	1.02
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			r	n = 1000				r	n = 2000		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	1000	0	0	0	0	1000	0	0	0
	T-S	0	0	0	1000	950	0	0	0	994	945
A-S 394.99 - 394.83 - 1.06 795 - 795 1 1.05	O-S	1000	0	1000	0	50	1000	0	1000	6	55
	1 . ~	394 99	_	394 83	_	1.06	795	_	795	1	1.05

Table 3: Selection times of the KOO methods with AIC, BIC, C_p thresholds and bootstrap methods under Settings (I) and (iii) based on 1,000 replications.

S1. ADDITIONAL SIMULATION RESULTS

				$\alpha =$	0.2, <i>c</i> =	= 0.2				
		1	n = 100	. (2)	. (5)			n = 500		. (2)
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	0	77	0	971	215	0	648	0	0	0
T-S	41	847	46	29	753	0	352	1	1000	967
O-S	959	76	954	0	32	1000	0	999	0	33
A-S	3.11	1.04	3.01	-	1	7.40	-	6.68	-	1.03
		r	n = 1000				r	n = 2000		
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^{C}_{*}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	0	993	0	0	0	0	1000	0	0	0
T-S	4	7	6	1000	944	177	0	267	997	942
O-S	996	0	994	0	56	823	0	733	3	58
A-S	5.51	-	4.65	-	1.04	2.11	-	1.77	1	1.02
				$\alpha =$	0.2, <i>c</i> =	= 0.4				
		1	n = 100				1	n = 500		
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	0	925	0	991	622	0	1000	0	2	0
T-S	2	74	0	9	361	0	0	0	997	934
O-S	998	1	1000	0	17	1000	0	1000	1	66
A-S	4.74	1	7	-	1.06	16.40	-	45.88	1	1
		r	n = 1000				r	n = 2000		
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	0	1000	0	0	0	0	1000	0	0	0
T-S	0	0	0	1000	938	0	0	0	999	924
O-S	1000	0	1000	0	62	1000	0	1000	1	76
A-S	18.74	-	95.36	-	1.03	13.77	-	194.65	1	1.01
				$\alpha =$	0.4, c =	= 0.2				
		1	n = 100	α =	0.4, c =	= 0.2	:	n = 500		(=)
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\frac{n = 100}{\hat{\mathbf{j}}_*^C}$	$\frac{\alpha}{\hat{\mathbf{j}}_{*}^{(0)}}$	0.4, c = $\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\frac{n = 500}{\hat{\mathbf{j}}_*^C}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	$\begin{array}{c} \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \end{array}$	$\hat{\mathbf{j}}^B_*$ 4	$\frac{n = 100}{\hat{\mathbf{j}}_*^C}$	$ \boldsymbol{\alpha} = \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 999 $	0.4, c = $\hat{\mathbf{j}}_{*}^{(5)}$ 597		$\hat{\mathbf{j}}^B_*$ 7	$\frac{n = 500}{\hat{\mathbf{j}}_*^C}$	$\hat{\mathbf{j}}_{*}^{(0)}$ 7	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S T-S	$\begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \end{array}$	$\hat{\mathbf{j}}_{*}^{B}$ 4 348	$\frac{n = 100}{\hat{\mathbf{j}}_{*}^{C}}$ 0 0	$\alpha = \frac{\hat{\mathbf{j}}_{*}^{(0)}}{999}$ 1	0.4, c = $\hat{\mathbf{j}}_{*}^{(5)}$ 597 386		$\hat{\mathbf{j}}_{*}^{B}$ 7 993	$\frac{n = 500}{\hat{\mathbf{j}}_*^C}$	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961
U-S T-S O-S		$\hat{\mathbf{j}}_{*}^{B}$ 4 348 648	$ \begin{array}{r} n = 100 \\ \hat{\mathbf{j}}_{*}^{C} \\ \hline 0 \\ \hline 0 \\ \hline 1000 \end{array} $	$\alpha = \frac{\hat{\mathbf{j}}_{*}^{(0)}}{\hat{\mathbf{j}}_{*}^{(0)}}$ $\frac{1}{0}$	0.4, c = $\hat{j}_{*}^{(5)}$ 597 386 17	$ \begin{array}{c} = 0.2 \\ \hat{j}_{*}^{A} \\ 0 \\ 0 \\ $	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0	$ \frac{n = 500}{\hat{\mathbf{j}}_{*}^{C}} $ $ 0 $ $ 0 $ $ 1000 $	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39
U-S T-S O-S A-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \end{array} $	$\hat{\mathbf{j}}_{*}^{B}$ 4 348 648 1.67	$ \begin{array}{r} n = 100 \\ \hat{\mathbf{j}}_{*}^{C} \\ \hline 0 \\ \hline 0 \\ \hline 1000 \\ 9.23 \end{array} $	$\alpha =$ $\hat{j}_{*}^{(0)}$ 999 1 0 -	$\begin{array}{c} 0.4, \ c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \end{array}$	$ \begin{array}{c} = 0.2 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ $	\hat{j}_{*}^{B} 7 993 0 -	$ \begin{array}{r} n = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ \hline 0 \\ 1000 \\ 28.61 \end{array} $	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 -	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1
U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 15.31	$\hat{\mathbf{j}}_{*}^{B}$ 4 348 648 1.67 n	$n = 100$ \hat{j}_{*}^{C} 0 0 1000 9.23 $n = 1000$	$\alpha = \frac{\hat{\mathbf{j}}_{*}^{(0)}}{999}$ $\frac{1}{0}$ $-$ $\hat{\mathbf{j}}_{*}^{(0)}$	0.4, c = $\hat{\mathbf{j}}_{*}^{(5)}$ 597 386 17 1	$ \begin{array}{c} = 0.2 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 94.64 \\ \end{array} $	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - r	$ \begin{array}{r} n = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 28.61 \\ n = 2000 \end{array} $	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 -	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1
U-S T-S O-S A-S	$ \begin{array}{c} \hat{j}_{*}^{A} \\ 0 \\ 0 \\ $	\hat{j}^B_* 4 348 648 1.67 \hat{j}^B_*	$n = 100$ \hat{j}_{c}^{C} 0 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C}		$\begin{array}{c} 0.4, \ c = \\ \hline \mathbf{\hat{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hline \mathbf{\hat{j}}_{*}^{(5)} \end{array}$			$ \begin{array}{r} n = 500 \\ \hat{\mathbf{j}}_{k}^{C} \\ 0 \\ 0 \\ 1000 \\ 28.61 \\ n = 2000 \\ \hat{\mathbf{j}}_{k}^{C} \end{array} $	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$
U-S T-S O-S A-S U-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ \end{array} $		$n = 100$ \hat{j}_{c}^{C} 0 0 1000 9.23 $n = 1000$ \hat{j}_{s}^{C} 0 0		$\begin{array}{c} 0.4, \ c = \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \end{array}$	$ \begin{array}{c} = 0.2 \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 94.64 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ \end{array} $	\hat{j}_{*}^{B} 7 993 0 - \hat{j}_{*}^{B} 354	$n = 500$ \hat{j}_{*}^{C} 0 0 1000 28.61 $n = 2000$ \hat{j}_{*}^{C} 0	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0
U-S T-S O-S A-S U-S T-S	$ \begin{array}{r} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ $		$n = 100$ \hat{j}_{*}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{*}^{C} 0 0 0 0	$ \alpha = \frac{\hat{j}_{*}^{(0)}}{999} \\ 1 \\ 0 \\ - \\ \hat{j}_{*}^{(0)} \\ 0 \\ 1000 $	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \end{array}$	$ \begin{array}{c} = 0.2 \\ & \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 94.64 \\ & \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ \end{array} $	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646	$n = 500$ \hat{j}_{*}^{C} 0 0 1000 28.61 $n = 2000$ \hat{j}_{*}^{C} 0 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956
U-S T-S O-S A-S U-S T-S O-S	$ \frac{\hat{\mathbf{j}}_{*}^{A}}{0} \\ 0 \\ 1000 \\ 15.31 \\ \frac{\hat{\mathbf{j}}_{*}^{A}}{0} \\ 0 \\ 1000 \\ 1000 \\ 1000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	$ \begin{array}{r} \hat{\mathbf{j}}_{*}^{0} \\ \hat{\mathbf{j}}_{*}^{0} \\ 4 \\ 348 \\ 648 \\ 1.67 \\ \hat{\mathbf{j}}_{*}^{B} \\ 27 \\ 973 \\ 0 \end{array} $	$n = 100$ \hat{j}_{*}^{C} 0 0 1000 9.23 $a = 1000$ \hat{j}_{*}^{C} 0 0 1000 0 1000 0 0 0 0 0 0 0 0 0 0 0	$ \alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}} 999 1 0 - \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}} 0 1000 0 $	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ \hline 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1 \\ 0 \\ \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 1000	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0	$n = 500$ \hat{j}_{*}^{C} 0 0 1000 28.61 $n = 2000$ \hat{j}_{*}^{C} 0 0 1000 20.00	$ \hat{\mathbf{j}}_{*}^{(0)} $ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0	
U-S T-S O-S A-S U-S T-S O-S A-S	$ \begin{array}{c} & \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \end{array} $	$\hat{\mathbf{j}}_{*}^{B}$ 4 348 648 1.67 $\hat{\mathbf{j}}_{*}^{B}$ 27 973 0 -	$n = 100$ \hat{j}_{*}^{C} 0 0 1000 9.23 $a = 1000$ \hat{j}_{*}^{C} 0 0 1000 31.12	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0 -	$n = 500$ $\hat{\mathbf{j}}_{k}^{C}$ 0 0 1000 28.61 $n = 2000$ $\hat{\mathbf{j}}_{k}^{C}$ 0 0 1000 21.32	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 -	
U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c c} & \hat{\mathbf{j}}_{*}^{A} \\ \hline 0 \\ 0 \\ 1000 \\ 15.31 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \end{array}$	$\begin{array}{c} \hat{\mathbf{j}}_{*}^{0} \\ \hat{\mathbf{j}}_{*}^{0} \\ \frac{4}{348} \\ 648 \\ 1.67 \\ \mathbf{j}_{*}^{0} \\ \frac{27}{973} \\ 0 \\ - \end{array}$	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 1000 31.12	$ \alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{1000} = \frac{\alpha}{\alpha} = \alpha $	$\begin{array}{c} 0.4, \ c = \\ \hline \mathbf{j}_{*}^{(5)} \\ 597 \\ \overline{386} \\ 17 \\ 1 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, \ c = \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0 -	$n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 0 1000 28.61 $n = 2000$ $\hat{\mathbf{j}}_{c}^{C}$ 0 0 1000 21.32	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 -	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02
U-S T-S O-S A-S U-S T-S O-S A-S	$\hat{\mathbf{j}}_{*}^{A}$ 0 1000 15.31 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 198.92	$\hat{\mathbf{j}}_{*}^{R}$ $\hat{\mathbf{j}}_{*}^{R}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{4}$ $\hat{5}$ 5 $\hat{5}$ \hat	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 1000 31.12 $n = 100$	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{\alpha} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^$	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ 2(5) \end{array}$	$= 0.2$ \hat{j}_{*}^{A} 0 0 1000 94.64 \hat{j}_{*}^{A} 0 0 1000 417.94 $= 0.4$ \hat{j}_{*}	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0	$n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 28.61 $n = 2000$ $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 21.32 $n = 500$	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 -	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 2
U-S T-S O-S A-S U-S T-S O-S A-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ \hat{\mathbf{j}}_{*}^{A} \end{array} $	\hat{j}_{*}^{0} \hat{j}_{*}^{0} $\frac{1}{4}$ $\frac{1}{348}$ $\frac{1}{648}$ $\frac{1}{647}$ $\frac{1}{7}$ \hat{j}_{*}^{B} $\frac{27}{973}$ 0 $ \hat{j}_{*}^{0}$ \hat{j}_{*}^{0}	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 1000 31.12 $n = 100$ \hat{j}_{c}^{C}		$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}^{B}$	$n = 500$ \hat{j}_{c}^{C} 0 1000 28.61 $n = 2000$ \hat{j}_{c}^{C} 0 1000 21.32 $n = 500$ \hat{j}_{c}^{C}	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1,000	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$
U-S T-S O-S A-S U-S C-S A-S	$ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	\hat{j}_{*}^{R} \hat{j}_{*}^{R} 4 348 648 1.67 \hat{r} \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 27 \hat{j}_{*}^{R} 27 \hat{j}_{*}^{R} 27 \hat{j}_{*}^{R} 27 \hat{j}_{*}^{R} 27 \hat{j}_{*}^{R} \hat{j}_{*}^{R} \hat{j}_{*}^{R} 243 \hat{j}_{*}^{R}	$n = 100$ \hat{j}_{c}^{C} 0 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 0 1000 31.12 $n = 100$ \hat{j}_{c}^{C} 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{0} = $	$\begin{array}{c} 0.4, c = \\ \hline \mathbf{j}_{*}^{(5)} \\ 597 \\ \overline{386} \\ 17 \\ 1 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hline \mathbf{j}_{*}^{(5)} \\ \mathbf{s}_{*} \\ 898 \\ \overline{\mathbf{s}_{*}} \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0 - $\hat{\mathbf{j}}_{*}^{B}$ 988	$n = 500$ \hat{j}_{c}^{C} 0 0 1000 28.61 $n = 2000$ \hat{j}_{c}^{C} 0 0 1000 21.32 $n = 500$ \hat{j}_{c}^{C} 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\hat{j}_{*}^{(0)}$ 7 993 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 - $\hat{j}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ 2 $\hat{\mathbf{j}}_{*}^{(5)}$
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S S-S	$ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000$	$\hat{\mathbf{j}}_{*}^{R}$ $\mathbf{$	$n = 100$ \hat{j}_{c}^{C} 0 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 0 1000 31.12 $n = 100$ \hat{j}_{c}^{C} 0 0 0 1000 31.12	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\alpha}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{1000} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}} = \frac{\hat{j}_{*}^{(0)}}{1000} = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \hat{$	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ \overline{\mathbf{j}}_{*} \end{array}$	$= 0.2$ \hat{j}_{*}^{A} 0 0 1000 94.64 \hat{j}_{*}^{A} 0 0 1000 417.94 $= 0.4$ \hat{j}_{*}^{A} 0 0 0 1000 417.94 $= 0.4$	$\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}$	$n = 500$ \hat{j}_{c}^{C} 0 0 1000 28.61 $n = 2000$ \hat{j}_{c}^{C} 0 0 1000 21.32 $n = 500$ \hat{j}_{c}^{C} 0 0 0 1000 21.32 $n = 500$	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 188 188 812	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 2 2 965 2
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \end{array} $ $ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \end{array} $ $ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ \hat{\mathbf{j}}_{*}^{A} \\ 0$	$\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{A}$ $\hat{\mathbf{j}}_{*}^{B}$ $\hat{\mathbf{j}}_{*}^{B}$ 27 973 0 - $\hat{\mathbf{j}}_{*}^{B}$ 243 233 524 $\hat{\mathbf{j}}_{*}^{C}$ $\hat{\mathbf{j}}_{*}^{$	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $a = 1000$ \hat{j}_{c}^{C} 0 0 1000 31.12 $n = 100$ \hat{j}_{c}^{C} 0 0 1000 20.00	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{1000} = \frac{\hat{j}_{*}^{(0)}}{1000} = \frac{\hat{j}_{*}^{(0)}}{0} = \frac{\hat{j}_{*}^{(0)}}{0}$	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.00 \\ \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 1000 1000 1000	$\hat{\mathbf{j}}_{*}^{B}$ $\overline{\mathbf{j}}_{*}$	$n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 0 1000 28.61 $n = 2000$ $\hat{\mathbf{j}}_{c}^{C}$ 0 0 1000 21.32 $n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 0 1000 100 20.22	$\begin{array}{c} \hat{\mathbf{j}}_{*}^{(0)} \\ 7 \\ 993 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \hline \hat{\mathbf{j}}_{*}^{(0)} \\ 188 \\ 812 \\ 0 \\ \end{array}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 33 1 2
U-S T-S O-S A-S U-S A-S U-S T-S O-S A-S	$ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \end{pmatrix} $	$\hat{\mathbf{j}}_{*}^{B}$ $\mathbf{$	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 0 1000 31.12 $n = 100$ \hat{j}_{s}^{C} 0 0 1000 26.89 1000	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0 - $\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 0 0 0 0	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.20 \\ \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 1000 191.27	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 $ \hat{\mathbf{j}}_{*}^{B}$ 354 646 0 $ \hat{\mathbf{j}}_{*}^{B}$ 988 12 0 $-$	$n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 28.61 $n = 2000$ $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 21.32 $n = 500$ $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 $\hat{\mathbf{j}}_{c}^{C}$ 0 1000 186.32 2000	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 188 812 0 -	$ \begin{array}{r} \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 961 \\ 39 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 956 \\ 44 \\ 1.02 \\ \hline \hat{\mathbf{j}}_{*}^{(5)} \\ 2 \\ 965 \\ 33 \\ 1.03 \\ \end{array} $
U-S T-S O-S A-S U-S T-S O-S A-S U-S A-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \hline \hat{\mathbf{i}}_{*}^{A} \\ \end{array} $	$\hat{\mathbf{j}}_{*}^{B}$ $\mathbf{$	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 1000 31.12 $n = 100$ \hat{j}_{s}^{C} 0 1000 26.89 $n = 1000$ \hat{j}_{c}	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0 - $\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 $\hat{j}_{*}^{(0)}$	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hline \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.20 \\ \hline \\ 2(5) \\ \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 191.27 $\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0 - $\hat{\mathbf{j}}_{*}^{B}$ 988 12 0 - $\hat{\mathbf{j}}_{*}^{B}$ 7 $\hat{\mathbf{j}}_{*}^{B}$ 12 0 - $\hat{\mathbf{j}}_{*}^{B}$ 7 $\hat{\mathbf{j}}_{*}^$	$n = 500$ \hat{j}_{c}^{C} 0 0 1000 28.61 $i = 2000$ \hat{j}_{c}^{C} 0 0 1000 21.32 $n = 500$ \hat{j}_{s}^{C} 0 0 1000 186.32 $i = 2000$ \hat{j}_{c}	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 188 812 0 - ; $\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	\hat{j}_{*}^{R} \hat{j}_{*}^{R} 4 348 648 1.67 \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 243 243 243 243 524 1.99 \hat{j}_{*}^{R} 1.922	$n = 100$ \hat{j}_{*}^{C} 0 0 1000 9.23 $i = 1000$ \hat{j}_{*}^{C} 0 0 1000 31.12 $n = 100$ \hat{j}_{*}^{C} 0 0 1000 26.89 $i = 1000$ \hat{j}_{*}^{C} 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 $\hat{j}_{*}^{(0)}$ 0	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.20 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 5 \\ 1.20 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 191.27 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 0 0 0 0 0 0 0 0	$\hat{\mathbf{j}}_{*}^{R}$ 7 993 0 $ \hat{\mathbf{j}}_{*}^{R}$ 354 646 0 $ \hat{\mathbf{j}}_{*}^{R}$ 988 12 0 $ \hat{\mathbf{j}}_{*}^{R}$ 122 0 $ 7$ $\hat{\mathbf{j}}_{*}^{R}$ 1000	$n = 500$ \hat{j}_{c}^{C} 0 0 1000 28.61 $i = 2000$ \hat{j}_{c}^{C} 0 0 1000 21.32 $n = 500$ \hat{j}_{s}^{C} 0 0 1000 186.32 $i = 2000$ \hat{j}_{c}^{C} 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ 2
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$ \begin{array}{c} & \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	\hat{j}_{*}^{R} \hat{j}_{*}^{R} 4 348 648 1.67 \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 243 243 233 524 1.99 \hat{j}_{*}^{R} \hat{j}_{*	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 1000 31.12 $n = 100$ \hat{j}_{c}^{C} 0 1000 26.89 $n = 1000$ \hat{j}_{c}^{C} 0 0 1000 26.89 $n = 1000$ \hat{j}_{c}^{C} 0 0 0 0 0 0 0 0 0 0	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 0 - $\hat{j}_{*}^{(0)}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.20 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 5 \\ 1.20 \\ 0 \\ 42 \\ \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 191.27 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 0 0 0 0 0 0 0 0	\hat{j}_{*}^{B} 7 993 0 - \hat{j}_{*}^{B} 354 646 0 - \hat{j}_{*}^{B} 988 12 0 - \hat{j}_{*}^{B} 122 0 - \hat{j}_{*}^{B} 120 0 - \hat{j}_{*}^{B} 1000 0 0	$n = 500$ \hat{j}_{k}^{C} 0 1000 28.61 $n = 2000$ \hat{j}_{k}^{C} 0 1000 21.32 $n = 500$ \hat{j}_{k}^{C} 0 1000 186.32 $n = 2000$ \hat{j}_{k}^{C} 0 0 0 0 0 0 0 0 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 2 2 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 2 2 2 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 0 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 33 3 3.03 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 0 33 3 3 3 3 3 3
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$ \begin{array}{c} \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 15.31 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 198.92 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 1000 \\ 28.58 \\ \hline \\ \end{array} $	\hat{j}_{*}^{R} \hat{j}_{*}^{R} 4 348 648 1.67 \hat{j}_{*}^{R} 27 973 0 - \hat{j}_{*}^{R} 243 233 524 1.99 \hat{j}_{*}^{R} 1000 0 0	$n = 100$ \hat{j}_{k}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{k}^{C} 0 1000 31.12 $n = 100$ \hat{j}_{k}^{C} 0 1000 26.89 $n = 1000$ \hat{j}_{k}^{C} 0 1000 26.89 $n = 1000$ \hat{j}_{k}^{C} 0 1000 26.89 $n = 1000$ \hat{j}_{k}^{C} 0 0 0 1000 26.89 $n = 1000$ \hat{j}_{k}^{C} 0 0 0 0 0 0 0 0 0 0	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 - $\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 0 0 0 $\hat{j}_{*}^{(0)}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.20 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 5 \\ 1.20 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 942 \\ 5 \\ 5 \\ 1.20 \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 191.27 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 191.27	\hat{j}_{*}^{B} 7 993 0 - \hat{j}_{*}^{B} 354 646 0 - \hat{j}_{*}^{B} 988 12 0 - \hat{j}_{*}^{B} 120 0 - \hat{j}_{*}^{B} 1000 0 0	$n = 500$ \hat{j}_{k}^{C} 0 1000 28.61 $n = 2000$ \hat{j}_{k}^{C} 0 1000 21.32 $n = 500$ \hat{j}_{k}^{C} 0 1000 186.32 $n = 2000$ \hat{j}_{k}^{C} 0 1000 186.32 $n = 2000$ \hat{j}_{k}^{C} 0 0 1000 0 0 0	$\hat{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ $\overline{\mathbf{j}}_{*}^{(0)}$ 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 188 812 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 -	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 928 72
U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$ \hat{\mathbf{j}}_{*}^{A} $ $ 0 $ $ 0 $ $ 1000 $ $ 15.31 $ $ \hat{\mathbf{j}}_{*}^{A} $ $ 0 $ $ 0 $ $ 1000 $ $ 198.92 $ $ \hat{\mathbf{j}}_{*}^{A} $ $ 0 $ $ 0 $ $ 1000 $ $ 28.58 $ $ \hat{\mathbf{j}}_{*}^{A} $ $ 0 $ $ 0 $ $ 1000 $ $ 28.58 $ $ \hat{\mathbf{j}}_{*}^{A} $ $ 0 $ $ 0 $ $ 1000 $ $ 200 $ $ 200 $	$\hat{\mathbf{j}}_{*}^{R}$ $\mathbf{$	$n = 100$ \hat{j}_{c}^{C} 0 1000 9.23 $n = 1000$ \hat{j}_{c}^{C} 0 1000 31.12 $n = 100$ \hat{j}_{c}^{C} 0 1000 26.89 $n = 1000$ \hat{j}_{c}^{C} 0 1000 201.74	$\alpha = \frac{\hat{j}_{*}^{(0)}}{\hat{j}_{*}^{(0)}}$ 999 1 0 - $\hat{j}_{*}^{(0)}$ 0 - $\hat{j}_{*}^{(0)}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 1000 0 - $\hat{j}_{*}^{(0)}$ 0 1000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\begin{array}{c} 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 597 \\ 386 \\ 17 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 939 \\ 61 \\ 1.03 \\ 0.4, c = \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 898 \\ 97 \\ 5 \\ 1.20 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 942 \\ 58 \\ 1.05 \\ \end{array}$	$= 0.2$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 94.64 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 417.94 $= 0.4$ $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 191.27 $\hat{\mathbf{j}}_{*}^{A}$ 0 0 1000 1000 1000 704.00	$\hat{\mathbf{j}}_{*}^{B}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{B}$ 354 646 0 - $\hat{\mathbf{j}}_{*}^{B}$ 988 12 0 - $\hat{\mathbf{j}}_{*}^{B}$ 120 0 - $\hat{\mathbf{j}}_{*}^{B}$ 1000 0 0	$n = 500$ \hat{j}_{k}^{C} 0 1000 28.61 $n = 2000$ \hat{j}_{k}^{C} 0 0 1000 21.32 $n = 500$ \hat{j}_{k}^{C} 0 1000 186.32 $n = 2000$ \hat{j}_{k}^{C} 0 0 1000 186.32 $n = 2000$ \hat{j}_{k}^{C} 0 0 1000 $704 74$	$\hat{\mathbf{j}}_{*}^{(0)}$ 7 993 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 0 1000 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ 188 812 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 0 0 0 - $\hat{\mathbf{j}}_{*}^{(0)}$ $\hat{\mathbf{j}}_{*}^{(0)}$ 0 0 0 0 0 0 0 0	$\hat{\mathbf{j}}_{*}^{(5)}$ 0 961 39 1 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 956 44 1.02 $\hat{\mathbf{j}}_{*}^{(5)}$ 2 965 33 1.03 $\hat{\mathbf{j}}_{*}^{(5)}$ 0 928 72 1.04

