

SUPPLEMENT

A CONTINUOUS-TIME STOCHASTIC PROCESS FOR HIGH-RESOLUTION NETWORK DATA IN SPORTS

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A. Figures

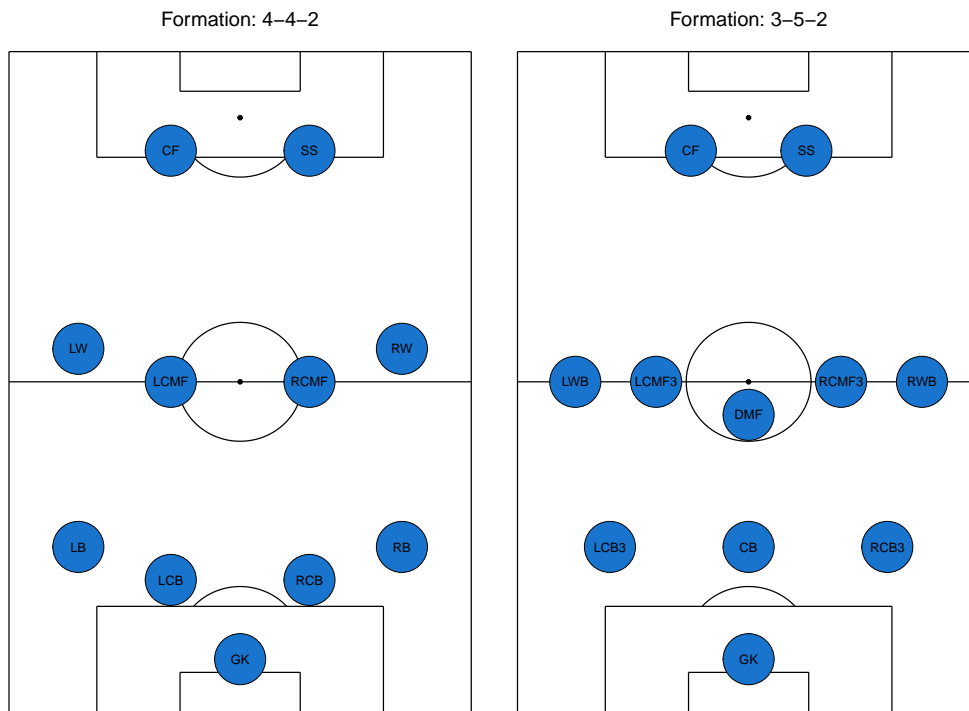
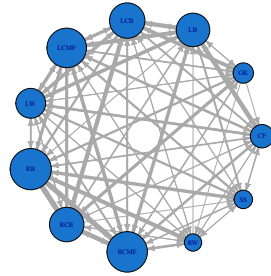
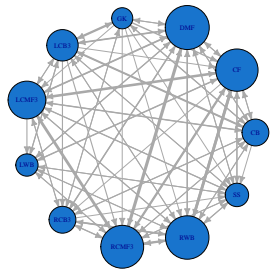


Figure 1: Two popular formations of soccer teams, known as 4-4-2 and 3-5-2. The abbreviations of player positions are detailed in Supplement B.



(a) Juventus Turin (4-4-2)



(b) Inter Milan (3-5-2)

Figure 2: The numbers of passes between the positions of (a) Juventus Turin (with 4-4-2 formation) and (b) Inter Milan (with 3-5-2 formation). These data are based on the home games of (a) Juventus Turin versus AC Milan and (b) Inter Milan versus AC Milan in 2020/21. The 4-4-2 and 3-5-2 formations are shown in Figure 1 in Supplement A. The sizes of the positions are proportional to the number of passes, while the widths of the edges are proportional to the number of passes between the positions.

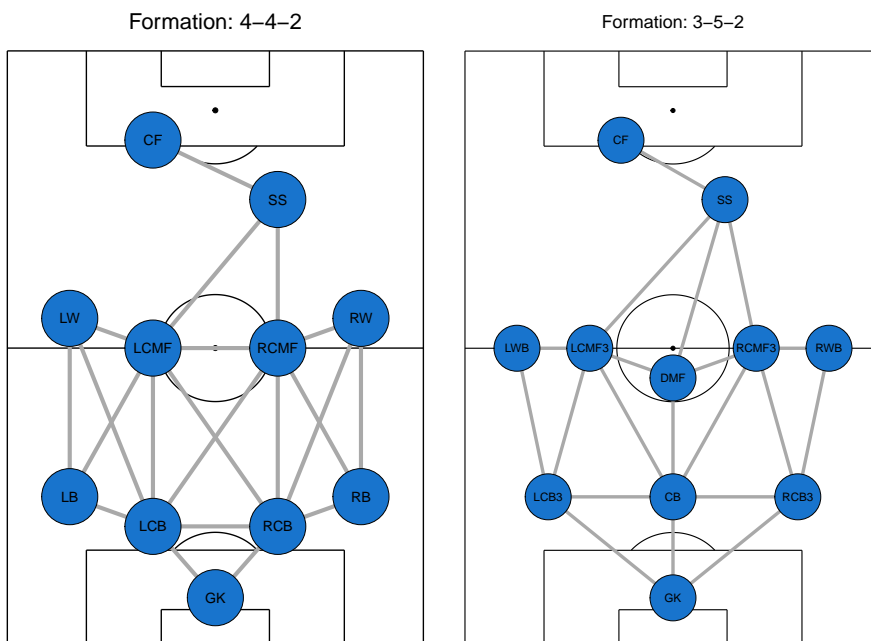


Figure 3: The nearest-neighbor graph, which connects pairs of positions that are considered to be nearest neighbors on the field. The graph distance between a pair of positions is the length of the shortest path between them. The abbreviations of player positions are detailed in Supplement B.

