OPTIMAL DESIGNS FOR FUNCTIONAL PRINCIPAL AND EMPIRICAL COMPONENT SCORES

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Supplementary Material

In Tables S1–S3, we present our obtained designs $\boldsymbol{d}^* = \{\boldsymbol{t}_1, ..., \boldsymbol{t}_n\}$ for predicting the functional empirical component (FEC) scores with the scenarios considered in Section 4 of the paper. The distinct K-point elemental designs for each $\boldsymbol{d}^*$ are listed, along with the number of replicates of these elemental designs in $\boldsymbol{d}^*$; $K = 3, 5, 7$. The elemental design is represented by the K indices of the sampling time points t_{ij} from the 21-point regular grid of $\mathcal{T} = [0, 1]$.

In Table S4, the elemental design for each single-support design $\boldsymbol{d}_s = \boldsymbol{d}_{fpc}$ is listed. $\boldsymbol{d}_s$ is obtained by minimizing Φ_A of Corollary 1, by an exhaustive search over all the single-support designs that are in Ξ_d. There are multiple $\boldsymbol{d}_s$ for each case. Except for $J = K = 7$, $\boldsymbol{d}_s$ is the same as $\boldsymbol{d}_{fpc}$ that minimizes Φ_{A1} among the single-support designs. The $\boldsymbol{d}_s$ and $\boldsymbol{d}_{fpc}$ for $J = K = 7$ can be found in Table 2 in the paper. As presented there, the former design depends on the number of subjects n.

Table S1: Obtained Designs for FEC scores with $J = 3$: elemental design $\times$ number of replicates

	n = 10	
$K = 3$	$K = 5$	$K = 7$
$(3, 7, 19) \times 3$	$(3, 5, 8, 13, 19) \times 2$	$(3, 4, 8, 9, 13, 14, 18) \times 2$
$(3, 15, 19) \times 2$	$(3, 9, 13, 15, 18) \times 3$	$(3, 4, 8, 9, 14, 18, 19) \times 3$
$(5, 9, 13) \times 2$	$(3, 9, 14, 17, 19) \times 3$	$(3, 4, 8, 13, 14, 18, 19) \times 3$
$(9, 13, 17) \times 3$	$(4, 7, 9, 13, 19) \times 2$	$(4, 8, 9, 13, 14, 18, 19) \times 2$
	n = 50	
$K = 3$	$K = 5$	$K = 7$
$(3, 7, 19) \times 13$	$(3, 5, 8, 13, 19) \times 12$	$(3, 4, 8, 9, 13, 14, 18) \times 13$
$(3, 15, 19) \times 12$	$(3, 9, 13, 15, 18) \times 13$	$(3, 4, 8, 9, 14, 18, 19) \times 12$
$(5, 9, 13) \times 12$	$(3, 9, 14, 17, 19) \times 13$	$(3, 4, 8, 13, 14, 18, 19) \times 12$
$(9, 13, 17) \times 13$	$(4, 7, 9, 13, 19) \times 12$	$(4, 8, 9, 13, 14, 18, 19) \times 13$
	n = 70	
$K = 3$	$K = 5$	$K = 7$
$(3, 7, 19) \times 17$	$(3, 5, 8, 13, 19) \times 18$	$(3, 4, 8, 9, 13, 14, 18) \times 18$
$(3, 15, 19) \times 18$	$(3, 9, 13, 15, 18) \times 17$	$(3, 4, 8, 9, 14, 18, 19) \times 17$
$(5, 9, 13) \times 18$	$(3, 9, 14, 17, 19) \times 17$	$(3, 4, 8, 13, 14, 18, 19) \times 17$
$(9, 13, 17) \times 17$	$(4, 7, 9, 13, 19) \times 18$	$(4, 8, 9, 13, 14, 18, 19) \times 18$

Table S2: Obtained Designs for FEC scores with $J = 5$: elemental design $\times$ number of replicates