Table 4: Selection times of the KOO methods with AIC, BIC, C_p thresholds and bootstrap methods under Settings (II) and (iv) based on 1,000 replications.

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$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$					$\alpha =$	0.2, c =	= 0.2				
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			1	n = 100					n = 500		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{i}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	63	0	66	0	0	732	0	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	127	936	144	933	963	169	268	245	998	969
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	O-S	873	1	856	1	37	831	0	755	2	31
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	A-S	2.38	1	2.27	1	1	2.16	_	1.92	1	1
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			n	n = 1000				r	n = 2000		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$\hat{\mathbf{i}}^A$	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}^C$	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$	\hat{i}^A	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{j}}^C$	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	<u>J*</u>	<u> </u>	0		<u></u>	<u>J*</u>	<u>J*</u> 1000	0	 	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	688	0	779	994	949	994	0	998	1000	937
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0-S	312	0	221	6	51	6	0	2	0	63
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	1.16	_	1.11	1	1.02	1	_	1	_	1.05
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		1110			$\alpha =$	0.2. c =	= 0.4		-		1.00
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			1	n = 100		0.2, 0	0.1		n = 500		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		ÎA	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	î ^A	$\hat{\mathbf{i}}^B_{i}$	$\hat{\mathbf{i}}_{c}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	973	0	615	100	0	1000	0	0	0
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	T-S	21	27	2	385	863	3	0	0	999	948
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	0-S	979	0	998	0	37	997	0	1000	1	52
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	3.86	_	6.98	_	1.05	8.99	_	46.12	1	1.02
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		0.00	n	n = 1000			0.00	r	n = 2000		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		îΑ	$\hat{\mathbf{i}}^B$	ĵC	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$	îΑ	îΒ	ĵC	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	U-S	J *	J * 1000	0			J *	J* 1000	0		J *
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	T-S	2	0	0	999	952	132	0	0	998	947
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0-5	998	0	1000	1	48	868	0	1000	2	53
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	A-S	6 64	_	95 24	1	1.02	2.30	_	193.88	1	1.06
$\begin{array}{c c c c c c } \hline & n = 100 & n = 500 \\ \hline \hat{j}_{A}^{A} & \hat{j}_{B}^{B} & \hat{j}_{C}^{C} & \hat{j}_{1}^{(0)} & \hat{j}_{1}^{(5)} & \hat{j}_{A}^{A} & \hat{j}_{B}^{B} & \hat{j}_{C}^{C} & \hat{j}_{1}^{(0)} & \hat{j}_{1}^{(5)} \\ \hline U-S & 0 & 1 & 0 & 384 & 17 & 0 & 1 & 0 & 0 & 0 \\ \hline T-S & 0 & 841 & 1 & 615 & 939 & 0 & 999 & 0 & 1000 & 944 \\ \hline O-S & 1000 & 158 & 999 & 1 & 44 & 1000 & 0 & 1000 & 0 & 56 \\ \hline A-S & 15.86 & 1.09 & 7.40 & 1 & 1.07 & 98.76 & - & 13.10 & - & 1.02 \\ \hline & & n = 1000 & & & n = 2000 \\ \hline \hat{j}_{A}^{A} & \hat{j}_{B}^{B} & \hat{j}_{C}^{C} & \hat{j}_{1}^{(0)} & \hat{j}_{1}^{(5)} & \hat{j}_{A}^{A} & \hat{j}_{B}^{B} & \hat{j}_{C}^{C} & \hat{j}_{1}^{(0)} & \hat{j}_{1}^{(5)} \\ \hline U-S & 0 & 6 & 0 & 0 & 0 & 0 & 374 & 0 & 0 & 0 \\ \hline T-S & 0 & 994 & 0 & 1000 & 954 & 0 & 666 & 197 & 999 & 954 \\ \hline O-S & 1000 & 0 & 1000 & 0 & 46 & 1000 & 0 & 803 & 1 & 46 \\ \hline A-S & 208.66 & - & 7.73 & - & 1.02 & 438.75 & - & 2.01 & 1 & 1.02 \\ \hline \hline \hat{j}_{A}^{A} & \hat{j}_{B}^{B} & \hat{j}_{C}^{C} & \hat{j}_{1}^{(0)} & \hat{j}_{1}^{(5)} & \hat{j}_{A}^{A} & \hat{j}_{B}^{B} & \hat{j}_{C}^{C} & \hat{j}_{1}^{(0)} & \hat{j}_{1}^{(5)} \\ \hline \hline V-S & 0 & 263 & 0 & 969 & 684 & 0 & 994 & 0 & 1 & 0 \\ \hline T-S & 0 & 540 & 0 & 31 & 302 & 0 & 6 & 0 & 999 & 948 \\ \hline O-S & 1000 & 197 & 1000 & 0 & 14 & 1000 & 0 & 1000 & 0 & 52 \\ \hline A-S & 30.52 & 1.38 & 28.73 & - & 1.07 & 194.24 & - & 192.21 & - & 1.04 \\ \hline \hline T-S & 0 & 540 & 0 & 31 & 302 & 0 & 6 & 0 & 999 & 948 \\ \hline O-S & 1000 & 197 & 1000 & 0 & 14 & 1000 & 0 & 1000 & 0 & 52 \\ \hline A-S & 30.52 & 1.38 & 28.73 & - & 1.07 & 194.24 & - & 192.21 & - & 1.04 \\ \hline \hline T-S & 0 & 1000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $		0.01		00.21		0.4 c =	- 0.2		100.00	-	1.00
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			1	n = 100	α –	0.4, C -	- 0.2		n = 500		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}_{*}^{C}$	$\hat{\mathbf{i}}^{(0)}$	$\hat{\mathbf{i}}^{(5)}$	îA.	ŝ₿	îC	î (0)	î (5)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TT O	J *			• •					1	lù ′
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S	0	1	0	384	<u>J*</u> 17	J *	<u>J</u> * 1	<u> </u>	J *´ 0	J * 7
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S	0	1 841	0 1	384 615	17 939	J * 0 0	$\frac{J_{*}}{1}$ 999	<u> </u>	J * 0 1000	J * 0 944
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	U-S T-S O-S	0 0 1000	1 841 158		384 615 1	17 939 44	J* 0 1000	J* 1 999 0	J* 0 0 1000	J * 7 0 1000 0	J * 0 944 56
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	U-S T-S O-S A-S	0 0 1000 15.86	J* 1 841 158 1.09		384 615 1 1	$ \begin{array}{r} 17 \\ 939 \\ 44 \\ 1.07 \end{array} $	J* 0 1000 98.76	<u>J*</u> 1 999 0 -	J* 0 0 1000 13.10	J * 7 0 1000 0 -	J * 0 944 56 1.02
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S	0 0 1000 15.86	J* 1 841 158 1.09	$ \begin{array}{r} 3 \\ 0 \\ 1 \\ 999 \\ 7.40 \\ n = 1000 \end{array} $	384 615 1 1	J* 17 939 44 1.07	J* 0 0 1000 98.76	<u>J*</u> 1 999 0 - r	$ \begin{array}{r} \mathbf{J}_{*} \\ \hline \\ 0 \\ \hline \\ 0 \\ \hline \\ 1000 \\ \\ 13.10 \\ \\ n = 2000 \\ \end{array} $	J * 7 0 1000 0 -	J* 0 944 56 1.02
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S	$0 \\ 0 \\ 1000 \\ 15.86 \\ \hat{\mathbf{j}}_{\star}^{A}$		$ \begin{array}{r} 3 \\ \hline 0 \\ 1 \\ 999 \\ 7.40 \\ n = 1000 \\ \hat{j}_{c}^{C} \\ \end{array} $	$ \frac{\mathbf{j}_{*}}{384} $ $ \frac{615}{1} $ 1 $ \hat{\mathbf{j}}_{*}^{(0)} $	$ \frac{\mathbf{j}_{*}}{17} $ 939 44 1.07 $ \hat{\mathbf{j}}_{*}^{(5)} $	J_{*} 0 1000 98.76 \hat{J}_{*}^{A}	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \end{array}$	$ \begin{array}{r} \mathbf{J}_{*} \\ 0 \\ 0 \\ $		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S	$ \begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \hat{j}_{*}^{A} \\ 0 \end{array} $		$ \begin{array}{r} 3 & * \\ 0 \\ 1 \\ 999 \\ 7.40 \\ i = 1000 \\ \hat{j}_{*}^{C} \\ 0 \end{array} $		$ \begin{array}{r} j_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ \end{array} $	$ \begin{array}{c} J_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hat{j}_{*}^{A} \\ 0 \end{array} $	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \hat{\mathbf{j}}_{*}^{B} \\ 374 \end{array}$	$ \begin{array}{r} \mathbf{J}_{*} \\ 0 \\ 0 \\ $		
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	U-S T-S O-S A-S U-S T-S	$ \begin{array}{r} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \hat{j}_{*}^{A} \\ 0 \\ 0 \\ \end{array} $	$ \begin{array}{r} J_{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \vec{r} \\ \hat{j}_{*}^{B} \\ \hline \hat{j}_{*} \\ 6 \\ 994 \\ \end{array} $	$ \begin{array}{r} 3 \\ 0 \\ $	$ \begin{array}{r} j_{*} \\ 384 \\ 615 \\ 1 \\ 1 \\ $	$ \begin{array}{r} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \mathbf{\hat{j}}_{*}^{(5)} \\ 0 \\ 954 \\ \end{array} $	$ \begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \end{array} $	$ \begin{array}{r} J_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \hat{j}_{*}^{B} \\ 374 \\ 626 \\ \end{array} $	$ \begin{array}{r} \mathbf{J}_{*} \\ 0 \\ 0 \\ $	$ \begin{array}{c} \underline{J}_{*} \\ 0 \\ 1000 \\ 0 \\ - \\ \underbrace{\hat{J}_{*}^{(0)}}_{\hat{J}_{*}^{(0)}} \\ 0 \\ 999 \end{array} $	$ \begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ \end{array} $
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	U-S T-S A-S U-S T-S O-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \end{array}$	$ \begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \hline \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \end{array} $	$\begin{array}{c} 0\\ 0\\ 1\\ 999\\ 7.40\\ a=1000\\ \hat{\mathbf{j}}_{*}^{C}\\ 0\\ 0\\ 1000 \end{array}$	$ \begin{array}{r} \mathbf{j}_{*} \\ 384 \\ 615 \\ 1 \\ 1 \\ \mathbf{j}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \end{array} $	$ \begin{array}{r} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 46 $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \end{array}$	$ \begin{array}{c} J_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \hat{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \end{array} $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ i = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \end{array}$	$ \begin{array}{r} \mathbf{j}_{*} \\ 0 \\ 1000 \\ $	$ \begin{array}{r} \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \mathbf{j}_{*}^{(5)} \\ \mathbf{j}_{*} \\ 0 \\ 954 \\ 46 \\ 46 \\ $
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \end{array}$		$\begin{array}{c} 3 \\ \hline 0 \\ 1 \\ 999 \\ 7.40 \\ a = 1000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \end{array}$		$ \begin{array}{r} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hat{\mathbf{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \end{array} $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \end{array}$	$ \begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \mathbf{\hat{j}}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \end{array} $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \end{array}$	$ \begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 1000 \\ 0 \\ - \\ \mathbf{\hat{j}}_{*}^{(0)} \\ 0 \\ 9999 \\ 1 \\ 1 \end{array} $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \\ \mathbf{\hat{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \end{array}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \hline \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \end{array}$	$ \begin{array}{c} 3_{*} \\ 0 \\ 1 \\ $	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}} = \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}} $ 0 1000 0	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \\ \mathbf{\hat{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \end{array}$	$ \begin{array}{r} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \end{array} $	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \mathbf{\hat{j}}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \end{array}$	\mathbf{j}_{*}^{*} 0 1000 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 1	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \end{array}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \end{array}$		$ \begin{array}{r} 3_{*} \\ 0 \\ 1 \\ $	$ \frac{\mathbf{j}_{*}}{384} \\ \frac{384}{615} \\ 1 \\ 1 \\ \hat{\mathbf{j}}_{*}^{(0)} \\ 0 \\ 1000 \\ 0 \\ - \\ \alpha = $	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \mathbf{\hat{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ 0.4, c = \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \end{array}$	$ \begin{array}{c} J_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \overline{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ 0 \\ 0 \\ - \\ 0 \\ 0 \\ - \\ 0 \\ 0 \\ 0 \\ $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline m = 500 \end{array}$	j_* , 0 1000 0 - $j_{(0)}$ j_{*} 0 999 1 1 1	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \hline \\ \mathbf{\hat{j}}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \end{array}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S U-S T-S O-S A-S	$ \begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ \hline \hat{\mathbf{j}}_{*}^{A} \\ \hline \end{array} $	$ \frac{\mathbf{J}_{*}}{1} $ $ \frac{1}{841} $ $ \frac{158}{1.09} $ $ \frac{\mathbf{J}_{*}}{\mathbf{J}_{*}} $ $ \frac{6}{994} $ $ \frac{994}{0} $ - $ \frac{1}{\mathbf{J}_{*}} $	$ \frac{3}{1} $ $ \frac{0}{1} $ $ \frac{1}{999} $ $ 7.40 $ $ \hat{j}_{*}^{C} $ $ \frac{1}{0} $ $ \frac{1}{1000} $ $ \frac{1}{7.73} $ $ \frac{1}{100} $ $ \frac{1}{100} $ $ \frac{1}{100} $ $ \frac{1}{100} $	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} $	$ \frac{\mathbf{j}_{*}}{17} \\ \frac{939}{939} \\ \frac{44}{1.07} \\ \frac{\mathbf{j}_{*}^{(5)}}{0} \\ \frac{954}{46} \\ \frac{46}{1.02} \\ \frac{0.4, c}{c} = \\ \frac{\mathbf{j}_{*}^{(5)}}{\mathbf{j}_{*}} $	$ \begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ \hline \mathbf{\hat{j}}_{*$	$ \begin{array}{c} J_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \overline{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \overline{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \overline{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \overline{j}_{*}^{B} \\ \overline{j}_{*}^$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \\ m = 500 \\ \hat{\mathbf{j}}_{*}^{C} \end{array}$	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}} $ 0 1000 0 $\mathbf{j}_{*}^{(0)}$ 999 1 1 - $\mathbf{j}_{*}^{(0)}$	\mathbf{J}_{*}^{*} 0 944 56 1.02 $\mathbf{\hat{j}}_{*}^{(5)}$ 0 954 46 1.02 $\mathbf{\hat{j}}_{*}^{(5)}$ $\mathbf{\hat{j}}_{*}^{(5)}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*}^{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \mathbf{\tilde{j}}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \mathbf{\tilde{j}}_{*}^{B} \\ 0 \\ \mathbf{\tilde{j}}_{*}^{B} \\ 263 \end{array}$	$\begin{array}{c} 3x \\ 0 \\ 1 \\ 999 \\ 7.40 \\ i = 1000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ n = 100 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ \end{array}$	$ \frac{3^{*}}{384} = \frac{384}{615} = \frac{384}{1} = \frac{384}{1}$	$ \frac{\mathbf{j}_{*}}{17} \\ \frac{17}{939} \\ \frac{39}{44} \\ \frac{1.07}{\mathbf{j}_{*}} \\ 0 \\ \frac{954}{46} \\ \frac{46}{1.02} \\ 0.4, c = \\ \mathbf{j}_{*}^{(5)} \\ 684 $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ \mathbf{j}_{*}^{B} \\ 994 \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline n = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ \end{array}$	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}} $ 0 1000 0 - \mathbf{j}_{*} 0 9999 1 1 \mathbf{j}_{*} \mathbf{j}_{*} 1	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ \mathbf{j}_{*} \\ 0 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ \mathbf{j}_{*} \\ 0 \end{array}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*}^{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 263 \\ 540 \end{array}$	$\begin{array}{c} 3x \\ 0 \\ 1 \\ 999 \\ 7.40 \\ i = 1000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ n = 100 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 0 \\ \end{array}$	$ \frac{384}{615} \\ \frac{384}{615} \\ \frac{1}{1} \\ \frac{1}{3} \\ \frac{1}{$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 684 \\ \hline \\ 302 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline n = 500 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 0 \end{array}$	\mathbf{j}_{*}	\mathbf{j}_{*} $\hat{\mathbf{j}}_{*}$ \mathbf