B. Abbreviations

Table 1: Abbreviations of player positions.

| Player position | Abbreviation |
|---|--------------|
| Center Back | CB |
| Right Center Back | RCB |
| Left Center Back | LCB |
| Left Defensive Midfielder | LDMF |
| Right Defensive Midfielder | RDMF |
| Right Center Back (3 at the back) | RCB3 |
| Goalkeeper | GK |
| Defensive Midfielder | DMF |
| Left Center Midfielder | LCMF |
| Left Center Back (3 at the back) | LCB3 |
| Right Center Midfielder | RCMF |
| Left Center Midfielder (3 at the back) | LCMF3 |
| Right Back | RB |
| Left Back | LB |
| Attacking Midfielder | AMF |
| Right Center Midfielder (3 at the back) | RCMF3 |
| Left Attacking Midfielder | LAMF |
| Left Wing Forward | LWF |
| Right Wing Forward | RWF |
| Left Wing | LW |
| Right Attacking Midfielder | RAMF |
| Right Wing Back | RWB |
| Second Striker | SS |
| Right Wing | RW |
| Left Wing Back | LWB |
| Striker | CF |
| Left Back (5 at the back) | LB5 |
| Right Back (5 at the back) | RB5 |

C. Descriptive statistics

Table 2: Proportion of successful passes by teams during the 2020/21 season of Serie A, the premier league of the Italian football league system.

| Team | Proportion of successful passes | | |
|----------------|---------------------------------|------------|-------------|
| | Total | First half | Second half |
| Sassuolo | 89.47% | 89.97% | 88.92% |
| Juventus Turin | 88.95% | 89.56% | 88.27% |
| Inter Milan | 88.55% | 88.69% | 88.40% |
| Napoli | 88.10% | 88.67% | 87.46% |
| Roma | 86.44% | 87.17% | 85.65% |
| Atalanta | 86.33% | 86.33% | 86.32% |
| Milan | 86.22% | 86.59% | 85.82% |
| Lazio | 85.91% | 86.36% | 85.43% |
| Parma | 85.07% | 85.86% | 84.27% |
| Udinese | 84.79% | 85.57% | 84.01% |
| Torino | 84.68% | 85.75% | 83.53% |
| Fiorentina | 84.50% | 85.37% | 83.64% |
| Bologna | 84.45% | 84.71% | 84.17% |
| Spezia | 84.39% | 84.84% | 83.91% |
| Crotone | 84.28% | 84.53% | 84.01% |
| Cagliari | 83.50% | 84.41% | 82.55% |
| Genoa | 83.20% | 84.13% | 82.27% |
| Benevento | 81.70% | 81.52% | 81.87% |
| Hellas Verona | 81.39% | 82.14% | 80.58% |
| Sampdoria | 81.35% | 81.97% | 80.70% |

Table 3: Number of passes and successful passes of Juventus Turin during the 2020/21 season, by formation.

| Formation | Number of passes | Proportion of successful passes |
|-----------|------------------|---------------------------------|
| 4-4-2 | 15832 | 89.30% |
| 4-4-1-1 | 1529 | 88.69% |
| 4-4-1 | 744 | 88.04% |
| 3-4-2-1 | 737 | 91.18% |
| 4-2-3-1 | 591 | 87.14% |
| 3-5-2 | 455 | 81.98% |
| 3-4-1-2 | 261 | 88.89% |
| 3-5-1-1 | 147 | 87.76% |
| 3-4-3 | 114 | 81.58% |
| 4-3-1-2 | 95 | 84.21% |
| 4-3-2 | 54 | 87.04% |
| 4-5-1 | 39 | 84.62% |
| 5-3-1 | 3 | 66.67% |
| 5-4-1 | 1 | 0.00% |

Table 4: Number of passes and successful passes of Juventus Turin during the 2020/21 season based on the 4-4-2 formation, by position and player. Danilo refers to the player Danilo Luiz da Silva.

| Position | Player | Number of passes | Proportion of successful passes |
|-----------|-----------------|------------------|---------------------------------|
| CF | C. Ronaldo | 741 | 81.38% |
| | A. Morata | 197 | 76.65% |
| | P. Dybala | 5 | 80.00% |
| GK | W. Szczesny | 559 | 95.53% |
| | G. Buffon | 173 | 91.33% |
| | C. Pinsoglio | 8 | 87.50% |
| LB | A. Sandro | 769 | 88.43% |
| | Danilo | 535 | 91.78% |
| | G. Frabotta | 239 | 87.45% |
| | F. Bernardeschi | 152 | 88.16% |
| LCB | J. Cuadrado | 26 | 92.31% |
| | G. Chiellini | 786 | 92.75% |
| | M. de Ligt | 518 | 94.59% |
| | L. Bonucci | 399 | 90.23% |
| | Danilo | 91 | 95.60% |
| LCMF | M. Demiral | 77 | 98.70% |
| | A. Sandro | 72 | 90.28% |
| | A. Rabiot | 765 | 92.16% |
| | R. Bentancur | 478 | 91.84% |
| | A. Melo | 368 | 95.65% |
| | W. McKennie | 100 | 87.00% |
| | N. Fagioli | 17 | 100.00% |
| LW | A. Ramsey | 9 | 100.00% |
| | A. Ramsey | 400 | 89.75% |
| | F. Chiesa | 345 | 79.13% |
| | F. Bernardeschi | 153 | 81.70% |
| | W. McKennie | 140 | 85.00% |
| | D. Kulusevski | 51 | 80.39% |
| | G. Frabotta | 23 | 78.26% |
| | F. Correia | 8 | 87.50% |
| RB | A. Rabiot | 4 | 100.00% |
| | J. Cuadrado | 1010 | 85.15% |
| | Danilo | 883 | 88.34% |
| RCB | M. Demiral | 5 | 80.00% |
| | M. de Ligt | 798 | 95.49% |
| | L. Bonucci | 627 | 92.50% |
| | M. Demiral | 328 | 96.95% |
| | Danilo | 35 | 97.14% |
| RCMF | R. Drăgușin | 2 | 50.00% |
| | R. Bentancur | 835 | 90.30% |
| | A. Melo | 501 | 94.81% |
| | A. Rabiot | 267 | 92.51% |
| | Danilo | 145 | 92.41% |
| | W. McKennie | 129 | 93.02% |
| | M. Portanova | 5 | 100.00% |
| RW | A. Ramsey | 3 | 100.00% |
| | D. Kulusevski | 386 | 80.05% |
| | F. Chiesa | 243 | 80.25% |
| | W. McKennie | 164 | 85.37% |
| | J. Cuadrado | 159 | 87.42% |
| | A. Ramsey | 82 | 85.37% |
| | F. Bernardeschi | 19 | 78.95% |
| | P. Dybala | 4 | 100.00% |
| | D. Costa | 3 | 33.33% |
| G. Vrioni | 2 | 100.00% | |
| SS | P. Dybala | 495 | 87.88% |
| | Á. Morata | 305 | 80.33% |
| | D. Kulusevski | 113 | 80.53% |
| | C. Ronaldo | 73 | 76.71% |
| | F. Chiesa | 1 | 0.00% |