n = 10			
$K = 3$		$K = 5$	$K = 7$
$(2, 6, 10) \times 1$	$(2, 7, 18) \times 1$		
$(3, 7, 19) \times 1$	$(3, 15, 19) \times 1$		$(2, 4, 6, 9, 13, 16, 19) \times 2$
$(4, 15, 20) \times 1$	$(5, 9, 13) \times 1$	$(2, 10, 13, 16, 19) \times 5$	$(3, 6, 8, 10, 13, 16, 19) \times 3$
$(5, 10, 14) \times 1$	$(8, 12, 17) \times 1$	$(3, 6, 9, 12, 20) \times 5$	$(3, 6, 9, 12, 14, 16, 19) \times 2$
$(9, 13, 17) \times 1$	$(12, 16, 20) \times 1$		$(3, 6, 9, 13, 16, 18, 20) \times 3$
n = 50			
$K = 3$		$K = 5$	$K = 7$
$(2, 7, 18) \times 8$	$(3, 7, 19) \times 5$		
$(3, 14, 18) \times 1$	$(3, 15, 19) \times 1$		$(2, 4, 6, 9, 13, 16, 19) \times 12$
$(4, 15, 20) \times 10$	$(5, 9, 13) \times 2$	$(2, 10, 13, 16, 19) \times 25$	$(3, 6, 8, 10, 13, 16, 19) \times 13$
$(5, 10, 14) \times 10$	$(8, 12, 17) \times 9$	$(3, 6, 9, 12, 20) \times 25$	$(3, 6, 9, 12, 14, 16, 19) \times 12$
$(9, 13, 17) \times 4$			$(3, 6, 9, 13, 16, 18, 20) \times 13$
n = 70			
$K = 3$		$K = 5$	$K = 7$
$(2, 7, 18) \times 12$	$(3, 7, 19) \times 1$		
$(3, 14, 18) \times 2$	$(3, 15, 19) \times 5$		$(2, 4, 6, 9, 13, 16, 19) \times 18$
$(4, 8, 13) \times 3$	$(4, 8, 19) \times 4$	$(2, 10, 13, 16, 19) \times 35$	$(3, 6, 8, 10, 13, 16, 19) \times 17$
$(4, 15, 20) \times 10$	$(5, 9, 13) \times 4$	$(3, 6, 9, 12, 20) \times 35$	$(3, 6, 9, 12, 14, 16, 19) \times 18$
$(5, 10, 14) \times 11$	$(8, 12, 17) \times 13$		$(3, 6, 9, 13, 16, 18, 20) \times 17$
$(9, 14, 18) \times 5$			

Table S3: Obtained Designs for FEC scores with $J = 7$: elemental design $\times$ number of replicates

n	$K = 3$		$K = 5$	$K = 7$
n = 10	$(2,5,8)\times 1$ $(2,14,18)\times 1$ $(4,8,12)\times 1$ $(10,14,18)\times 1$ $(12,16,20)\times 1$	$(2,6,10)\times 1$ $(4,7,10)\times 1$ $(4,8,20)\times 1$ $(12,15,18)\times 1$ $(14,17,20)\times 1$	$(1,9,12,15,18)\times 1$ $(2,5,8,16,19)\times 3$ $(3,6,14,17,20)\times 1$ $(3,11,14,17,20)\times 1$ $(4,7,10,13,16)\times 3$ $(6,9,12,15,18)\times 1$	$(2,5,7,10,13,16,19)\times 35$ $(3,6,9,12,15,17,20)\times 35$
n = 50	$(2,6,10)\times 4$ $(3,7,19)\times 1$ $(4,8,20)\times 10$ $(10,14,18)\times 10$	$(2,14,18)\times 10$ $(4,8,12)\times 10$ $(9,13,17)\times 1$ $(12,16,20)\times 4$	$(2,5,8,16,19)\times 13$ $(3,6,14,17,20)\times 13$ $(4,7,10,13,16)\times 12$ $(6,9,12,15,18)\times 12$	$(2,5,7,10,13,16,19)\times 35$ $(3,6,9,12,15,17,20)\times 35$
n = 70	$(2,6,10)\times 3$ $(3,7,19)\times 2$ $(4,8,12)\times 14$ $(5,9,13)\times 2$ $(10,14,18)\times 14$	$(2,14,18)\times 14$ $(3,15,19)\times 2$ $(4,8,20)\times 14$ $(9,13,17)\times 2$ $(12,16,20)\times 3$	$(2,5,8,16,19)\times 18$ $(3,6,14,17,20)\times 18$ $(4,7,10,13,16)\times 17$ $(6,9,12,15,18)\times 17$	$(2,5,7,10,13,16,19)\times 35$ $(3,6,9,12,15,17,20)\times 35$

Table S4: Elemental designs for single-support designs

J=3		
$K = 3$	$K = 5$	$K = 7$
$(3, 7, 19)$	$(3, 5, 8, 13, 19)$	$(3, 4, 8, 9, 13, 14, 18)$
$(3, 15, 19)$	$(3, 9, 13, 15, 18)$	$(3, 4, 8, 9, 14, 18, 19)$
$(5, 9, 13)$	$(3, 9, 14, 17, 19)$	$(3, 4, 8, 13, 14, 18, 19)$
$(9, 13, 17)$	$(4, 7, 9, 13, 19)$	$(4, 8, 9, 13, 14, 18, 19)$
J=5		
$K = 3$	$K = 5$	$K = 7$
$(3, 14, 18)$		$(2, 4, 6, 9, 13, 16, 19)$
$(4, 8, 13)$	$(2, 10, 13, 16, 19)$	$(3, 6, 8, 10, 13, 16, 19)$
$(4, 8, 19)$	$(3, 6, 9, 12, 20)$	$(3, 6, 9, 12, 14, 16, 19)$
$(9, 14, 18)$		$(3, 6, 9, 13, 16, 18, 20)$
J=7		
$K = 3$	$K = 5$	$K = 7$
$(3, 14, 18)$	$(3, 6, 9, 13, 16)$	See Table 2 in the paper
$(4, 8, 13)$	$(3, 6, 9, 16, 19)$	
$(4, 8, 19)$	$(3, 6, 13, 16, 19)$	
$(9, 14, 18)$	$(6, 9, 13, 16, 19)$	