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \mathbf{\tilde{j}}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \mathbf{\tilde{j}}_{*}^{B} \\ 263 \\ \mathbf{\tilde{j}}_{*}^{B} \\ 263 \\ 540 \\ 197 \end{array}$	$\begin{array}{c} 3x \\ 0 \\ 1 \\ 999 \\ 7.40 \\ n = 1000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ n = 100 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \end{array}$	$ \frac{3}{384} = \frac{3}{384} = \frac{3}{384} = \frac{3}{384} = \frac{3}{384} = \frac{3}{384} = \frac{3}{31} = \frac{3}{384} = \frac{3}$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 684 \\ \hline \\ 302 \\ 14 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \mathbf{m} = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \end{array}$	\mathbf{j}_{*}^{*} 0 1000 0 - $\mathbf{j}_{*}^{(0)}$ 9999 1 1 $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ 1 9999 0	\mathbf{j}_{*} $\hat{\mathbf{j}}_{*}$ \mathbf
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 30.52 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*}^{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 263 \\ \mathbf{j}_{*}^{B} \\ 263 \\ 540 \\ 197 \\ 1.38 \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1 \\ 999 \\ 7.40 \\ \mathbf{n} = 1000 \\ \mathbf{j}_{*}^{C} \\ 0 \\ 0 \\ 0 \\ 000 \\ 7.73 \\ \mathbf{n} = 100 \\ \mathbf{j}_{*}^{C} \\ 0 \\ $	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}} = \mathbf$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 684 \\ 302 \\ \hline \\ 14 \\ 1.07 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \\ \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.24 \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 9999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \mathbf{m} = 500 \\ \hat{\mathbf{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 192.21 \end{array}$	\mathbf{j}_{*} \mathbf{j}_{*} 0 1000 0 - \mathbf{j}_{*} 0 999 1 1 \mathbf{j}_{*} 0 999 1 1 \mathbf{j}_{*} 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	\mathbf{j}_{*}^{*} 0 944 56 1.02 $\mathbf{j}_{*}^{(5)}$ 0 954 46 1.02 $\mathbf{j}_{*}^{(5)}$ $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 30.52 \\ \hline \end{array}$	$ \begin{array}{c} J_{*} \\ 1 \\ 1841 \\ 158 \\ 1.09 \\ \overline{n} \\ \hat{j}_{*} \\ 6 \\ 994 \\ 0 \\ - \\ $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1 \\ 999 \\ 7.40 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ \mathbf{n} = 100 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 28.73 \\ \mathbf{n} = 1000 \end{array}$	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\alpha}{\mathbf{j}_{*}^{(0)}} =$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ 0.4, c = \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 684 \\ 302 \\ 14 \\ 1.07 \\ \hline \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.24 \\ \hline \end{array}$	$\begin{array}{c c} \mathbf{J}_{*} \\ 1 \\ 9999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{r} \\ \mathbf{r} \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \mathbf{n} = 500 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 192.21 \\ a = 2000 \end{array}$	\mathbf{j}_{*} \mathbf{j}_{*} \mathbf{j}_{*} 0 1000 0 $ \mathbf{j}_{*}$ 0 0 \mathbf{j}_{*} 0 999 1 1 \mathbf{j}_{*} 0 1 1 1 1 0 1 $$	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \\ \mathbf{j}_{*}^{(5)} \\ 0 \\ 948 \\ 52 \\ 1.04 \\ \hline \end{array}$
	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 30.52 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \end{array}$	$ \begin{array}{c} j_{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \overline{n} \\ \hat{j}_{*} \\ 6 \\ 994 \\ 0 \\ - \\ $	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 1 \\ 999 \\ 7.40 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ \mathbf{n} = 100 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 10000 \\ 28.73 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \end{array}$	$ \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ 0.4, c = \\ \hline \mathbf{j}_{*}^{(5)} \\ 684 \\ 302 \\ 14 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ \mathbf{j}_{*}^{(5)} \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.24 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ \hline \mathbf{\hat{j}}_{*}^{A} \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 1 \\ 9999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline n = 500 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 192.21 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \end{array}$	\mathbf{j}_{*}^{*} 0 1000 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 $\mathbf{j}_{*}^{(0)}$ 1 1 9999 0 - $\mathbf{j}_{*}^{(0)}$	\mathbf{j}_{*}^{*} 0 944 56 1.02 $\mathbf{j}_{*}^{(5)}$ 0 954 46 1.02 $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$
O-S 1000 0 1000 1 47 1000 0 1000 1 50 A-S 394.98 - 394.64 1 1.02 795 - 795 1 1.02	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 30.52 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*}^{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \hline \\ \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \hline \\ \mathbf{j}_{*}^{B} \\ 263 \\ 540 \\ 197 \\ 1.38 \\ \hline \\ \mathbf{j}_{*}^{B} \\ 1000 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 0 \\ 1 \\ 999 \\ 7.40 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ \mathbf{n} = 100 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 10000 \\ 28.73 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ \mathbf$	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}}$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ 0.4, c = \\ \hline \mathbf{j}_{*}^{(5)} \\ 684 \\ 302 \\ 14 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.24 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c c} \mathbf{J}_{*} \\ 1 \\ 9999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 1000 \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \mathbf{n} = 500 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 192.21 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	\mathbf{j}_{*}^{*} 0 1000 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 $\mathbf{j}_{*}^{(0)}$ 1 1 9999 0 - $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ 1 $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 0 - $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ 0 0 - $\mathbf{j}_{*}^{(0)}$ 0 0 0 - $\mathbf{j}_{*}^{(0)}$ 0 0 0 - $\mathbf{j}_{*}^{(0)}$ 0 0 0 - 0 0 0 0 0 0 0 0	$\begin{array}{c} \mathbf{j}_{*} \\ \mathbf{j}_{*} \\ 0 \\ 944 \\ 56 \\ 1.02 \\ \hline \mathbf{j}_{*} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline \mathbf{j}_{*} \\ 6 \\ 1.02 \\ \hline \mathbf{j}_{*} \\ 0 \\ 948 \\ 52 \\ 1.04 \\ \hline \mathbf{j}_{*} \\ \mathbf{j}_{*} \\ 0 \\ \hline \mathbf{j}_{*} \\ 0 \\ \hline \mathbf{j}_{*} \\ \mathbf{j}_{*} \\$
A-S 394.98 - 394.64 1 1.02 795 - 795 1 1.02	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 30.52 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$	$\begin{array}{c} \mathbf{j}_{*}^{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \hline \\ \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \hline \\ \mathbf{j}_{*}^{B} \\ 263 \\ 540 \\ 197 \\ 1.38 \\ \hline \\ \mathbf{j}_{*}^{B} \\ 1000 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 0 \\ 1 \\ 999 \\ 7.40 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ \mathbf{n} = 100 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 10000 \\ 28.73 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ \mathbf$	$ \begin{array}{r} \mathbf{j}_{*}^{(0)} \\ \mathbf{j}_{*}^{(0)} \\ \mathbf{j}_{*}^{(0)} \\ 0 \\ 0 \\ $	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline 0.4, c = \\ \hline \mathbf{j}_{*}^{(5)} \\ 684 \\ 302 \\ 14 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 953 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.24 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c c} \mathbf{J}_{*} \\ 1 \\ 9999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{r} \\ \mathbf{j}_{*}^{B} \\ 1000 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \mathbf{n} = 500 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 192.21 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \end{array}$	\mathbf{j}_{*}^{*} 0 1000 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 $\mathbf{j}_{*}^{(0)}$ 1 $\mathbf{j}_{*}^{(0)}$ 1 1 $\mathbf{j}_{*}^{(0)}$ 1 $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ 1 $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 0 - 0 0 0 - 0 0 0 - 0 0 0 - 0 0 0 - 0 0 0 0 0 0 0 0	\mathbf{j}_{*}^{*} 0 944 56 1.02 $\mathbf{j}_{*}^{(5)}$ 0 954 46 1.02 $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$ $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$ $\mathbf{j}_{*}^{(5)}$ $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$
	U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S U-S T-S O-S A-S	$\begin{array}{c} 0 \\ 0 \\ 1000 \\ 15.86 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 208.66 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 30.52 \\ \hline \\ \hat{\mathbf{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 1000 \end{array}$	$\begin{array}{c} \mathbf{j}_{*}^{*} \\ 1 \\ 841 \\ 158 \\ 1.09 \\ \hline \\ \mathbf{j}_{*}^{B} \\ 6 \\ 994 \\ 0 \\ - \\ \hline \\ \mathbf{j}_{*}^{B} \\ 263 \\ 540 \\ 197 \\ 1.38 \\ \hline \\ \mathbf{j}_{*}^{B} \\ 1000 \\ \hline \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{j}_{*} \\ 0 \\ 0 \\ 1 \\ 999 \\ 7.40 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 7.73 \\ \mathbf{n} = 100 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 10000 \\ 28.73 \\ \mathbf{n} = 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 0 \\ 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 0 \\ 10000 \end{array}$	$ \frac{\mathbf{j}_{*}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}} = \frac{\mathbf{j}_{*}^{(0)}}{\mathbf{j}_{*}^{(0)}}$	$\begin{array}{c} \mathbf{j}_{*} \\ 17 \\ 939 \\ 44 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 954 \\ 46 \\ 1.02 \\ \hline 0.4, c = \\ \hline \mathbf{j}_{*}^{(5)} \\ 684 \\ 302 \\ 14 \\ 1.07 \\ \hline \mathbf{j}_{*}^{(5)} \\ 0 \\ 953 \\ 47 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 98.76 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 438.75 \\ = 0.4 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.24 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ 194.04 \\ \hline \mathbf{\hat{j}}_{*}^{A} \\ 0 \\ 0 \\ 1000 \\ \end{array}$	$\begin{array}{c c} \mathbf{J}_{*} \\ 1 \\ 9999 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 374 \\ 626 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{j}_{*}^{B} \\ 994 \\ 6 \\ 0 \\ - \\ \mathbf{r} \\ \mathbf{j}_{*}^{B} \\ 1000 \\ 0 \\ 0 \\ 0 \\ \end{array}$	$\begin{array}{c} \mathbf{J}_{*} \\ 0 \\ 0 \\ 1000 \\ 13.10 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 197 \\ 803 \\ 2.01 \\ \hline \mathbf{n} = 500 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ 192.21 \\ a = 2000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \\ \mathbf{\hat{j}}_{*}^{C} \\ 0 \\ 0 \\ 1000 \end{array}$	\mathbf{j}_{*}^{*} 0 1000 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 $\mathbf{j}_{*}^{(0)}$ 1 999 0 - $\mathbf{j}_{*}^{(0)}$ 0 - $\mathbf{j}_{*}^{(0)}$ 1 999 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 1 999 0 - $\mathbf{j}_{*}^{(0)}$ 0 999 1 1 1 1 999 1 1 1 1 999 1 1 1 1 1 1 1 1 1 1 1 1 1	\mathbf{j}_{*}^{*} 0 944 56 1.02 $\mathbf{j}_{*}^{(5)}$ 0 954 46 1.02 $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$ $\mathbf{j}_{*}^{(5)}$ 0 948 52 1.04 $\mathbf{j}_{*}^{(5)}$ \mathbf{j}_{*}