Table 5: Number of passes and successful passes of Juventus Turin based on the 4-4-2 formation during the 2020/21 season, by player and position.

| Player | Position | Number of passes | Proportion of successful passes |
|---------------|----------|------------------|---------------------------------|
| C. Ronaldo | CF | 908 | 81.28% |
| | SS | 80 | 76.25% |
| | AMF | 18 | 88.89% |
| D. Kulusevski | RW | 386 | 80.05% |
| | SS | 136 | 80.88% |
| | LW | 87 | 79.31% |
| | AMF | 67 | 83.58% |
| | RAMF | 26 | 80.77% |
| | RWF | 12 | 75.00% |
| | CF | 9 | 66.67% |
| | RCMF | 4 | 75.00% |
| | LCMF3 | 3 | 33.33% |
| F. Chiesa | LW | 367 | 79.56% |
| | RW | 294 | 80.95% |
| | RWB | 44 | 61.36% |
| | LAMF | 22 | 72.73% |
| | LWB | 4 | 50.00% |
| | RCMF3 | 4 | 50.00% |
| | RAMF | 2 | 50.00% |
| | SS | 1 | 0.00% |
| M. de Ligt | RCB | 964 | 95.64% |
| | LCB | 528 | 94.51% |
| | RCB3 | 64 | 92.19% |
| | CB | 39 | 92.31% |
| P. Dybala | SS | 512 | 87.70% |
| | AMF | 81 | 91.36% |
| | CF | 10 | 90.00% |
| | LW | 7 | 85.71% |
| | RW | 4 | 100.00% |
| R. Bentancur | RCMF | 1042 | 90.60% |
| | LCMF | 548 | 91.97% |
| | DMF | 64 | 85.94% |
| | RCMF3 | 20 | 90.00% |
| | LCMF3 | 9 | 77.78% |

Table 6: Number of passes and successful passes of Inter Milan during the 2020/21 season, by formation.

| Formation | Number of passes | Proportion of successful passes |
|-----------|------------------|---------------------------------|
| 3-5-2 | 13564 | 88.85% |
| 3-4-1-2 | 3329 | 88.80% |
| 5-3-2 | 1098 | 85.70% |
| 3-4-3 | 485 | 86.19% |
| 4-3-1-2 | 262 | 93.51% |
| 5-4-1 | 172 | 78.49% |
| 3-4-2-1 | 110 | 87.27% |
| 3-4-2 | 60 | 90.00% |
| 3-5-1-1 | 57 | 89.47% |
| 4-4-1-1 | 28 | 85.71% |
| 4-3-2 | 14 | 85.71% |

Table 7: Number of passes and successful passes of Inter Milan during the 2020/21 season based on the 3-5-2 formation, by position and player.

| Position | Player | Number of passes | Proportion of successful passes |
|----------|----------------|------------------|---------------------------------|
| CB | S. de Vrij | 1371 | 96.21% |
| | A. Ranocchia | 296 | 95.95% |
| CF | R. Lukaku | 305 | 76.72% |
| | A. Sánchez | 174 | 81.61% |
| | L. Martínez | 136 | 80.88% |
| | I. Perišić | 5 | 60.00% |
| | A. Pinamonti | 1 | 100.00% |
| DMF | M. Brozović | 1518 | 91.77% |
| | C. Eriksen | 204 | 88.73% |
| | N. Barella | 60 | 88.33% |
| | A. Vidal | 28 | 89.29% |
| | R. Gagliardini | 23 | 91.30% |
| GK | S. Handanović | 557 | 90.84% |
| | I. Radu | 28 | 100.00% |
| | D. Padelli | 9 | 100.00% |
| LCB3 | A. Bastoni | 1578 | 92.27% |
| | M. Škriniar | 86 | 94.19% |
| | A. Kolarov | 43 | 86.05% |
| | M. Darmian | 25 | 100.00% |
| LCMF3 | C. Eriksen | 419 | 88.78% |
| | R. Gagliardini | 347 | 90.78% |
| | A. Vidal | 245 | 89.39% |
| | S. Sensi | 204 | 88.73% |
| | N. Barella | 102 | 90.20% |
| LWB | I. Perišić | 390 | 77.18% |
| | A. Young | 354 | 82.20% |
| | M. Darmian | 97 | 81.44% |
| | D. D'Ambrosio | 5 | 80.00% |
| RCB3 | M. Škriniar | 1526 | 94.82% |
| | D. D'Ambrosio | 250 | 93.60% |
| | S. de Vrij | 21 | 95.24% |
| RCMF3 | N. Barella | 1114 | 84.11% |
| | A. Vidal | 157 | 84.71% |
| | M. Vecino | 97 | 86.60% |
| | S. Sensi | 30 | 86.67% |
| | C. Eriksen | 25 | 92.00% |
| | R. Gagliardini | 12 | 83.33% |
| | R. Nainggolan | 2 | 100.00% |
| RWB | A. Hakimi | 950 | 82.32% |
| | M. Darmian | 166 | 83.13% |
| | A. Young | 12 | 100.00% |
| | D. D'Ambrosio | 5 | 100.00% |
| SS | L. Martínez | 228 | 71.49% |
| | A. Sánchez | 165 | 80.00% |
| | R. Lukaku | 164 | 71.95% |
| | A. Pinamonti | 26 | 76.92% |

Table 8: Number of passes and successful passes of Inter Milan during the 2020/21 season by famous players in different positions.

| Player | Position | Number of passes | Proportion of successful passes |
|-------------|----------|------------------|---------------------------------|
| C. Eriksen | LCMF3 | 507 | 88.76% |
| | DMF | 212 | 89.15% |
| | AMF | 141 | 82.98% |
| | RCMF3 | 25 | 92.00% |
| | LCMF | 8 | 75.00% |
| | RCMF | 4 | 100.00% |
| | SS | 4 | 50.00% |
| L. Martínez | SS | 313 | 69.33% |
| | CF | 195 | 78.97% |
| | LWF | 19 | 73.68% |
| | LW | 2 | 100.00% |
| M. Brozović | DMF | 1661 | 91.75% |
| | RCMF | 272 | 87.50% |
| | LCMF | 108 | 89.81% |
| M. Škriniar | RCB3 | 1950 | 94.82% |
| | LCB3 | 87 | 93.10% |
| | RCB | 18 | 94.44% |
| N. Barella | RCMF3 | 1245 | 84.58% |
| | RCMF | 176 | 91.48% |
| | LCMF | 140 | 88.57% |
| | LCMF3 | 114 | 90.35% |
| | DMF | 80 | 88.75% |
| | AMF | 70 | 88.57% |
| | RW | 8 | 75.00% |
| | LWF | 4 | 100.00% |
| R. Lukaku | CF | 462 | 76.41% |
| | SS | 244 | 75.00% |

D. Properties of stochastic process

We discuss basic properties of the continuous-time stochastic process specified in Sections 3.1 and 3.2. Throughout Supplement D, we suppress the notational dependence of all quantities on the parameters $\alpha, \beta, \gamma, \omega, \Sigma$ and the random effects η_1, η_2, \dots .

Suppose that the continuous-time stochastic process satisfies two assumptions:

A.1 In a time interval $[t_1, t_2]$, the compositions of teams $\mathcal{T}_{1,t}$ and $\mathcal{T}_{2,t}$ are constant, in the sense that $\mathcal{T}_{1,t} \equiv \mathcal{T}_1$ and $\mathcal{T}_{2,t} \equiv \mathcal{T}_2$ for all $t \in [t_1, t_2]$, and the 22 players of the two teams are labeled $1, \dots, 22$.

A.2 In a time interval $[t_1, t_2]$, the attributes of players and teams, the rates λ_i , the success probabilities $\mathbb{P}(S_i = s_i)$, and the pass probabilities $\mathbb{P}(i \rightarrow j \mid S_i = s_i)$ are time-invariant.

Assumptions A.1 and A.2 are concerned with the behavior of the continuous-time stochastic process in a time interval $[t_1, t_2]$, which can be a short interval (e.g., the time interval may be one time unit long: $t_2 - t_1 = 1$). Assumption A.1 states that the compositions of the teams do not change in a short time interval, that is, the two teams do not substitute play-

ers. Assumption A.2 ensures that the continuous-time stochastic process is time-homogeneous in a short time interval. The assumption that the continuous-time stochastic process is time-homogeneous in a short time interval is not unreasonable, because soccer teams consist of humans, and humans are incapable of instantaneous changes. We hasten to point out that the stochastic modeling framework is not restricted to time-homogeneous stochastic processes: It does allow the attributes of players and teams, the rates λ_i , the success probabilities $\mathbb{P}(S_i = s_i)$, and the pass probabilities $\mathbb{P}(i \rightarrow j \mid S_i = s_i)$ to change over time. The purpose of the following proposition is to shed light on the behavior of the continuous-time stochastic process in a short time interval, during which the stochastic process can be approximated by a time-homogeneous stochastic process.

Proposition 1. *Consider the continuous-time stochastic process described in Sections 3.1 and 3.2 satisfying Assumptions A.1 and A.2. Then the stochastic process is a right-continuous and time-homogeneous Markov process $\{Y(t), t \in [t_1, t_2]\}$ with finite state space $\mathcal{Y} := \{1, \dots, 22\}$ during a time interval $[t_1, t_2)$, where the state $Y(t) \in \mathcal{Y}$ of the Markov process at time t indicates which player is in control of the ball at time t . The elements $q_{i,j}$*

of the generator matrix $\mathbf{Q} \in \mathbb{R}^{|\mathcal{Y}| \times |\mathcal{Y}|}$ of the Markov process are

$$q_{i,j} := \begin{cases} \lambda_i \mathbb{P}(S_i = 0) \mathbb{P}(i \rightarrow j \mid S_i = 0) & \text{if } i \neq j \text{ and } j \notin \mathcal{I}_i \\ \lambda_i \mathbb{P}(S_i = 1) \mathbb{P}(i \rightarrow j \mid S_i = 1) & \text{if } i \neq j \text{ and } j \in \mathcal{I}_i \\ -\lambda_i & \text{if } i = j, \end{cases}$$

where \mathcal{I}_i denotes the team of player $i \in \mathcal{Y}$. Consider any $t \in [t_1, t_2)$ and any $h \in (0, t_2 - t)$. Then, for all $(i, j) \in \mathcal{Y}^2$, conditional on $\{Y(t) = i\}$, the event $\{Y(t+h) = j\}$ is independent of $\{Y(s), s \leq t\}$ and, as $h \downarrow 0$, the conditional probability of event $\{Y(t+h) = j\}$ given $\{Y(t) = i\}$ is

$$\mathbb{P}(Y(t+h) = j \mid Y(t) = i) = \delta_{i,j} + q_{i,j} h + o(h),$$

where $\delta_{i,j} := 1$ if $i = j$ and $\delta_{i,j} := 0$ otherwise.

The proposition is a straightforward consequence of the construction of the continuous-time stochastic process and Theorem 2.8.2 of Norris (1997, p. 94). The proposition shows that the continuous-time stochastic process focuses on ball control and who passes the ball to whom, by specifying the rates $q_{i,j}$ of passing the ball between pairs of players $(i, j) \in \mathcal{Y}^2$.