Table 5: Selection times of the KOO methods with AIC, BIC, C_p thresholds and bootstrap methods under Settings (II) and (v) based on 1,000 replications.

S1. ADDITIONAL SIMULATION RESULTS

				$\alpha =$	0.2, c =	= 0.2				
		1	n = 100				1	n = 500		
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^{C}_{*}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^{C}_{*}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	0	61	0	0	0	0	788	0	0	0
T-S	483	939	534	999	974	960	212	977	1000	956
O-S	517	0	466	1	26	40	0	23	0	44
A-S	1.49	-	1.44	1	1.08	1	-	1	-	1
		n	u = 1000				r	n = 2000		
	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$	$\hat{\mathbf{j}}_*^A$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{\mathbf{j}}_{*}^{(5)}$
U-S	0	1000	0	0	0	0	1000	0	0	0
T-S	1000	0	1000	1000	937	1000	0	1000	999	951
O-S	0	0	0	0	63	0	0	0	1	49
A-S	-	-	-	-	1.10	-	-	-	1	1.02
				$\alpha =$	0.2, c =	= 0.4				
		1	n = 100				1	n = 500		
	$\hat{\mathbf{j}}^A_*$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$
U-S	0	987	0	161	13	0	1000	0	0	0
T-S	60	13	0	838	951	28	0	0	1000	958
O-S	940	0	1000	1	36	972	0	1000	0	42
A-S	3.01	-	6.83	1	1.06	3.94	-	45.52	-	1.05
		n	a = 1000				r	n = 2000		
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{j}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{j}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$
U-S	0	1000	0	0	0	0	1000	0	0	0
T-S	237	0	0	999	949	870	0	0	999	954
O-S	763	0	1000	1	51	130	0	1000	1	46
A-S	1.92	_	94.99	1	1	1.05	-	193.56	1	1
				$\alpha =$	0.4, c =	= 0.2				
		1	n = 100		,		1	n = 500		
	$\hat{\mathbf{i}}^A$	$\hat{\mathbf{i}}^B_{\pm}$	$\hat{\mathbf{i}}_{*}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	$\hat{\mathbf{i}}^A$	$\hat{\mathbf{i}}_{\pm}^{B}$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$
U-S	0	0	0	3	0	0	0	0	0	0
T-S	0	995	17	996	941	0	1000	82	998	943
O-S	1000	5	983	1	59	1000	0	918	2	57
A-S	16.54	1	4.84	1	1.07	102.68	-	2.96	1	1.05
		n	a = 1000				r	n = 2000		
	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}_{*}^{C}$	$\hat{\mathbf{j}}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$	$\hat{\mathbf{j}}_{*}^{A}$	$\hat{\mathbf{j}}^B_*$	$\hat{\mathbf{j}}^C_*$	$\hat{j}_{*}^{(0)}$	$\hat{j}_{*}^{(5)}$
U-S	0	3	0	0	0	0	345	0	0	0
T-S	0	997	655	1000	946	0	655	992	1000	951
O-S	1000	0	345	0	54	1000	0	8	0	49
A-S	217.91	—	1.17	-	1.07	465.89	_	1	_	1.06
				$\alpha =$	0.4, c =	= 0.4				
		1	n = 100		, -		1	n = 500		
	Î.	$\hat{\mathbf{i}}^B_i$	$\hat{\mathbf{i}}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	î ^A	$\hat{\mathbf{i}}^B_{i}$	$\hat{\mathbf{i}}_{c}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$
U-S	J *	237	0			0	997	 	 	<u>J</u> *
T-S	0	658	0	39	505	0	3	0	1000	952
O-S	1000	105	1000	0	31	1000	0	1000	0	48
A-S	32.02	1.32	30.24	_	1.13	194.90	_	194.26	_	1.02
	-	- 	a = 1000		-		r	n = 2000		-
	î.A	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}^C$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$	î ^A	$\hat{\mathbf{i}}^B$	$\hat{\mathbf{i}}_{c}^{C}$	$\hat{i}_{*}^{(0)}$	$\hat{i}_{*}^{(5)}$
U-S	0	1000	0	0	0	0	1000	0	0	0
	-	0	0	999	937	0	0	0	999	939
T-S	0	0	0	000	001				ALC: 1 AL	
T-S O-S	0	0	1000	1	63	1000	0	1000	1	61
T-S O-S A-S	$ \begin{array}{c} 0 \\ 1000 \\ 395 \end{array} $	0 -	1000 394.98	1 1	63 1.02	1000 795	0	1000 795	1 1	61 1.05

Table 6: Selection times of the KOO methods with AIC, BIC, C_p thresholds and bootstrap methods under Settings (II) and (vi) based on 1,000 replications.

S2 Additional real data analysis

The second example is chemometrics data taken from Skagerberg et al. (1992) (we replaced the value 19203 with 1.9203 in the 37th observation). The data are taken from a simulation of a low-density tubular polyethylene reactor studying the relationship between polymer properties and the process. The predictor variables consist of 20 temperatures measured at equal distances along the low-density polyethylene reactor section, together with the wall temperature of the reactor and the solvent feed rate. The responses are the output characteristics of the polymers, including two molecular weights, two branching frequencies and the contents of two groups. This data set has been studied by Breiman and Friedman (1997) and Similä and Tikka (2007). Similar to Breiman and Friedman (1997), we log-transformed the response values because they are highly skewed to the right. In total, there are n = 56 observations with k = 22 predictor variables and p = 6 responses.

We present the scatterplot of $\{\mathcal{K}_j\}$ in descending order in Figure 1. We also indicate the critical values of KAIC, KBIC and KCp, and \hat{K}_0 , $\hat{K}_{0.05}$ estimated by Algorithm 1 with standard normal distribution and N =1,000. Since the dimension is relatively small, we recommend using a larger significance level ν to prevent under-specifying. It seems that the variables {22,3,4} are significant and variables {21,11} are potentially significant too. KAIC and KCp, however, select many more variables, which are likely to be spurious.



Figure 1: Scatterplots for the chemometrics dataset.

S3 Proofs of Theorems 1-4

In this appendix, we present the proofs of Theorems 1–4 under general distributions by random matrix theory. Before that, we first give some notation and preliminary results which will be used in the sequel frequently. For simplicity, we denote $\mathbf{M} = p^{-1} \mathbf{E}' \mathbf{Q} \mathbf{E}$ and $\mathbf{M}_l = \frac{1}{p} \mathbf{E}'_l \mathbf{Q} \mathbf{E}_l$, where \mathbf{E}_l is

the $n \times (p-1)$ submatrix of **E** with the *l*-th column removed. Denote by \mathbb{E}_l the conditional expectation given $\{\mathbf{e}_1, \ldots, \mathbf{e}_l\}$ and by $\mathbb{E}_0 = \mathbb{E}$ the unconditional expectation, where \mathbf{e}_i is the *n*-vector of the *i*-th column of **E**. Let $\mathbf{b} = \mathbf{\Sigma}^{-1/2} \mathbf{\Theta}_* \mathbf{X}'_* \mathbf{a}_1$ and \mathbf{b}_l be the p-1 sub-vector of **b** with the *l*-th entry b_l removed. Then we have

$$\mathbf{a}_1'\mathbf{Y}\mathbf{\Sigma}^{-1/2}(\mathbf{E}'\mathbf{Q}\mathbf{E})^{-1}\mathbf{\Sigma}^{-1/2}\mathbf{Y}'\mathbf{a}_1 = p^{-1}(\mathbf{b}' + \mathbf{a}_1'\mathbf{E})\mathbf{M}^{-1}(\mathbf{E}'\mathbf{a}_1 + \mathbf{b}).$$

Modifying the truncation argument of Bai et al. (2018), we can assume that the variables $\{e_{ij}, i = 1...n, j = 1...p\}$ satisfy the following additional condition:

$$|e_{ij}| < \eta_n \sqrt{n}, \quad \text{for all } i, j,$$
 (S3.1)

where $\eta_n \to 0$ slowly enough. By the theorem in the appendix of Bai and Silverstein (2004), we know for any positive constant $d < (1 - \sqrt{c})^2$ and any given t > 0, $\lambda_{\min}^{\frac{1}{n}\mathbf{E'E}} \stackrel{\mathbf{E}}{\to} (1 - \sqrt{c})^2$ and

$$\mathbb{P}(\lambda_{\min}^{\frac{1}{n}\mathbf{E}'\mathbf{E}} < d) = o(n^{-t}).$$

Moreover, by Theorem 1.2 in Bai and Silverstein (1999), we conclude that for any positive constant $d < (1 - \sqrt{c/(1-\alpha)})^2$ and any given t > 0, $\lambda_{\min}^{\frac{1}{n}\mathbf{E}'\mathbf{QE}} \xrightarrow{a.s.} (1 - \sqrt{c/(1-\alpha)})^2$ and

$$\mathbb{P}(\lambda_{\min}^{\frac{1}{n}\mathbf{E}'\mathbf{Q}\mathbf{E}} < d) = o(n^{-t}).$$

Denote

$$\beta_l = \frac{1}{p} \mathbf{e}'_l \mathbf{Q} \mathbf{e}_l - \frac{1}{p^2} \mathbf{e}'_l \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}'_l \mathbf{Q} \mathbf{e}_l$$

and

$$\beta_1^{tr} = \operatorname{tr}[\frac{1}{p}\mathbf{Q} - \frac{1}{p^2}\mathbf{Q}\mathbf{E}_l\mathbf{M}_l^{-1}\mathbf{E}_l'\mathbf{Q}] = \frac{n-k-p+1}{p}.$$

It follows that

$$\frac{1}{\beta_l} = \frac{1}{\beta_1^{tr}} - \frac{\xi_l}{\beta_l \beta_1^{tr}},\tag{S3.2}$$

where $\xi_l = \beta_l - \beta_1^{tr}$. By Lemma 7.2 in Bai and Yao (2005) (see Lemma 4), we have that for any $2 \le \ell \le \log(n)$,

$$\mathbb{E}|\xi_l|^{\ell} = O(p^{-1}\eta_n^{2\ell-4}), \tag{S3.3}$$

which indicate that ξ_l tends to 0 in probability with order of $o(n^{-t})$ for any t > 0. Analogously, for application later, together with the condition that $p^{-1/2}\mathbf{b}$ is bounded in Euclidean norm, we conclude that for $2 \le \ell \le \log(n)$,

$$\max\{\mathbb{E}|p^{-1}\mathbf{b}_{l}'\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}|^{\ell}, \mathbb{E}|p^{-1}\mathbf{a}_{l}\mathbf{E}_{l}'\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}|^{\ell}\} = O(p^{\ell/2-1}\eta_{n}^{\ell-2}) \quad (S3.4)$$

and

$$\max\{\mathbb{E}|p^{-2}\mathbf{b}_{l}'\boldsymbol{\Psi}_{l}\mathbf{b}_{l}|^{\ell}, \mathbb{E}|p^{-2}\mathbf{a}_{1}\mathbf{E}_{l}'\boldsymbol{\Psi}_{l}\mathbf{b}_{l}|^{\ell}, \mathbb{E}|p^{-2}\mathbf{a}_{1}\mathbf{E}_{l}'\boldsymbol{\Psi}_{l}\mathbf{E}_{l}\mathbf{a}_{1}|^{\ell}\} = O(p^{\ell-2}\eta_{n}^{2\ell-4})$$
(S3.5)

where

$$\boldsymbol{\Psi}_l = \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q} \mathbf{e}_l \mathbf{e}_l' \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} - \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1}.$$

As we only need to prove the weak convergence conclusion and $\beta_1^{tr} \rightarrow (1-\alpha-c)/c > 0$, thus throughout the proofs, we can safely assume $\|\mathbf{M}^{-1}\|$, $\|\mathbf{M}_l^{-1}\|$ and $|1/\beta_l|$ are all bounded for large n.