E. Posterior summaries

Table 9: Posterior summaries for Fiorentina, Crotone, and Inter Milan (with 3-5-2 formation): M refers to posterior medians and CI refers to 95% posterior credible intervals.

| | Fiorentina | | Crotone | | Inter Milan | |
|--|------------|-------------------|---------|-------------------|-------------|-------------------|
| | M | CI | M | CI | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | | | | | |
| Intercept | 2.93 | (2.47, 3.39) | 3.27 | (2.85, 3.68) | 3.34 | (2.82, 3.86) |
| Length of pass | 0.00 | (-0.01, 0.00) | 0.00 | (-0.01, 0.00) | 0.00 | (0.00, 0.01) |
| Forward pass | -0.57 | (-0.74, -0.40) | -0.88 | (-1.07, -0.70) | -0.84 | (-0.99, -0.70) |
| Start: half | 0.17 | (-0.02, 0.36) | -0.03 | (-0.23, 0.17) | 0.29 | (0.12, 0.46) |
| End: third | -0.67 | (-0.85, -0.49) | -0.64 | (-0.84, -0.45) | -0.79 | (-0.96, -0.62) |
| Air pass | -1.76 | (-1.93, -1.59) | -1.90 | (-2.07, -1.73) | -1.84 | (-1.98, -1.70) |
| Winning | -0.13 | (-0.30, 0.04) | -0.25 | (-0.46, -0.04) | -0.13 | (-0.26, 0.00) |
| Losing | -0.01 | (-0.17, 0.15) | -0.11 | (-0.26, 0.04) | 0.02 | (-0.16, 0.21) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | | | | | |
| Graph distance | -0.69 | (-0.73, -0.65) | -0.70 | (-0.74, -0.65) | -0.98 | (-1.02, -0.95) |
| Pass received | 0.00 | (-2.3e-3, 2.0e-3) | 0.00 | (-1.7e-3, 3.2e-4) | 0.00 | (-1.6e-3, 8.6e-5) |
| Holding times h_m: | | | | | | |
| GK | -3.23 | (-3.34, -3.13) | -2.86 | (-2.95, -2.77) | -2.98 | (-3.06, -2.90) |
| LCB | -2.62 | (-2.68, -2.56) | -2.77 | (-2.83, -2.71) | -2.32 | (-2.37, -2.27) |
| CB | -2.62 | (-2.70, -2.55) | -2.70 | (-2.76, -2.64) | -2.39 | (-2.44, -2.34) |
| RCB | -2.86 | (-2.92, -2.79) | -2.44 | (-2.51, -2.37) | -2.34 | (-2.39, -2.30) |
| LWB | -2.33 | (-2.40, -2.26) | -2.61 | (-2.69, -2.53) | -2.27 | (-2.34, -2.20) |
| LCMF | -2.51 | (-2.59, -2.43) | -2.50 | (-2.57, -2.42) | -2.07 | (-2.13, -2.01) |
| DMF | -2.62 | (-2.68, -2.55) | -2.42 | (-2.49, -2.36) | -2.13 | (-2.18, -2.08) |
| RCMF | -2.34 | (-2.41, -2.26) | -2.62 | (-2.70, -2.54) | -2.21 | (-2.26, -2.16) |
| RWB | -2.48 | (-2.56, -2.40) | -2.29 | (-2.37, -2.20) | -2.01 | (-2.07, -1.95) |
| SS | -2.63 | (-2.72, -2.54) | -2.37 | (-2.46, -2.28) | -2.11 | (-2.19, -2.03) |
| CF | -2.62 | (-2.71, -2.54) | -2.98 | (-3.08, -2.88) | -2.30 | (-2.38, -2.22) |
| Winning | -0.42 | (-0.47, -0.36) | -0.37 | (-0.44, -0.30) | -0.36 | (-0.40, -0.33) |
| Losing | 0.05 | (0.00, 0.10) | -0.01 | (-0.05, 0.04) | -0.08 | (-0.13, -0.03) |
| Random effects: | | | | | | |
| Correlation | -0.36 | (-0.83, 0.12) | -0.25 | (-0.75, 0.25) | -0.03 | (-0.53, 0.47) |
| SD: success | 0.58 | (0.31, 0.86) | 0.62 | (0.34, 0.89) | 0.80 | (0.44, 1.17) |
| SD: pass | 0.51 | (0.28, 0.74) | 0.24 | (0.13, 0.36) | 0.47 | (0.25, 0.69) |

Table 10: Posterior summaries for Juventus Turin (with 4-4-2 formation): M refers to posterior medians and CI refers to 95% posterior credible intervals.

| Juventus Turin | | |
|--|-------|--------------------|
| | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | |
| Intercept | 3.36 | (2.90, 3.81) |
| Length of pass | 0.00 | (-0.01, 0.00) |
| Forward pass | -0.61 | (-0.75, -0.47) |
| Start: half | 0.26 | (0.10, 0.42) |
| End: third | -0.92 | (-1.07, -0.76) |
| Air pass | -2.04 | (-2.18, -1.89) |
| Winning | -0.04 | (-0.17, 0.09) |
| Losing | 0.04 | (-0.13, 0.20) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | |
| Graph distance | -0.70 | (-0.73, -0.67) |
| Pass received | 0.00 | (-1.6e-3, 8.6e-05) |
| Holding times h_m: | | |
| GK | -2.80 | (-2.88, -2.72) |
| LB | -2.08 | (-2.13, -2.03) |
| LCB | -2.38 | (-2.42, -2.33) |
| RCB | -2.37 | (-2.41, -2.32) |
| RB | -2.09 | (-2.14, -2.05) |
| LW | -2.06 | (-2.12, -2.00) |
| LCMF | -2.19 | (-2.24, -2.14) |
| RCMF | -2.26 | (-2.30, -2.21) |
| RW | -2.17 | (-2.23, -2.11) |
| SS | -1.81 | (-1.87, -1.74) |
| CF | -1.94 | (-2.01, -1.87) |
| Winning | -0.20 | (-0.23, -0.16) |
| Losing | -0.15 | (-0.19, -0.10) |
| Random effects: | | |
| Correlation | -0.33 | (-0.82, 0.15) |
| SD: success | 0.69 | (0.38, 0.99) |
| SD: pass | 0.44 | (0.24, 0.64) |

F. Posterior sensitivity checks

Table 11: Posterior summaries for Fiorentina, Crotone, and Inter Milan (with 3-5-2 formation) under Prior 1 described in Section 6.1: M refers to posterior medians and CI refers to 95% posterior credible intervals.

| | Fiorentina | | Crotone | | Inter Milan | |
|--|------------|---------------------|---------|---------------------|-------------|----------------------|
| | M | CI | M | CI | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | | | | | |
| Intercept | 2.94 | (2.56, 3.31) | 3.23 | (2.82, 3.64) | 3.24 | (2.74, 3.75) |
| Length of pass | 0.00 | (-0.01, 0.00) | 0.00 | (-0.01, 0.00) | 0.00 | (-0.01, 0.00) |
| Forward pass | -0.56 | (-0.73, -0.4) | -0.88 | (-1.06, -0.71) | -0.84 | (-0.99, -0.68) |
| Start: half | 0.15 | (-0.04, 0.35) | -0.02 | (-0.22, 0.17) | 0.29 | (0.11, 0.47) |
| End: third | -0.68 | (-0.87, -0.49) | -0.62 | (-0.82, -0.43) | -0.79 | (-0.97, -0.62) |
| Air pass | -1.76 | (-1.93, -1.59) | -1.90 | (-2.07, -1.73) | -1.84 | (-1.99, -1.7) |
| Winning | -0.15 | (-0.32, 0.03) | -0.23 | (-0.45, -0.02) | -0.12 | (-0.25, 0.02) |
| Losing | -0.01 | (-0.19, 0.16) | -0.12 | (-0.27, 0.02) | 0.01 | (-0.17, 0.19) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | | | | | |
| Graph distance | -0.69 | (-0.73, -0.65) | -0.70 | (-0.74, -0.65) | -0.99 | (-1.02, -0.95) |
| Pass received | 0.00 | (-2.09e-3, 1.92e-3) | 0.00 | (-1.68e-3, 3.13e-4) | 0.00 | (-1.52e-3, 4.37e-05) |
| Holding times h_m: | | | | | | |
| GK | -3.23 | (-3.33, -3.13) | -2.86 | (-2.94, -2.77) | -2.98 | (-3.06, -2.90) |
| LCB | -2.62 | (-2.68, -2.56) | -2.77 | (-2.83, -2.7) | -2.32 | (-2.37, -2.27) |
| CB | -2.62 | (-2.69, -2.55) | -2.69 | (-2.76, -2.63) | -2.39 | (-2.44, -2.34) |
| RCB | -2.86 | (-2.93, -2.79) | -2.44 | (-2.5, -2.37) | -2.34 | (-2.39, -2.30) |
| LWB | -2.33 | (-2.4, -2.26) | -2.61 | (-2.69, -2.53) | -2.27 | (-2.34, -2.20) |
| LCMF | -2.51 | (-2.59, -2.43) | -2.50 | (-2.57, -2.42) | -2.07 | (-2.13, -2.01) |
| DMF | -2.62 | (-2.68, -2.55) | -2.42 | (-2.49, -2.36) | -2.13 | (-2.18, -2.09) |
| RCMF | -2.34 | (-2.41, -2.26) | -2.62 | (-2.7, -2.54) | -2.20 | (-2.26, -2.15) |
| RWB | -2.48 | (-2.56, -2.4) | -2.29 | (-2.38, -2.19) | -2.01 | (-2.07, -1.95) |
| SS | -2.62 | (-2.71, -2.54) | -2.38 | (-2.46, -2.29) | -2.12 | (-2.21, -2.04) |
| CF | -2.62 | (-2.71, -2.53) | -2.99 | (-3.09, -2.89) | -2.30 | (-2.38, -2.22) |
| Winning | -0.41 | (-0.47, -0.35) | -0.38 | (-0.45, -0.31) | -0.36 | (-0.4, -0.33) |
| Losing | 0.06 | (0, 0.11) | -0.01 | (-0.05, 0.04) | -0.07 | (-0.12, -0.02) |
| Random effects: | | | | | | |
| Correlation | -0.36 | (-0.82, 0.09) | -0.26 | (-0.75, 0.23) | -0.05 | (-0.56, 0.46) |
| SD: success | 0.58 | (0.3, 0.85) | 0.62 | (0.33, 0.91) | 0.82 | (0.45, 1.19) |
| SD: pass | 0.50 | (0.27, 0.73) | 0.24 | (0.12, 0.36) | 0.46 | (0.25, 0.67) |

Table 12: Posterior summaries for Juventus Turin (with 4-4-2 formation) under Prior 1 described in Section 6.1: M refers to posterior medians and CI refers to 95% posterior credible intervals.

| Juventus Turin | | |
|--|-------|---------------------|
| | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | |
| Intercept | 3.38 | (2.96, 3.80) |
| Length of pass | 0.00 | (-0.01, 0.00) |
| Forward pass | -0.62 | (-0.76, -0.48) |
| Start: half | 0.25 | (0.09, 0.41) |
| End: third | -0.92 | (-1.07, -0.76) |
| Air pass | -2.04 | (-2.18, -1.90) |
| Winning | -0.05 | (-0.17, 0.08) |
| Losing | 0.04 | (-0.13, 0.20) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | |
| Graph distance | -0.70 | (-0.73, -0.67) |
| Pass received | 0.00 | (-8.12e-4, 6.42e-4) |
| Holding times h_m : | | |
| GK | -2.80 | (-2.88, -2.73) |
| LB | -2.08 | (-2.13, -2.03) |
| LCB | -2.38 | (-2.42, -2.33) |
| RCB | -2.36 | (-2.41, -2.31) |
| RB | -2.09 | (-2.14, -2.05) |
| LW | -2.05 | (-2.11, -1.99) |
| LCMF | -2.20 | (-2.25, -2.15) |
| RCMF | -2.26 | (-2.3, -2.21) |
| RW | -2.16 | (-2.23, -2.1) |
| SS | -1.80 | (-1.86, -1.74) |
| CF | -1.93 | (-2, -1.86) |
| Winning | -0.20 | (-0.23, -0.16) |
| Losing | -0.15 | (-0.19, -0.10) |
| Random effects: | | |
| Correlation | -0.30 | (-0.77, 0.17) |
| SD: success | 0.68 | (0.39, 0.97) |
| SD: pass | 0.43 | (0.24, 0.62) |

Table 13: Posterior summaries for Fiorentina, Crotone, and Inter Milan (with 3-5-2 formation) under Prior 3 described in Section 6.1: M refers to posterior medians and CI refers to 95% posterior credible intervals.