S3.1 Proof of Theorem 1

Theorem 1 can be obtained from Proposition 3.1 in Bai et al. (2022) with letting $z \downarrow 0$ directly. That is, for any non-random vectors \mathbf{r}_1 , $\mathbf{r}_2 \mathbf{r}_3$ and \mathbf{r}_4 with suitable dimensions and bounded in Euclidean norm, under conditions in Theorem 1, we have that for any t > 0 and $\varepsilon > 0$,

$$\mathbb{P}\left(\left|\mathbf{r}_{1}'\mathbf{M}^{-1}\mathbf{r}_{2} - \frac{c_{n}\mathbf{r}_{1}'\mathbf{r}_{2}}{1 - c_{n} - \alpha_{n}}\right| \ge \varepsilon\right) = o(n^{-t}), \qquad (S3.6)$$

$$\mathbb{P}\left(\left|\frac{1}{\sqrt{p}}\mathbf{r}_{1}'\mathbf{M}^{-1}\mathbf{E}'\mathbf{r}_{3}\right| \geq \varepsilon\right) = o(n^{-t}),\tag{S3.7}$$

and

$$\mathbb{P}\left(\left|\frac{1}{p}\mathbf{r}_{3}'\mathbf{E}\mathbf{M}^{-1}\mathbf{E}'\mathbf{r}_{4}-\frac{c_{n}\mathbf{r}_{3}'\mathbf{r}_{4}}{1-c_{n}-\alpha_{n}}+\frac{c_{n}^{2}\mathbf{r}_{3}'\mathbf{Q}\mathbf{r}_{4}}{(1-c_{n}-\alpha_{n})(1-\alpha_{n})}\right|\geq\varepsilon\right)=o(n^{-t}).$$
(S3.8)

Then the proof of Theorem 1 is complete.

S3.2 Proof of Theorem 2

For simple presentation, in the following we assume $\{1, \ldots, q\} \subset [k] \setminus \mathbf{j}_*$ and $j_i = i$. To prove Theorem 2, it is sufficient to show that for any non-null vector $\mathbf{h} = (h_1, \ldots, h_q)', \sqrt{p}[(\mathcal{K}_1, \ldots, \mathcal{K}_q)\mathbf{h} - \frac{c_n}{1-c_n-\alpha_n}\mathbf{1}'_q\mathbf{h}]$ converges weakly to a normal distribution with mean zero and variance $\frac{c^2}{(1-\alpha_n-c_n)^2}[\frac{2(1-\alpha_n)}{(1-\alpha_n-c_n)}\mathbf{h}'(\mathcal{A}'_q\mathcal{A}_q)^2\mathbf{h} + \tau\mathbf{h}'(\mathcal{A}_q \circ \mathcal{A}_q)'(\mathcal{A}_q \circ \mathcal{A}_q)\mathbf{h}]$, where $\mathcal{A}_q = (\mathbf{a}_1, \ldots, \mathbf{a}_q)$.

We split the proof of this theorem into two parts. First, we show the asymptotic normality of the sequence of random variables

$$\mathcal{M}_1^{(n)} := \sqrt{p}[(\mathcal{K}_1, \dots, \mathcal{K}_q)\mathbf{h} - \mathbb{E}(\mathcal{K}_1, \dots, \mathcal{K}_q)\mathbf{h}].$$

Second, we prove the non-random sequence

$$\mathcal{M}_2^{(n)} = \sqrt{p} [\mathbb{E}(\mathcal{K}_1, \dots, \mathcal{K}_q)\mathbf{h} - \frac{c_n \mathbf{1}'_q \mathbf{h}}{1 - c_n - \alpha_n}]$$

tends to zero. Note that for notational simplicity the superscript $^{(n)}$ in $\mathcal{M}_1^{(n)}$ and $\mathcal{M}_2^{(n)}$ are suppressed in the sequel.

We start to consider \mathcal{M}_1 . Let $\mathcal{H} = Diag(h_1, \ldots, h_q)$. It follows that

$$\mathcal{M}_{1} = p^{-1/2} \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \operatorname{tr}(\mathbf{E}\mathbf{M}^{-1}\mathbf{E}'\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}')$$
$$= p^{-1/2} \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \operatorname{tr}[(\mathbf{E}\mathbf{M}^{-1}\mathbf{E}' - \mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}')\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}'].$$

By the inversion formula of block matrix, we obtain

$$\mathbf{E}\mathbf{M}^{-1}\mathbf{E}' - \mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}' = \frac{1}{\beta_{l}p^{2}}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}\mathbf{e}_{l}'\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'$$
$$-\frac{1}{\beta_{l}p}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}\mathbf{e}_{l}' - \frac{1}{\beta_{l}p}\mathbf{e}_{l}\mathbf{e}_{l}'\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}' + \frac{\mathbf{e}_{l}\mathbf{e}_{l}'}{\beta_{l}}.$$
(S3.9)

Then, by the equation (S3.2), we can rewrite \mathcal{M}_1 as

$$\mathcal{M}_1 = \frac{1}{p^{1/2}\beta_1^{tr}} \sum_{l=1}^p \mathbb{E}_l(\mathbf{e}_l' \mathbf{\Gamma}_l \mathbf{e}_l - \mathrm{tr} \mathbf{\Gamma}_l) - \frac{1}{p^{1/2}(\beta_1^{tr})^2} \sum_{l=1}^p \mathbb{E}_l(\xi_l \mathrm{tr} \mathbf{\Gamma}_l) + \mathcal{M}_{10},$$

where

$$egin{aligned} & \mathbf{\Gamma}_l = p^{-2} \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}_l' \mathcal{A}_q \mathcal{H} \mathcal{A}_q' \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q} - p^{-1} \mathcal{A}_q \mathcal{H} \mathcal{A}_q' \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q} \ & - p^{-1} \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}_l \mathcal{A}_q \mathcal{H} \mathcal{A}_q' + \mathcal{A}_q \mathcal{H} \mathcal{A}_q' \end{aligned}$$

and

$$\mathcal{M}_{10} = -\sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l} (\mathbf{e}_{l}^{\prime} \Gamma_{l} \mathbf{e}_{l} - \operatorname{tr} \Gamma_{l})}{p^{1/2} (\beta_{1}^{tr})^{2}} + \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}^{2} \mathbf{e}_{l}^{\prime} \Gamma_{l} \mathbf{e}_{l}}{p^{1/2} \beta_{l} (\beta_{1}^{tr})^{2}}.$$

It follows from (S3.3) that

$$\mathbb{E}\left|\frac{1}{p^{1/2}}\sum_{l=1}^{p}\mathbb{E}_{l}(\xi_{l}\mathrm{tr}\boldsymbol{\Gamma}_{l})\right|^{2} = \frac{1}{p}\sum_{l=1}^{p}\mathbb{E}|\mathbb{E}_{l}(\xi_{l}\mathrm{tr}\boldsymbol{\Gamma}_{l})|^{2} = O(p^{-1}).$$

By (S3.4), (S3.5) and the BurkHölder's inequality (see Lemma 2) we have that $\mathcal{M}_{10} = o_p(1)$. Applying Lemma 2.7 in Bai and Silverstein (1998) (see Lemma 3), we have that

$$\mathbb{E}|\mathbb{E}_l(\mathbf{e}_l'\boldsymbol{\Gamma}_l\mathbf{e}_l - \mathrm{tr}\boldsymbol{\Gamma}_l)|^4 \le \mathbb{E}|\mathbf{e}_l'\boldsymbol{\Gamma}_l\mathbf{e}_l - \mathrm{tr}\boldsymbol{\Gamma}_l|^4 = O(p\eta_n^4)$$

which verifies the condition (ii) in Lemma 1. Thus, what we need is to obtain the limit of

$$\frac{1}{p(\beta_1^{tr})^2} \sum_{l=1}^p \mathbb{E}_{l-1} \{ \mathbb{E}_l [\mathbf{e}_l' \boldsymbol{\Gamma}_l \mathbf{e}_l - \mathrm{tr} \boldsymbol{\Gamma}_l] \}^2.$$

By Lemma 5, we have that

$$\mathbb{E}_{l-1}\{\mathbb{E}_{l}[\mathbf{e}_{l}'\mathbf{\Gamma}_{l}\mathbf{e}_{l}-\mathrm{tr}\mathbf{\Gamma}_{l}]\}^{2}=2\mathbb{E}_{l-1}\mathrm{tr}(\mathbb{E}_{l}\mathbf{\Gamma}_{l}\mathbb{E}_{l}\mathbf{\Gamma}_{l})+\tau\mathbb{E}_{l-1}\mathrm{tr}(\mathbb{E}_{l}\mathbf{\Gamma}_{l}\circ\mathbb{E}_{l}\mathbf{\Gamma}_{l}),$$

where \circ stands for the Hadamard product. Notice that

$$\operatorname{tr}(\mathbb{E}_{l}\Gamma_{l}\mathbb{E}_{l}\Gamma_{l}) = p^{-4}\operatorname{tr}(\mathbb{E}_{l}\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}'\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q})^{2}$$
(S3.10)
+ $p^{-2}\operatorname{2tr}[\mathbb{E}_{l}(\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}'\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q})\mathbb{E}_{l}(\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}')]$ (S3.11)

$$+\operatorname{tr}(\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}')^{2}.$$

Let $\widetilde{\mathbf{E}}_l$ be \mathbf{E}_l by replacing $\{\mathbf{e}_{l+1}, \ldots, \mathbf{e}_p\}$ with $\{\widetilde{\mathbf{e}}_{l+1}, \ldots, \widetilde{\mathbf{e}}_p\}$, where $\{\widetilde{\mathbf{e}}_i\}$ are i.i.d. copies of \mathbf{e}_1 . We define $\widetilde{\mathbf{M}}_l = \frac{1}{p} \widetilde{\mathbf{E}}'_l \mathbf{Q} \widetilde{\mathbf{E}}_l$, correspondingly. As \mathcal{H} is a diagonal matrix, thus we have that

$$\mathbb{E}_{l-1} \operatorname{tr}(\mathbb{E}_{l} \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \boldsymbol{\mathcal{A}}_{q} \boldsymbol{\mathcal{H}} \boldsymbol{\mathcal{A}}_{q}^{\prime} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \mathbf{Q})^{2}$$
$$= \mathbb{E}_{l} \operatorname{tr}[\boldsymbol{\mathcal{H}} \boldsymbol{\mathcal{A}}_{q}^{\prime} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \widetilde{\boldsymbol{\Xi}}_{l} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \boldsymbol{\mathcal{A}}_{q}]$$
$$= \mathbb{E}_{l} \sum_{i=1}^{q} h_{i} \mathbf{a}_{i}^{\prime} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \widetilde{\boldsymbol{\Xi}}_{l} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \mathbf{a}_{i},$$

where $\widetilde{\Xi}_{l} = \mathbf{Q}\widetilde{\mathbf{E}}_{l}\widetilde{\mathbf{M}}_{l}^{-1}\widetilde{\mathbf{E}}_{l}'\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}'\widetilde{\mathbf{E}}_{l}\widetilde{\mathbf{M}}_{l}^{-1}\widetilde{\mathbf{E}}_{l}'\mathbf{Q}$. By applying the inversion formula of block matrix to \mathbf{M}_{l} , similar to (S3.9), we have that

$$\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}' - \mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}' = \frac{1}{\beta_{lp}p^{2}}\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'\mathbf{Q}\mathbf{e}_{p}\mathbf{e}_{p}'\mathbf{Q}\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'$$
$$-\frac{1}{\beta_{lp}p}\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'\mathbf{Q}\mathbf{e}_{p}\mathbf{e}_{p}' - \frac{1}{\beta_{lp}p}\mathbf{e}_{p}\mathbf{e}_{p}'\mathbf{Q}\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}' + \frac{\mathbf{e}_{p}\mathbf{e}_{p}'}{\beta_{lp}}, \qquad (S3.12)$$

where \mathbf{E}_{li} is the $n \times (i-2)$ submatrix of \mathbf{E} with the columns $\{\mathbf{e}_l, \mathbf{e}_i, \dots, \mathbf{e}_p\}$ removed, $\mathbf{M}_{li} = \frac{1}{p} \mathbf{E}'_{li} \mathbf{Q} \mathbf{E}_{li}$ and

$$\beta_{li} = \frac{1}{p} \mathbf{e}'_i \mathbf{Q} \mathbf{e}_i - \frac{1}{p^2} \mathbf{e}'_i \mathbf{Q} \mathbf{E}_{li} \mathbf{M}_{li}^{-1} \mathbf{E}'_{li} \mathbf{Q} \mathbf{e}_i.$$
(S3.13)

Denote

$$\beta_i^{tr} = \operatorname{tr}\left[\frac{1}{p}\mathbf{Q} - \frac{1}{p^2}\mathbf{Q}\mathbf{E}_{li}\mathbf{M}_{li}^{-1}\mathbf{E}_{li}'\mathbf{Q}\right] = \frac{n-k-p+i}{p}$$

and

$$\xi_{li} = \beta_{li} - \beta_i^{tr}.$$

We can easily check that the orders of (S3.3)–(S3.5) hold for replacing the subscripts l by li. Thus, analogous to the above discussion, we have that

$$\mathbb{E}_{l}\mathbf{a}_{i}'\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\widetilde{\Xi}_{l}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{a}_{i}$$
$$=\mathbb{E}_{l}[\mathbf{a}_{i}'\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'\widetilde{\Xi}_{l}\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'\mathbf{a}_{i}] + o_{p}(p^{3})$$
$$=\mathbb{E}_{l}[\mathbf{a}_{i}'\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'\widetilde{\Xi}_{lp}\mathbf{E}_{lp}\mathbf{M}_{lp}^{-1}\mathbf{E}_{lp}'\mathbf{a}_{i}] + o_{p}(p^{3}),$$

where $\widetilde{\Xi}_{lp}$ is defined by removing b_p and $\tilde{\mathbf{e}}_p$ from $\widetilde{\Xi}_l$. We then repeat the procedure that remove b_i , \mathbf{e}_i and $\tilde{\mathbf{e}}_i$, $i = l + 1, \ldots, p - 1$ from Ξ_{lp} and $\widetilde{\Xi}_{lp}$,

respectively. Then applying Proposition 3.1 in (Bai et al., 2022), we finally obtain that

$$(\mathbf{S3.18}) = \sum_{i,j}^{q} h_i h_j p^{-2} [\mathbf{a}'_i \mathbf{E}_{l(l-1)} \mathbf{M}_{l(l-1)}^{-1} \mathbf{E}'_{l(l-1)} \mathbf{a}_j)]^2 + o_p(1)$$
$$= \frac{(l-1)^2}{(n-k-l+1)^2} \mathbf{h}' (\mathcal{A}'_q \mathcal{A}_q)^2 \mathbf{h} + o_p(1).$$
(S3.14)

Analogously, we have that

$$(\mathbf{S3.11}) = p^{-2} \sum_{i=1}^{q} h_i h_j \mathbb{E}_l(\mathbf{a}_i' \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q}) \mathbb{E}_l(\mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} \mathbf{E}_l \mathbf{a}_i)$$
$$= \frac{(l-1)\mathbf{h}'(\mathcal{A}_q' \mathcal{A}_q)^2 \mathbf{h}}{n-k-l+1} + o_p(1),$$

which together with (S3.14) and the fact that $\operatorname{tr}(\mathcal{A}_q \mathcal{H} \mathcal{A}'_q)^2 = \mathbf{h}'(\mathcal{A}'_q \mathcal{A}_q)^2 \mathbf{h}$ implies

$$\begin{split} \frac{1}{p} \sum_{l=1}^{p} \mathbb{E}_{l-1} \mathrm{tr} (\mathbb{E}_{l} \boldsymbol{\Gamma}_{l} \mathbb{E}_{l} \boldsymbol{\Gamma}_{l}) = & \frac{\mathbf{h}' (\boldsymbol{\mathcal{A}}'_{q} \boldsymbol{\mathcal{A}}_{q})^{2} \mathbf{h}}{p} \sum_{l=1}^{p} \left(\frac{l-1}{n-k-l+1} + 1 \right)^{2} + o_{p}(1) \\ = & \mathbf{h}' (\boldsymbol{\mathcal{A}}'_{q} \boldsymbol{\mathcal{A}}_{q})^{2} \mathbf{h} \frac{1-\alpha_{n}}{1-\alpha_{n}-c_{n}} + o_{p}(1). \end{split}$$

We now turn to prove the term $\mathbb{E}_{l-1} \operatorname{tr}(\mathbb{E}_l \Gamma_l \circ \mathbb{E}_l \Gamma_l) = o_p(1)$. Let \mathbf{u}_j be an *n*-dimensional column vector with the j-th element being 1 and 0 otherwise. Then we have that

$$\mathbb{E}(\mathbb{E}_{l-1}\operatorname{tr}(\mathbb{E}_{l}\boldsymbol{\Gamma}_{l}\circ\mathbb{E}_{l}\boldsymbol{\Gamma}_{l})) = \sum_{j=1}^{n} \mathbb{E}(\mathbf{u}_{j}'\mathbb{E}_{l}\boldsymbol{\Gamma}_{l}\mathbf{u}_{j})^{2}$$
$$= \sum_{j=1}^{n} (\mathbb{E}\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j})^{2} + \sum_{j=1}^{n} \mathbb{E}(\mathbb{E}_{l}\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j} - \mathbb{E}\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j})^{2}.$$

By BurkHölder's inequality, we have that

$$\mathbb{E}(\mathbb{E}_{l}\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j} - \mathbb{E}\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j})^{2} \leq \sum_{s\neq l}^{p} \mathbb{E}(\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j} - \mathbf{u}_{j}'\boldsymbol{\Gamma}_{l\cdot s}\mathbf{u}_{j})^{2},$$

where $\Gamma_{l \cdot s}$ is the submatrix of Γ_l with \mathbf{e}_s removed. Applying the inversion formula of block matrix (S3.12) again, we have that

$$\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}' = \mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}' + \frac{\mathbf{Q}\mathbf{e}_{s}\mathbf{e}_{s}'}{\beta_{l\cdot s}}$$

$$+ \frac{1}{\beta_{l\cdot s}p^{2}}\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'\mathbf{Q}\mathbf{e}_{s}\mathbf{e}_{s}'\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'$$

$$- \frac{1}{\beta_{l\cdot s}p}\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'\mathbf{Q}\mathbf{e}_{s}\mathbf{e}_{s}' - \frac{1}{\beta_{l\cdot s}p}\mathbf{Q}\mathbf{e}_{s}\mathbf{e}_{s}'\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'$$

$$:= \mathcal{U}_{ls0} + \mathcal{U}_{ls1} + \mathcal{U}_{ls2} - \mathcal{U}_{ls3} - \mathcal{U}_{ls4}$$
(S3.15)

and

$$\mathbb{E}(\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j} - \mathbf{u}_{j}'\boldsymbol{\Gamma}_{ls}\mathbf{u}_{j})^{2}$$

$$= \mathbb{E}\{p^{-4}\sum_{i=1}^{q}h_{i}[\mathbf{u}_{j}'(\boldsymbol{\mathcal{U}}_{ls1} + \boldsymbol{\mathcal{U}}_{ls2} - \boldsymbol{\mathcal{U}}_{ls3} - \boldsymbol{\mathcal{U}}_{ls4})\mathbf{a}_{i}]^{2}$$

$$+ 2p^{-2}\sum_{i=1}^{q}h_{i}\mathbf{u}_{j}'\boldsymbol{\mathcal{U}}_{ls0}\mathbf{a}_{i}\mathbf{u}_{j}'(\boldsymbol{\mathcal{U}}_{ls1} + \boldsymbol{\mathcal{U}}_{ls2} - \boldsymbol{\mathcal{U}}_{ls3} - \boldsymbol{\mathcal{U}}_{ls4})\mathbf{a}_{i}$$

$$- 2p^{-1}\sum_{i=1}^{q}h_{i}\mathbf{u}_{j}'\mathbf{a}_{i}\mathbf{u}_{j}'(\boldsymbol{\mathcal{U}}_{ls1} + \boldsymbol{\mathcal{U}}_{ls2} - \boldsymbol{\mathcal{U}}_{ls3} - \boldsymbol{\mathcal{U}}_{ls4})\mathbf{a}_{i}\}^{2}.$$

We first consider $\mathbb{E}(\mathbf{u}'_{j}\boldsymbol{\mathcal{U}}_{ls1}\mathbf{a}_{i}\mathbf{a}'_{i}\boldsymbol{\mathcal{U}}'_{ls1}\mathbf{u}_{j})^{2}$. Notice that

$$\begin{split} & \mathbb{E}(\mathbf{e}_{s}^{\prime}\mathbf{Q}\mathbf{u}_{j}\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{e}_{s}\mathbf{e}_{s}^{\prime}\mathbf{a}_{i}\mathbf{a}_{i}^{\prime}\mathbf{e}_{s})^{2} \\ = & \mathbb{E}[(\mathbf{e}_{s}^{\prime}\mathbf{Q}\mathbf{u}_{j}\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{e}_{s}-\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{u}_{j}+\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{u}_{j})(\mathbf{e}_{s}^{\prime}\mathbf{a}_{i}\mathbf{a}_{i}^{\prime}\mathbf{e}_{s}-1+1)]^{2} \\ = & \mathbb{E}[(\mathbf{e}_{s}^{\prime}\mathbf{Q}\mathbf{u}_{j}\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{e}_{s}-\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{u}_{j})(\mathbf{e}_{s}^{\prime}\mathbf{a}_{i}\mathbf{a}_{i}^{\prime}\mathbf{e}_{s}-1)+\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{u}_{j}(\mathbf{e}_{s}^{\prime}\mathbf{a}_{i}\mathbf{a}_{i}^{\prime}\mathbf{e}_{s}-1) \\ & +(\mathbf{e}_{s}^{\prime}\mathbf{Q}\mathbf{u}_{j}\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{e}_{s}-\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{u}_{j})+\mathbf{u}_{j}^{\prime}\mathbf{Q}\mathbf{u}_{j}]^{2}. \end{split}$$

From Lemma 4 we have that for $\ell \geq 2$,

$$\mathbb{E}(\mathbf{e}'_{s}\mathbf{Q}\mathbf{u}_{j}\mathbf{u}'_{j}\mathbf{Q}\mathbf{e}_{s}-\mathbf{u}'_{j}\mathbf{Q}\mathbf{u}_{j})^{\ell}=O(n^{\ell-1}\eta_{n}^{2\ell-4})$$

and

$$\mathbb{E}(\mathbf{e}'_{s}\mathbf{a}_{i}\mathbf{a}'_{i}\mathbf{e}_{s}-1)^{\ell}=O(n^{\ell-1}\eta_{n}^{2\ell-4}).$$

Then, together with the fact that $\frac{1}{\beta_{l\cdot s}}$ and $\mathbf{u}'_{j}\mathbf{Q}\mathbf{u}_{j}$ are both bounded, and the c_{r} -inequality, we obtain

$$\mathbb{E}(\mathbf{u}_{j}^{\prime}\boldsymbol{\mathcal{U}}_{ls1}\mathbf{a}_{i}\mathbf{a}_{i}^{\prime}\boldsymbol{\mathcal{U}}_{ls1}^{\prime}\mathbf{u}_{j})^{2}=O(n^{3}\eta_{n}^{-4}).$$

Next, we consider the term $\mathbb{E}(\mathbf{u}'_{j}\mathcal{U}_{ls0}\mathbf{a}_{i}\mathbf{a}'_{i}\mathcal{U}'_{ls1}\mathbf{u}_{j})^{2}$. It follows $\mathbf{a}'_{i}\mathbf{Q} = \mathbf{0}$ that

$$\mathbb{E}(\mathbf{u}_{j}'\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'\mathbf{a}_{i}\mathbf{a}_{i}'\mathbf{e}_{s}\mathbf{e}_{s}'\mathbf{Q}\mathbf{u}_{j})^{2}$$
$$=\mathbb{E}(\mathbf{e}_{s}'\mathbf{Q}\mathbf{u}_{j}\mathbf{u}_{j}'\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'\mathbf{a}_{i}\mathbf{a}_{i}'\mathbf{e}_{s}-\mathbf{a}_{i}'\mathbf{Q}\mathbf{u}_{j}\mathbf{u}_{j}'\mathbf{Q}\mathbf{E}_{l\cdot s}\mathbf{M}_{l\cdot s}^{-1}\mathbf{E}_{l\cdot s}'\mathbf{a}_{i})^{2}=O(1)$$

As other terms are analogous, thus by combining the above argument, we conclude that $\sum_{j=1}^{n} \mathbb{E}(\mathbb{E}_{l}\mathbf{u}'_{j}\boldsymbol{\Gamma}_{l}\mathbf{u}_{j} - \mathbb{E}\mathbf{u}'_{j}\boldsymbol{\Gamma}_{l}\mathbf{u}_{j})^{2} = o(1).$

For $\sum_{j=1}^{n} (\mathbb{E}\mathbf{u}'_{j}\mathbf{\Gamma}_{l}\mathbf{u}_{j})^{2}$, it follows from the assumption that $\{e_{ij}\}$ are i.i.d.,

$$\sum_{j=1}^{n} (\mathbb{E}\mathbf{u}_{j}'\boldsymbol{\Gamma}_{l}\mathbf{u}_{j})^{2} = \sum_{j=1}^{n} (n^{-1}p^{-1}\mathbb{E}\mathrm{tr}\mathcal{H}\mathcal{A}_{q}'\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathcal{A}_{q} + \mathbf{u}_{j}'\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}'\mathbf{u}_{j})^{2}$$
$$= \sum_{j=1}^{n} (\mathbf{u}_{j}'\mathcal{A}_{q}\mathcal{H}\mathcal{A}_{q}'\mathbf{u}_{j})^{2} + O(n^{-1})$$
$$= \mathbf{h}'(\mathcal{A}_{q} \circ \mathcal{A}_{q})'(\mathcal{A}_{q} \circ \mathcal{A}_{q})\mathbf{h} + O(n^{-1}).$$

Here we use a result similar to (S3.8), that is

$$\mathbb{E}\frac{1}{p}\mathbf{r}_{3}'\mathbf{E}\mathbf{M}^{-1}\mathbf{E}'\mathbf{r}_{4} - \frac{c_{n}\mathbf{r}_{3}'\mathbf{r}_{4}}{1 - c_{n} - \alpha_{n}} + \frac{c_{n}^{2}\mathbf{r}_{3}'\mathbf{Q}\mathbf{r}_{4}}{(1 - c_{n} - \alpha_{n})(1 - \alpha_{n})} \to 0,$$

and the proof can be found in the proof of Proposition 3.1 in Bai et al. (2022). Then we conclude that

$$\sum_{l=1}^{p} \mathbb{E}_{l-1} \operatorname{tr}(\mathbb{E}_{l} \boldsymbol{\Gamma}_{l} \circ \mathbb{E}_{l} \boldsymbol{\Gamma}_{l}) = \mathbf{h}'(\boldsymbol{\mathcal{A}}_{q} \circ \boldsymbol{\mathcal{A}}_{q})'(\boldsymbol{\mathcal{A}}_{q} \circ \boldsymbol{\mathcal{A}}_{q})\mathbf{h} + o_{p}(1).$$

Next, we will prove that the non-random sequence

$$\mathcal{M}_2 = \mathcal{M}_2^{(n)} = o(1).$$

Write $\mathbf{M}^{-1} = (M^{ij})$. Without loss of generality, we only need to prove $p^{-1}\mathbb{E}\mathbf{a}'_1\mathbf{E}\mathbf{M}^{-1}\mathbf{E}'\mathbf{a}_1 - \frac{c_n}{1-c_n-\alpha_n} = o(p^{-1/2})$. Because the entries of \mathbf{E} are i.i.d., we have

$$p^{-1}\mathbb{E}\mathbf{a}_{1}'\mathbf{E}\mathbf{M}^{-1}\mathbf{E}'\mathbf{a}_{1} = p^{-1}\sum_{i,j=1}^{p}\mathbb{E}\mathbf{a}_{1}'\mathbf{e}_{i}M^{ij}\mathbf{e}_{j}'\mathbf{a}_{1}$$
$$=\mathbb{E}\mathbf{e}_{1}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}M^{11} + (p-1)\mathbb{E}\mathbf{a}_{1}'\mathbf{e}_{1}M^{12}\mathbf{e}_{2}'\mathbf{a}_{1}.$$
(S3.16)

From the inverse matrix formula, we know that

$$M^{11} = \frac{1}{\beta_1} = \frac{1}{\beta_1^{tr}} - \frac{\xi_1}{(\beta_1^{tr})^2} + \frac{\xi_1^2}{\beta_1(\beta_1^{tr})^2}.$$

and

$$M^{12} = \frac{\mathbf{e}_1' \mathbf{Q} \mathbf{E}_1 \mathbf{M}_1^{-1} \mathbf{u}_1}{p \beta_1^{tr}} - \frac{\xi_1 \mathbf{e}_1' \mathbf{Q} \mathbf{E}_1 \mathbf{M}_1^{-1} \mathbf{u}_1}{p (\beta_1^{tr})^2} + \frac{\xi_1^2 \mathbf{e}_1' \mathbf{Q} \mathbf{E}_1 \mathbf{M}_1^{-1} \mathbf{u}_1}{p \beta_1 (\beta_1^{tr})^2}.$$
 (S3.17)

Then it follows from (S3.2), (S3.3) and the Hölder's inequality that

$$\mathbb{E}\mathbf{e}_{1}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}M^{11} - \frac{c_{n}}{1 - c_{n} - \alpha_{n}} = \mathbb{E}\frac{\mathbf{e}_{1}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}\xi_{1}^{2}}{\beta_{1}(\beta_{1}^{tr})^{2}} = o(p^{-1/2}).$$

Moreover, substituting (S3.17) into the second term of (S3.16), we have three terms. The first one is

$$\mathbb{E}\frac{\mathbf{e}_{2}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{E}_{1}\mathbf{M}_{1}^{-1}\mathbf{u}_{1}}{p\beta_{1}^{tr}} = \mathbb{E}\frac{\mathbf{e}_{2}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{Q}\mathbf{E}_{1}\mathbf{M}_{1}^{-1}\mathbf{u}_{1}}{p\beta_{1}^{tr}} = 0,$$

because of $\mathbf{a}_1'\mathbf{Q} = \mathbf{0}$. Applying the inversion formula to \mathbf{M}_1^{-1} again, we obtain that

$$\mathbb{E} \frac{\xi_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{E}_{1}\mathbf{M}_{1}^{-1}\mathbf{u}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{2}'\mathbf{a}_{1}}{p}$$

=
$$\mathbb{E} \frac{\xi_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{e}_{2}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{2}'\mathbf{a}_{1}}{\beta_{1\cdot2}p} - \mathbb{E} \frac{\xi_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{E}_{1\cdot2}\mathbf{M}_{1\cdot2}^{-1}\mathbf{E}_{1\cdot2}'\mathbf{Q}\mathbf{e}_{2}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{2}'\mathbf{a}_{1}}{\beta_{1\cdot2}p^{2}}$$

=
$$\mathbb{E} \frac{\xi_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{e}_{2}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{2}'\mathbf{a}_{1}}{\beta_{1\cdot2}p}.$$

Rewrite $1/\beta_{1\cdot 2}$ as

$$\frac{1}{\beta_{1\cdot 2}} = \frac{1}{\beta_2^{tr}} - \frac{\xi_{1\cdot 2}}{\beta_{1\cdot 2}\beta_2^{tr}}$$

and by the fact that $\mathbf{Qa}_1 = \mathbf{0}$, we have that

$$\mathbb{E}\frac{\xi_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{e}_{2}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{2}'\mathbf{a}_{1}}{\beta_{1\cdot 2}} = -\mathbb{E}\frac{\xi_{1\cdot 2}\xi_{1}\mathbf{e}_{1}'\mathbf{Q}\mathbf{e}_{2}\mathbf{a}_{1}'\mathbf{e}_{1}\mathbf{e}_{2}'\mathbf{a}_{1}}{\beta_{2}^{tr}\beta_{1\cdot 2}} = o(p^{-1/2}).$$

Therefore, by combining the above results, we conclude that

$$\mathcal{M}_2 = o(1),$$

and we complete the proof of the theorem.

S3.3 Proof of Theorem 3

Note that

$$\operatorname{tr}[(\mathbf{Y}'\mathbf{Q}\mathbf{Y} - (n-k)\mathbf{I}) \circ (\mathbf{Y}'\mathbf{Q}\mathbf{Y} - (n-k)\mathbf{I})] = \sum_{i=1}^{p} (\mathbf{e}'_{i}\mathbf{Q}\mathbf{e}_{i} - (n-k))^{2}$$

and

$$\mathbb{E}(\mathbf{e}'_{i}\mathbf{Q}\mathbf{e}_{i}-(n-k))^{2}=2(n-k)+\tau\mathrm{tr}(\mathbf{Q}\circ\mathbf{Q}).$$

Thus by the definition of $\hat{\tau}$ and $\{\mathbf{e}_i\}$ are i.i.d., we have $\mathbb{E}\hat{\tau} = \tau$. Next we will show that

$$\mathbb{E}(\hat{\tau} - \tau)^2 \to 0.$$

It follows from (S3.1) and Lemma 3 that

$$\begin{split} & \mathbb{E}(\hat{\tau} - \tau)^2 = \mathbb{E}(\hat{\tau} - \mathbb{E}\hat{\tau})^2 \\ &= p^{-1}\mathbb{E}[(\mathbf{e}_1'\mathbf{Q}\mathbf{e}_1 - (n-k))^2 - \mathbb{E}(\mathbf{e}_1'\mathbf{Q}\mathbf{e}_1 - (n-k))]^2/\mathrm{tr}^2(\mathbf{Q}\circ\mathbf{Q}) \\ &= \{p^{-1}\mathbb{E}(\mathbf{e}_1'\mathbf{Q}\mathbf{e}_1 - (n-k))^4 - p^{-1}[\mathbb{E}(\mathbf{e}_1'\mathbf{Q}\mathbf{e}_1 - (n-k))]^2\}/\mathrm{tr}^2(\mathbf{Q}\circ\mathbf{Q}) \\ &\leq Kp^{-1}[((n-k)^2 + n^2(n-k)\eta_n^4 + (n-k)^2 + \tau^2\mathrm{tr}^2(\mathbf{Q}\circ\mathbf{Q})]/\mathrm{tr}^2(\mathbf{Q}\circ\mathbf{Q}), \end{split}$$

where K is a positive constant. By c_r -inequality, we have that $\operatorname{tr}(\mathbf{Q} \circ \mathbf{Q}) \geq n^{-1}(n-k)^2$, which together with condition (C1) implies $\mathbb{E}(\hat{\tau} - \tau)^2 \to 0$. Then we complete the proof of this theorem.