| | Fiorentina | | Crotone | | Inter Milan | |
|--|------------|-------------------|---------|-------------------|-------------|--------------------|
| | M | CI | M | CI | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | | | | | |
| Intercept | 2.94 | (2.54, 3.34) | 3.23 | (2.82, 3.65) | 3.26 | (2.75, 3.76) |
| Length of pass | 0.00 | (-0.01, 0) | 0.00 | (-0.01, 0) | 0.00 | (0, 0.01) |
| Forward pass | -0.56 | (-0.73, -0.39) | -0.88 | (-1.05, -0.7) | -0.84 | (-0.99, -0.69) |
| Start: half | 0.17 | (-0.02, 0.37) | -0.01 | (-0.21, 0.18) | 0.30 | (0.12, 0.47) |
| End: third | -0.69 | (-0.87, -0.5) | -0.63 | (-0.82, -0.44) | -0.79 | (-0.97, -0.62) |
| Air pass | -1.77 | (-1.93, -1.6) | -1.90 | (-2.06, -1.73) | -1.84 | (-1.99, -1.69) |
| Winning | -0.13 | (-0.3, 0.04) | -0.25 | (-0.45, -0.05) | -0.12 | (-0.25, 0.02) |
| Losing | -0.02 | (-0.18, 0.14) | -0.12 | (-0.27, 0.02) | 0.02 | (-0.16, 0.21) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | | | | | |
| Graph distance | -0.69 | (-0.73, -0.65) | -0.70 | (-0.74, -0.66) | -0.99 | (-1.02, -0.95) |
| Pass received | 0.00 | (-2.0e-3, 2.2e-3) | 0.00 | (-1.6e-3, 3.8e-4) | 0.00 | (-1.5e-3, 6.2e-05) |
| Holding times h_m: | | | | | | |
| GK | -3.23 | (-3.34, -3.13) | -2.86 | (-2.95, -2.77) | -2.98 | (-3.06, -2.9) |
| LCB | -2.62 | (-2.69, -2.56) | -2.77 | (-2.83, -2.7) | -2.32 | (-2.37, -2.27) |
| CB | -2.62 | (-2.69, -2.55) | -2.70 | (-2.76, -2.64) | -2.39 | (-2.44, -2.34) |
| RCB | -2.86 | (-2.93, -2.79) | -2.44 | (-2.5, -2.37) | -2.34 | (-2.39, -2.3) |
| LWB | -2.33 | (-2.4, -2.26) | -2.61 | (-2.69, -2.53) | -2.28 | (-2.34, -2.21) |
| LCMF | -2.50 | (-2.58, -2.42) | -2.50 | (-2.57, -2.42) | -2.08 | (-2.13, -2.02) |
| DMF | -2.62 | (-2.68, -2.55) | -2.42 | (-2.49, -2.36) | -2.13 | (-2.18, -2.09) |
| RCMF | -2.34 | (-2.41, -2.26) | -2.62 | (-2.7, -2.55) | -2.20 | (-2.26, -2.15) |
| RWB | -2.47 | (-2.55, -2.39) | -2.28 | (-2.37, -2.19) | -2.01 | (-2.07, -1.95) |
| SS | -2.63 | (-2.72, -2.54) | -2.38 | (-2.46, -2.29) | -2.12 | (-2.2, -2.03) |
| CF | -2.62 | (-2.72, -2.53) | -2.99 | (-3.09, -2.89) | -2.30 | (-2.38, -2.22) |
| Winning | -0.42 | (-0.47, -0.36) | -0.38 | (-0.45, -0.3) | -0.36 | (-0.4, -0.33) |
| Losing | 0.05 | (0, 0.1) | -0.01 | (-0.05, 0.04) | -0.08 | (-0.13, -0.02) |
| Random effects: | | | | | | |
| Correlation | -0.40 | (-0.87, 0.07) | -0.32 | (-0.83, 0.18) | -0.08 | (-0.61, 0.45) |
| SD: success | 0.58 | (0.32, 0.84) | 0.60 | (0.34, 0.86) | 0.78 | (0.45, 1.11) |
| SD: pass | 0.50 | (0.28, 0.71) | 0.23 | (0.12, 0.35) | 0.44 | (0.24, 0.64) |

Table 14: Posterior summaries for Juventus Turin (with 4-4-2 formation) using Prior 3 described in Section 6.1: M refers to posterior medians and CI refers to 95% posterior credible intervals.

| Juventus Turin | | |
|--|-------|-------------------|
| | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | |
| Intercept | 3.35 | (2.93, 3.77) |
| Length of pass | 0.00 | (-0.01, 0) |
| Forward pass | -0.62 | (-0.75, -0.48) |
| Start: half | 0.25 | (0.09, 0.41) |
| End: third | -0.92 | (-1.07, -0.76) |
| Air pass | -2.05 | (-2.19, -1.91) |
| Winning | -0.05 | (-0.18, 0.08) |
| Losing | 0.04 | (-0.13, 0.2) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | |
| Graph distance | -0.70 | (-0.73, -0.67) |
| Pass received | 0.00 | (-7.7e-4, 7.1e-4) |
| Holding times h_m: | | |
| GK | -2.80 | (-2.88, -2.73) |
| LB | -2.08 | (-2.13, -2.03) |
| LCB | -2.38 | (-2.42, -2.33) |
| RCB | -2.36 | (-2.41, -2.31) |
| RB | -2.09 | (-2.14, -2.05) |
| LW | -2.06 | (-2.12, -2) |
| LCMF | -2.20 | (-2.25, -2.15) |
| RCMF | -2.26 | (-2.31, -2.21) |
| RW | -2.17 | (-2.23, -2.11) |
| SS | -1.81 | (-1.88, -1.74) |
| CF | -1.94 | (-2.00, -1.87) |
| Winning | -0.20 | (-0.23, -0.16) |
| Losing | -0.15 | (-0.19, -0.1) |
| Random effects: | | |
| Correlation | -0.30 | (-0.77, 0.18) |
| SD: success | 0.66 | (0.37, 0.94) |
| SD: pass | 0.42 | (0.23, 0.61) |