S3.4 Proof of Theorem 4

For simple presentation, in the following we assume $\{1\} \subset \mathbf{j}_*$ and let j = 1. Then by the notation $\mathbf{b} = \Sigma^{-1/2} \Theta_* \mathbf{X}'_* \mathbf{a}_1$, $\mathcal{K}_1 = p^{-1} (\mathbf{b}' + \mathbf{a}'_1 \mathbf{E}) \mathbf{M}^{-1} (\mathbf{E}' \mathbf{a}_1 + \mathbf{b})$. Note that the proof procedure of Theorem 4 is the same as that of Theorem 2. And the difference is that Theorem 4 requires the consideration of linear combinations of three different forms of random variables, namely $\mathbf{a}'_1 \mathbf{E} \mathbf{M}^{-1} \mathbf{E}' \mathbf{a}_1$, $\mathbf{a}'_1 \mathbf{E} \mathbf{M}^{-1} \mathbf{b}$ and $\mathbf{b}' \mathbf{M}^{-1} \mathbf{b}$. As the asymptotic normality of $\mathbf{a}'_1 \mathbf{E} \mathbf{M}^{-1} \mathbf{E}' \mathbf{a}_1$ is proved in last subsection, in the sequel we only focus on the other two terms and their correlations.

Analogously, we split the proof of this theorem into two parts. It is

worthy noting that next we may use the same notation as in the proof of Theorem 2, but they represent a little different content. First, we show the asymptotic normality of the sequence of random variables

$$\mathcal{M}_3 := \sqrt{p}[p^{-1}(\mathbf{b}' + \mathbf{a}'_1 \mathbf{E})\mathbf{M}^{-1}(\mathbf{E}'\mathbf{a}_1 + \mathbf{b}) - \mathbb{E}p^{-1}(\mathbf{b}' + \mathbf{a}'_1 \mathbf{E})\mathbf{M}^{-1}(\mathbf{E}'\mathbf{a}_1 + \mathbf{b})].$$

Second, we prove the non-random sequence

$$\mathcal{M}_4 = \sqrt{p} [\mathbb{E}p^{-1}(\mathbf{b}' + \mathbf{a}_1'\mathbf{E})\mathbf{M}^{-1}(\mathbf{E}'\mathbf{a}_1 + \mathbf{b}) - \frac{c_n(1+\delta_1)}{1-c_n-\alpha_n}]$$

tends to zero. It follows that

$$\mathcal{M}_{1} = p^{-1/2} \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) (\mathbf{b}' + \mathbf{a}_{1}' \mathbf{E}) \mathbf{M}^{-1} (\mathbf{E}' \mathbf{a}_{1} + \mathbf{b})$$

= $p^{-1/2} \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) [(\mathbf{b}' + \mathbf{a}_{1}' \mathbf{E}) \mathbf{M}^{-1} (\mathbf{E}' \mathbf{a}_{1} + \mathbf{b}) - (\mathbf{b}_{l}' + \mathbf{a}_{1}' \mathbf{E}_{l}) \mathbf{M}_{l}^{-1} (\mathbf{E}_{l}' \mathbf{a}_{1} + \mathbf{b}_{l})].$

By the inversion formula of block matrix, we obtain

$$\begin{aligned} (\mathbf{b}' + \mathbf{a}_1'\mathbf{E})\mathbf{M}^{-1}(\mathbf{E}'\mathbf{a}_1 + \mathbf{b}) &- (\mathbf{b}_l' + \mathbf{a}_1'\mathbf{E}_l)\mathbf{M}_l^{-1}(\mathbf{E}_l'\mathbf{a}_1 + \mathbf{b}_l) \\ = & \frac{1}{\beta_l p^2} (\mathbf{b}_l' + \mathbf{a}_1'\mathbf{E}_l)\mathbf{M}_l^{-1}\mathbf{E}_l'\mathbf{Q}\mathbf{e}_l\mathbf{e}_l'\mathbf{Q}\mathbf{E}_l\mathbf{M}_l^{-1}(\mathbf{E}_l'\mathbf{a}_1 + \mathbf{b}_l) \\ &- \frac{2}{\beta_l p} (\mathbf{b}_l' + \mathbf{a}_1'\mathbf{E}_l)\mathbf{M}_l^{-1}\mathbf{E}_l'\mathbf{Q}\mathbf{e}_l(\mathbf{e}_l'\mathbf{a}_1 + b_l) + \frac{(\mathbf{e}_l'\mathbf{a}_1 + b_l)^2}{\beta_l}. \end{aligned}$$

Then, by the equation (S3.2), we can rewrite \mathcal{M}_3 as

$$\mathcal{M}_{3} = \frac{1}{p^{1/2}\beta_{1}^{tr}} \sum_{l=1}^{p} \mathbb{E}_{l} [\mathbf{e}_{l}' \boldsymbol{\Gamma}_{l} \mathbf{e}_{l} - \mathrm{tr} \boldsymbol{\Gamma}_{l} + 2\mathbf{e}_{l}' \boldsymbol{\gamma}_{l}] + \mathcal{M}_{30},$$

where

$$\Gamma_l = p^{-2} \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} (\mathbf{b}_l + \mathbf{E}_l' \mathbf{a}_1) (\mathbf{b}_l + \mathbf{E}_l' \mathbf{a}_1)' \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q}$$
$$- p^{-1} \mathbf{a}_1 (\mathbf{b}_l + \mathbf{E}_l' \mathbf{a}_1)' \mathbf{M}_l^{-1} \mathbf{E}_l' \mathbf{Q} - p^{-1} \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} (\mathbf{b}_l + \mathbf{E}_l' \mathbf{a}_1) \mathbf{a}_1' + \mathbf{a}_1 \mathbf{a}_1',$$

$$\boldsymbol{\gamma}_l = -p^{-1}b_l \mathbf{Q} \mathbf{E}_l \mathbf{M}_l^{-1} (\mathbf{b}_l + \mathbf{E}_l' \mathbf{a}_1) + \mathbf{a}_1 b_l$$

and

$$\mathcal{M}_{30} = -\sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l} (\mathbf{b}_{l}' + \mathbf{a}_{1}' \mathbf{E}_{l}) \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{Q} \mathbf{e}_{l} \mathbf{e}_{l}' \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} (\mathbf{E}_{l}' \mathbf{a}_{1} + \mathbf{b}_{l})}{p^{5/2} \beta_{l} \beta_{1}^{tr}} + 2\sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l} (\mathbf{b}_{l}' + \mathbf{a}_{1}' \mathbf{E}_{l}) \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{Q} \mathbf{e}_{l} (\mathbf{e}_{l}' \mathbf{a}_{1} + b_{l})}{p^{3/2} \beta_{l} \beta_{1}^{tr}} - \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l} (\mathbf{e}_{l}' \mathbf{a}_{1} + b_{l})^{2}}{p^{1/2} \beta_{l} \beta_{1}^{tr}} := -\mathcal{M}_{301} + 2\mathcal{M}_{302} - \mathcal{M}_{303}.$$

Next we will prove $\mathcal{M}_{10} = o_p(1)$. Substitute (S3.2) into \mathcal{M}_{101} , \mathcal{M}_{102} and

 \mathcal{M}_{103} respectively, we then have that

$$\mathcal{M}_{301} = \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}(\mathbf{b}_{l}' + \mathbf{a}_{1}'\mathbf{E}_{l})\Psi_{l}(\mathbf{E}_{l}'\mathbf{a}_{1} + \mathbf{b}_{l})}{p^{5/2}(\beta_{1}^{tr})^{2}} + \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}(\mathbf{b}_{l}' + \mathbf{a}_{1}'\mathbf{E}_{l})\mathbf{M}_{l}^{-1}(\mathbf{E}_{l}'\mathbf{a}_{1} + \mathbf{b}_{l})}{p^{3/2}(\beta_{1}^{tr})^{2}} \\ - \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}^{2}(\mathbf{b}_{l}' + \mathbf{a}_{1}'\mathbf{E}_{l})\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}\mathbf{e}_{l}'\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}(\mathbf{E}_{l}'\mathbf{a}_{1} + \mathbf{b}_{l})}{p^{5/2}\beta_{l}(\beta_{1}^{tr})^{2}}, \\ \mathcal{M}_{302} = \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}(\mathbf{b}_{l}' + \mathbf{a}_{1}'\mathbf{E}_{l})\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}(\mathbf{e}_{l}'\mathbf{a}_{1} + b_{l})}{p^{3/2}(\beta_{1}^{tr})^{2}} \\ - \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}^{2}(\mathbf{b}_{l}' + \mathbf{a}_{1}'\mathbf{E}_{l})\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}(\mathbf{e}_{l}'\mathbf{a}_{1} + b_{l})}{p^{3/2}(\beta_{1}^{tr})^{2}}, \\ \mathcal{M}_{303} = \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}^{2}(\mathbf{b}_{l}' + \mathbf{a}_{1}'\mathbf{E}_{l})\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{e}_{l}(\mathbf{e}_{l}'\mathbf{a}_{1} + b_{l})}{p^{3/2}(\beta_{1}^{tr})^{2}}, \\ \mathcal{M}_{303} = \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}(\mathbf{e}_{l}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{l} - 1 + 2b_{l}\mathbf{a}_{1}'\mathbf{e}_{l} + b_{l}^{2} + 1)}{p^{1/2}(\beta_{1}^{tr})^{2}} - \sum_{l=1}^{p} (\mathbb{E}_{l} - \mathbb{E}_{l-1}) \frac{\xi_{l}^{2}(\mathbf{e}_{l}'\mathbf{a}_{1} + b_{l})^{2}}{p^{1/2}\beta_{l}(\beta_{1}^{tr})^{2}}.$$

These together with (S3.4), (S3.5) and the BurkHölder's inequality (see Lemma 2) implies that $\mathcal{M}_{10} = o_p(1)$. Note that here we used the fact $\mathbf{Qa}_1 = \mathbf{0}$.

Applying Lemma 3, we have that

$$\mathbb{E}|\mathbb{E}_l(\mathbf{e}_l'\boldsymbol{\Gamma}_l\mathbf{e}_l - \mathrm{tr}\boldsymbol{\Gamma}_l)|^4 \le \mathbb{E}|\mathbf{e}_l'\boldsymbol{\Gamma}_l\mathbf{e}_l - \mathrm{tr}\boldsymbol{\Gamma}_l|^4 = O(p\eta_n^4)$$

and

$$\mathbb{E}|\mathbb{E}_{l}(\mathbf{e}_{l}'\boldsymbol{\gamma}_{l})|^{4} \leq \mathbb{E}|\mathbf{e}_{l}'\boldsymbol{\gamma}_{l}\boldsymbol{\gamma}_{l}^{*}\mathbf{e}_{l}|^{2} = O(p^{-2}b_{l}^{4}),$$

which verify the condition (ii) in Lemma 1. Thus, what we need is to obtain the limit of

$$\frac{1}{p(\beta_1^{tr})^2} \sum_{l=1}^p \mathbb{E}_{l-1} \{ \mathbb{E}_l [\mathbf{e}_l' \boldsymbol{\Gamma}_l \mathbf{e}_l - \mathrm{tr} \boldsymbol{\Gamma}_l + 2\mathbf{e}_l' \boldsymbol{\gamma}_l] \}^2.$$

By Lemma 5, we have that

$$\mathbb{E}_{l-1} \{ \mathbb{E}_{l} [\mathbf{e}_{l}^{\prime} \boldsymbol{\Gamma}_{l} \mathbf{e}_{l} - \operatorname{tr} \boldsymbol{\Gamma}_{l} + 2 \mathbf{e}_{l}^{\prime} \boldsymbol{\gamma}_{l}] \}^{2}$$

$$= 2 \mathbb{E}_{l-1} \operatorname{tr} (\mathbb{E}_{l} \boldsymbol{\Gamma}_{l} \mathbb{E}_{l} \boldsymbol{\Gamma}_{l}) + 4 \mathbb{E}_{l-1} (\mathbb{E}_{l} \boldsymbol{\gamma}_{l} \mathbb{E}_{l} \boldsymbol{\gamma}_{l}^{\prime})$$

$$+ \tau \mathbb{E}_{l-1} \operatorname{tr} (\mathbb{E}_{l} \boldsymbol{\Gamma}_{l} \circ \mathbb{E}_{l} \boldsymbol{\Gamma}_{l}) + 4 \mathbb{E} e_{11}^{3} \mathbb{E}_{l-1} \operatorname{tr} (\mathbb{E}_{l} \boldsymbol{\Gamma}_{l} \circ \mathbb{E}_{l} \boldsymbol{\gamma}_{l} \mathbf{1}^{\prime}).$$

Notice that

$$\operatorname{tr}(\mathbb{E}_{l}\Gamma_{l}\mathbb{E}_{l}\Gamma_{l}) = p^{-4}\operatorname{tr}(\mathbb{E}_{l}\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}(\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})(\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})'\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q})^{2}$$

$$(S3.18)$$

$$+ p^{-2}2\mathbb{E}_{l}((\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})'\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q})\mathbb{E}_{l}(\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}(\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})) + 1.$$

In the proof of Theorem 2, we have shown that

$$p^{-4} \operatorname{tr}(\mathbb{E}_{l} \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{a}_{1} \mathbf{a}_{1} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{Q})^{2}$$
$$+ p^{-2} 2 \mathbb{E}_{l} (\mathbf{a}_{1} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{Q}) \mathbb{E}_{l} (\mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{a}_{1}) + 1$$
$$= \left(\frac{l-1}{n-k-l+1} + 1\right)^{2} + o_{p}(1).$$

By the same procedure and the assumptions in Theorem 4, we can also have that

$$p^{-4} \operatorname{tr}(\mathbb{E}_{l} \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{b}_{l} \mathbf{b}_{l}' \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{Q})^{2} = \left(\frac{\sum_{i=1}^{l-1} b_{i}^{2}}{n-k-l+1}\right)^{2} + o_{p}(1),$$
$$p^{-4} \operatorname{tr}(\mathbb{E}_{l} \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{b}_{l} \mathbf{a}_{1} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} \mathbf{E}_{l}' \mathbf{Q})^{2} = \frac{(l-1) \sum_{i=1}^{l-1} b_{i}^{2}}{n-k-l+1} + o_{p}(1),$$

$$p^{-2}\mathbb{E}_l(\mathbf{b}_l\mathbf{M}_l^{-1}\mathbf{E}_l'\mathbf{Q})\mathbb{E}_l(\mathbf{Q}\mathbf{E}_l\mathbf{M}_l^{-1}\mathbf{E}_l'\mathbf{a}_1) = o_p(1),$$

and

$$p^{-2}\mathbb{E}_{l}(\mathbf{b}_{l}\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q})\mathbb{E}_{l}(\mathbf{Q}\mathbf{E}_{l}\mathbf{M}_{l}^{-1}\mathbf{b}_{l}) = \frac{\sum_{i=1}^{l-1}b_{i}^{2}}{n-k-l+1} + o_{p}(1),$$

which together with (S3.18) and (S3.14) implies

$$\frac{1}{p}\sum_{l=1}^{p}\mathbb{E}_{l-1}\mathrm{tr}(\mathbb{E}_{l}\boldsymbol{\Gamma}_{l}\mathbb{E}_{l}\boldsymbol{\Gamma}_{l}) = \frac{1}{p}\sum_{l=1}^{p}\left(\frac{(l-1+\sum_{i=1}^{l-1}b_{i}^{2})}{n-k-l+1}+1\right)^{2} + o_{p}(1).$$