G. Posterior predictive checks

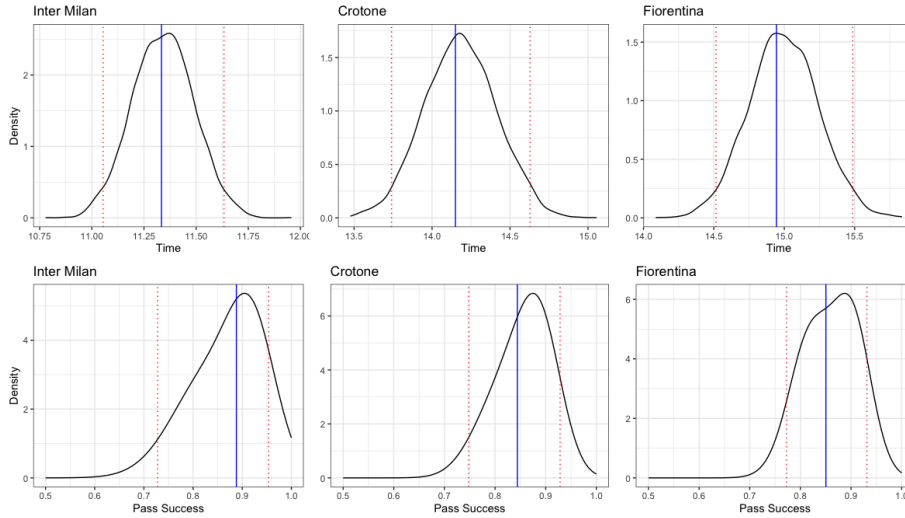


Figure 4: Posterior predictions of the waiting times between passes and the proportions of successful passes by Inter Milan, Crotone, and Fiorentina during the 2020/21 season. The blue-colored solid vertical lines represent the mean of the observed waiting times and the observed proportions of successful passes, while the red-colored dotted vertical lines represent the 2.5% and 97.5% percentiles of the posterior predictions.

H. Simulation results

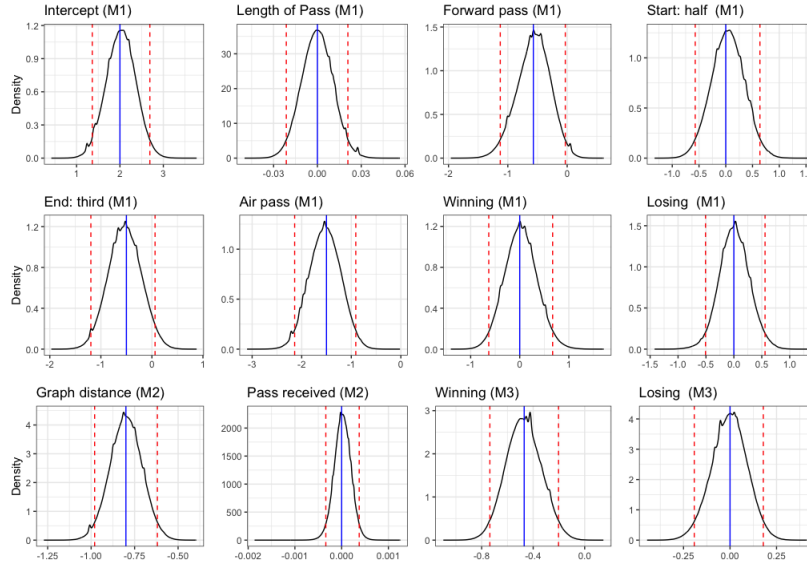


Figure 5: Simulation results: marginal posteriors of selected parameters based on 100 simulated soccer seasons, each with 1,000 passes. The blue-colored solid lines represent the data-generating parameters, while the red-colored dashed lines represent the 2.5% and 97.5% percentiles. M1, M2, and M3 refer to Module M1, M2, and M3 of the stochastic modeling framework specified in Section 6, respectively.

Table 15: Simulation results: data-generating parameters and posterior summaries of parameters based on one of the 100 simulated soccer seasons with 1,000 passes. M is the median of the posterior means. CI shows the interval consisting of the 2.5% and 97.5% quantiles of the posterior means.

| | Simulation | | |
|--|-------------------|-------|-------------------|
| | Truth | M | CI |
| Successful passes $\{S_{i_m} = 1\}$: | | | |
| Intercept | 2.00 | 2.26 | (1.78, 2.74) |
| Length of pass | 0.00 | 0.00 | (-0.02, 0.01) |
| Forward pass | -0.57 | -0.72 | (-1.1, -0.33) |
| Start: half | 0.00 | 0.00 | (-0.43, 0.43) |
| End: third | -0.50 | -0.43 | (-0.88, 0.02) |
| Air pass | -1.50 | -1.28 | (-1.72, -0.85) |
| Winning | 0.00 | -0.11 | (-0.57, 0.34) |
| Losing | 0.00 | -0.14 | (-0.49, 0.22) |
| Passes $\{i_m \rightarrow j_m\}$ given $\{S_{i_m} = 1\}$: | | | |
| Graph distance | -0.80 | -0.83 | (-0.96, -0.69) |
| Pass received | 0.00 | 0.00 | (-3.2e-4, 2.1e-4) |
| Holding times h_m: | | | |
| GK | -2.70 | -2.66 | (-2.94, -2.37) |
| LCB | -2.70 | -2.84 | (-3.01, -2.66) |
| CB | -2.70 | -2.66 | (-2.85, -2.48) |
| RCB | -2.70 | -2.60 | (-2.8, -2.41) |
| LWB | -2.70 | -2.71 | (-2.98, -2.44) |
| LCMF | -2.70 | -2.74 | (-2.94, -2.55) |
| DMF | -2.70 | -2.78 | (-2.96, -2.59) |
| RCMF | -2.70 | -2.52 | (-2.72, -2.32) |
| RWB | -2.70 | -2.70 | (-2.92, -2.48) |
| SS | -2.70 | -2.55 | (-2.83, -2.28) |
| CF | -2.70 | -2.70 | (-2.97, -2.43) |
| Winning | -0.47 | -0.58 | (-0.77, -0.39) |
| Losing | 0.00 | 0.03 | (-0.12, 0.17) |
| Random effects: | | | |
| Correlation | 0.00 | -0.09 | (-0.88, 0.71) |
| SD: success | 0.00 | 0.20 | (0.01, 0.38) |
| SD: pass | 0.00 | 0.09 | (0, 0.17) |

References

Norris, J. R. (1997). *Markov Chains*. Cambridge: Cambridge University Press.