For $\mathbb{E}_l \gamma_l \mathbb{E}_l \gamma'_l$, by the notation $\widetilde{\mathbf{M}}_l = \frac{1}{p} \widetilde{\mathbf{E}}'_l \mathbf{Q} \widetilde{\mathbf{E}}_l$, we have that

$$\begin{split} & \mathbb{E}_{l} \boldsymbol{\gamma}_{l}^{\prime} \mathbb{E}_{l} \boldsymbol{\gamma}_{l} \\ &= \mathbb{E}_{l} [(p^{-1}b_{l} \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} (\mathbf{b}_{l} + \mathbf{E}_{l}^{\prime} \mathbf{a}_{1}) - \mathbf{a}_{1} b_{l})^{\prime} ((p^{-1}b_{l} \mathbf{Q} \mathbf{E}_{l} \mathbf{M}_{l}^{-1} (\mathbf{b}_{l} + \mathbf{E}_{l}^{\prime} \mathbf{a}_{1}) - \mathbf{a}_{1} b_{l})] \\ &= \mathbb{E}_{l} [p^{-2}b_{l}^{2} (\mathbf{b}_{l} + \mathbf{a}_{1}^{\prime} \mathbf{E}_{l}) \mathbf{M}_{l}^{-1} \mathbf{E}_{l}^{\prime} \mathbf{Q} \widetilde{\mathbf{E}}_{l} \widetilde{\mathbf{M}}_{l}^{-1} (\mathbf{b}_{l} + \widetilde{\mathbf{E}}_{l}^{\prime} \mathbf{a}_{1}) + b_{l}^{2}]. \end{split}$$

Applying the inversion formula of block matrix (S3.12) again, we obtain that

$$p^{-1}\sum_{l=1}^{p} \mathbb{E}_{l-1}(\mathbb{E}_{l}\boldsymbol{\gamma}_{l}^{\prime}\mathbb{E}_{l}\boldsymbol{\gamma}_{l}) = p^{-1}\sum_{l=1}^{p} b_{l}^{2} \left(\frac{(l-1+\sum_{i=1}^{l-1}b_{i}^{2})}{n-k-l+1}+1\right) + o_{p}(1).$$

Then by applying Lemma 6, we have that as $n \to \infty$,

$$\begin{split} &\frac{1}{p}\sum_{l=1}^{p} \mathbb{E}_{l-1}(\operatorname{tr}(\mathbb{E}_{l}\Gamma_{l}\mathbb{E}_{l}\Gamma_{l}) + 2\mathbb{E}_{l}\gamma_{l}\mathbb{E}_{l}\gamma_{l}') \\ &= \frac{1}{p}\sum_{l=1}^{p}\left(\left(\frac{l-1+\sum_{i=1}^{l-1}b_{i}^{2}}{n-k-l+1} + 1\right)^{2} + 2b_{l}^{2}\left(\frac{l-1+\sum_{i=1}^{l-1}b_{i}^{2}}{n-k-l+1} + 1\right)\right) + o_{p}(1) \\ &= \frac{n}{p}\int_{0}^{c_{n}}\left[\left(\frac{t(1+\delta_{1})}{1-\alpha_{n}-t} + 1\right)^{2} + 2\delta_{1}\left(\frac{t(1+\delta_{1})}{1-\alpha_{n}-t} + 1\right)\right]dt + o_{p}(1) \\ &= \frac{(1-\alpha_{n})(1+2\delta_{1}) + c_{n}\delta_{1}^{2}}{1-\alpha_{n}-c_{n}} + o_{p}(1). \end{split}$$

We now turn to the term \mathbb{E}_{l-1} tr $(\mathbb{E}_l\Gamma_l \circ \mathbb{E}_l\Gamma_l)$. By the notation that \mathbf{u}_j is an *n*-dimensional column vector with the j-th element being 1 and 0 otherwise and repeating the same argument in the proof of Theorem 2, we can obtain that

$$\mathbb{E}_{l-1} \operatorname{tr}(\mathbb{E}_{l} \Gamma_{l} \circ \mathbb{E}_{l} \Gamma_{l}) = \sum_{j=1}^{n} (\mathbb{E} \mathbf{u}_{j}' \Gamma_{l} \mathbf{u}_{j})^{2} + o_{p}(1).$$

As $\{e_{ij}\}$ are i.i.d., thus from the assumptions of this theorem, we have that

$$\sum_{j=1}^{n} (\mathbb{E}\mathbf{u}_{j}'\mathbf{\Gamma}_{l}\mathbf{u}_{j})^{2}$$

=
$$\sum_{j=1}^{n} (n^{-1}p^{-1}\mathbb{E}(\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})'\mathbf{M}_{l}^{-1}(\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})$$

$$- 2p^{-1}\mathbf{u}_{j}'\mathbf{a}_{1}\mathbb{E}(\mathbf{b}_{l} + \mathbf{E}_{l}'\mathbf{a}_{1})'\mathbf{M}_{l}^{-1}\mathbf{E}_{l}'\mathbf{Q}\mathbf{u}_{j} + (\mathbf{u}_{j}'\mathbf{a}_{1})^{2}]^{2}$$

= $o(1).$

Then we conclude that

$$\frac{1}{p}\sum_{l=1}^{p}\mathbb{E}_{l-1}\mathrm{tr}(\mathbb{E}_{l}\boldsymbol{\Gamma}_{l}\circ\mathbb{E}_{l}\boldsymbol{\Gamma}_{l})=o_{p}(1).$$

Next, we will prove that the non-random sequence

$$\mathcal{M}_4 = o(1).$$

By the notation $\mathbf{M}^{-1} = (M^{ij})$ and $\{e_{ij}\}$ are i.i.d., we have that

$$\mathbb{E}(\mathbf{b}' + \mathbf{a}_{1}'\mathbf{E})\mathbf{M}^{-1}(\mathbf{E}'\mathbf{a}_{1} + \mathbf{b}) = \sum_{i,j=1}^{p} \mathbb{E}(b_{i} + \mathbf{a}_{1}'\mathbf{e}_{i})M^{ij}(b_{j} + \mathbf{a}_{1}'\mathbf{e}_{j})$$

= $\mathbf{b}'\mathbf{b}\mathbb{E}M^{11} + p\mathbb{E}\mathbf{e}_{1}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}M^{11} + 2\mathbb{E}\mathbf{a}_{1}'\mathbf{e}_{1}M^{11}\sum_{i=1}^{p}b_{i}$
+ $\sum_{i\neq j}^{p}b_{i}b_{j}\mathbb{E}M^{12} + p(p-1)\mathbb{E}\mathbf{a}_{1}'\mathbf{e}_{1}M^{12}\mathbf{e}_{2}'\mathbf{a}_{1} + 2(p-1)\mathbb{E}\mathbf{a}_{1}'\mathbf{e}_{1}M^{12}\sum_{i=1}^{p}b_{i}.$

From the inverse matrix formula, we know that

$$M^{11} = \frac{1}{\beta_1} = \frac{1}{\beta_1^{tr}} - \frac{\xi_1}{(\beta_1^{tr})^2} + \frac{\xi_1^2}{\beta_1(\beta_1^{tr})^2}.$$

and

$$M^{12} = \frac{\mathbf{e}_1' \mathbf{Q} \mathbf{E}_1 \mathbf{M}_1^{-1} \mathbf{u}_1}{p \beta_1^{tr}} - \frac{\xi_1 \mathbf{e}_1' \mathbf{Q} \mathbf{E}_1 \mathbf{M}_1^{-1} \mathbf{u}_1}{p (\beta_1^{tr})^2} + \frac{\xi_1^2 \mathbf{e}_1' \mathbf{Q} \mathbf{E}_1 \mathbf{M}_1^{-1} \mathbf{u}_1}{p \beta_1 (\beta_1^{tr})^2}.$$

Then it follows from (S3.2), (S3.3) and the Hölder's inequality that

$$p^{-1}\mathbf{b}'\mathbf{b}\mathbb{E}M^{11} - \frac{c_n\delta_1}{1 - c_n - \alpha_n} = o(p^{-1/2})$$

and

$$\mathbb{E}\mathbf{e}_{1}'\mathbf{a}_{1}\mathbf{a}_{1}'\mathbf{e}_{1}M^{11} - \frac{c_{n}}{1 - c_{n} - \alpha_{n}} = o(p^{-1/2}).$$

By the facts that

$$|\mathbb{E}\mathbf{a}_1'\mathbf{e}_1\xi_1| \le \sqrt{\mathbb{E}|\xi_1|^2} = O(p^{-1/2})$$

and

$$\sum_{i=1}^{p} b_i | = O(p^{1/2}),$$

we can obtain that

$$\mathbb{E}\mathbf{a}_1'\mathbf{e}_1 M^{11} \sum_{i=1}^p b_i = O(1).$$

It follows from

$$|p^{-1}\sum_{i\neq j}^{p}b_ib_j| = O(1), |\mathbb{E}\xi_1\mathbf{e}_1'\mathbf{Q}\mathbf{E}_1\mathbf{M}_1^{-1}\mathbf{u}_1| = O(1)$$

and the Hölder's inequality, we have that

$$\sum_{i \neq j}^{p} b_i b_j \mathbb{E} M^{12} = o(p^{-1/2}).$$

Therefore, similar to the proof of Theorem 2, we conclude that

$$\mathcal{M}_2 = o(1),$$

and we complete the proof of this theorem.

S3.5 Some useful lemmas

Lemma 1 (Theorem 35.12 of Billingsley (1995)). Suppose that for each n, $Y_{n1}, Y_{n2}, \ldots, Y_{nr_n}$ is a real martingale difference sequence with respect to the increasing σ -field $\{\mathcal{F}_{nj}\}$ having second moments. If as $n \to \infty$, for each $\varepsilon > 0$,

(i)
$$\sum_{j=1}^{r_n} \mathbb{E}\left(Y_{nj}^2 | \mathcal{F}_{n,j-1}\right) \xrightarrow{p} \sigma^2$$
, where σ^2 is a positive constant;

(*ii*)
$$\sum_{j=1}^{r_n} \mathbb{E}\left(Y_{nj}^2 I_{(|Y_{nj}| \ge \varepsilon)}\right) \to 0,$$

then we have that

$$\sum_{j=1}^{r_n} Y_{nj} \xrightarrow{\mathcal{D}} N\left(0, \sigma^2\right).$$

Lemma 2 (Burkholder (1971)). Let $\{Y_k\}$ be a martingale difference sequence with respect to the increasing σ -field $\{\mathcal{F}_k\}$. Then, for $\ell > 1$,

$$\mathbb{E}\left|\sum X_{k}\right|^{\ell} \leq K_{\ell} \left(\mathbb{E}\left(\sum \mathbb{E}(|X_{k}|^{2} |\mathcal{F}_{k-1})\right)^{\ell/2} + \sum \mathbb{E}|X_{k}|^{\ell}\right).$$

Lemma 3 (Lemma 2.7 of Bai and Silverstein (1998)). For $\mathbf{e} = (e_1, \ldots, e_n)'$ *i.i.d. standardized entries,* \mathbf{A} *a* $n \times n$ *matrix, we have, for any* $\ell \geq 2$

$$\mathbb{E} \left| \mathbf{e}' \mathbf{A} \mathbf{e} - \operatorname{tr} \mathbf{A} \right|^{\ell} \leq K_{\ell} \left(\left(\mathbb{E} \left| e_{1} \right|^{4} \operatorname{tr} \mathbf{A} \mathbf{A}' \right)^{\ell/2} + \mathbb{E} \left| e_{1} \right|^{2\ell} \operatorname{tr} (\mathbf{A} \mathbf{A}')^{\ell/2} \right)$$

Lemma 4 (Lemma 7.2 of Bai and Yao (2005)). Let $\mathbf{e} = (e_1, \ldots, e_n)'$ be a random n-vector with i.i.d. standardized entries. Suppose $E |e_i|^4 < \infty$ and $|e_i| \leq \eta_n \sqrt{n}$ with $\eta_n \to 0$ slowly. Assume that \mathbf{A} is a symmetric matrix of

order n bounded in norm by M. Then, for any given $2 \leq \ell \leq b \log(n\eta_n^2)$ with some b > 1, there exists a constant K such that

$$\mathbb{E}\left|\mathbf{e}'\mathbf{A}\mathbf{e}-\mathrm{tr}(\mathbf{A})\right|^{\ell} \leq n^{\ell} \left(n\eta_{n}^{4}\right)^{-1} \left(MK\eta_{n}^{2}\right)^{\ell}$$

Lemma 5. Let **B** and **C** be $n \times n$ matrices. Let **d** be a n-vector. Let $\mathbf{e} = (e_1, \dots, e_n)'$ be a random n-vector with i.i.d. standardized entries. Let $\tau := \mathbb{E}e_i^4 - 3$ Then, we have that

$$\mathbb{E}\left\{\left(\mathbf{e'Be} - \operatorname{tr} \mathbf{B}\right)\left(\mathbf{e'Ce} - \operatorname{tr} \mathbf{C}\right)\right\} = \operatorname{tr}(\mathbf{BC}) + \operatorname{tr}(\mathbf{BC'}) + \tau \sum_{i=1}^{n} b_{ii}c_{ii}$$

and

$$\mathbb{E}(\mathbf{e'Bee'd}) = \mathbb{E}e_1^3 \sum_{i=1}^n b_{ii} d_i.$$

Lemma 6 (Lemma 3.1 of Bai and Pan (2012)). Let $\{\mathbf{d}_n = (d_1, \ldots, d_n)'\}$ be a sequence of unit vectors with $\max_{k \leq n} |d_k| \to 0$. There is a permutation m of $\{1, \ldots, n\}$ given by

$$\left(\begin{array}{cccc} 1 & 2 & \cdots & n \\ m(1) & m(2) & \cdots & m(n) \end{array}\right)$$

such that $\mathbf{d}_f = (d_{f(1)}, \dots, d_{f(n)})$ and F_{nm} tends to a uniform distribution over the interval (0,1), where F_{nm} is a distribution function defined by

$$F_{nm}(t) = \sum_{i \le nt} \left| d_{f(i)} \right|^2.$$

Bibliography

- Bai, Z., K. P. Choi, Y. Fujikoshi, and J. Hu (2022). Asymptotics of AIC, BIC and Cp model selection rules in high-dimensional regression. Bernoulli 28(4), 2375–2403.
- Bai, Z. D., K. P. Choi, and Y. Fujikoshi (2018). Limiting behavior of eigenvalues in high-dimensional MANOVA via RMT. <u>The Annals of</u> Statistics 46(6A), 2985–3013.
- Bai, Z. D. and G. M. Pan (2012). Limiting Behavior of Eigenvectors of Large Wigner Matrices. Journal of Statistical Physics 146(3), 519–549.
- Bai, Z. D. and J. W. Silverstein (1998). No eigenvalues outside the support of the limiting spectral distribution of large-dimensional sample covariance matrices. The Annals of Probability 26(1), 316–345.
- Bai, Z. D. and J. W. Silverstein (1999). Exact separation of eigenvalues of large dimensional sample covariance matrices. <u>The Annals of</u> Probability 27(3), 1536–1555.
- Bai, Z. D. and J. W. Silverstein (2004). CLT for linear spectral statistics of large-dimensional sample covariance matrices. <u>The Annals of</u> Probability 32(1), 553–605.

- Bai, Z. D. and J. F. Yao (2005). On the convergence of the spectral empirical process of Wigner matrices. Bernoulli 11(6), 1059–1092.
- Billingsley, P. (1995). <u>Probability and Measure</u>. John Wiley&Sons, New York.
- Breiman, L. and J. H. Friedman (1997). Predicting Multivariate Responses in Multiple Linear Regression. <u>Journal of the Royal Statistical Society</u>: Series B (Statistical Methodology) 59(1), 3–54.
- Burkholder, D. L. (1971). Distribution function inequalities for martingales.The Annals of Probability 1(1), 19–42.
- Similä, T. and J. Tikka (2007). Input selection and shrinkage in multiresponse linear regression. <u>Computational Statistics & Data Analysis 52(1)</u>, 406–422.
- Skagerberg, B., J. F. MacGregor, and C. Kiparissides (1992). Multivariate data analysis applied to low-density polyethylene reactors. <u>Chemometrics</u> and Intelligent Laboratory Systems 14(1), 341–356.