Statistica Sinica: Supplement

REGRESSION ANALYSIS OF RANDOMIZED RESPONSE EVENT TIME DATA

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Supplementary Material

The Matlab codes and the TSCS data analysis.

S1 Code for the unrelated-question RRT estimation

```
%%%% unrelated-question RRT estimation

%%%% (1) Run

% Model=RRT(Data);

%%%% (2) Input

% Data.v=1 if PO; 0.001 if PH

% Data.Qp= probability of choosing sensitive question

% Data.Wp= proportion of answering 'yes' to innocuous question

% Data.C= survey time (nx1 column vector)

% Data.Y= observed response (nx1 column vector)

% Data.Z= covariate (nxd matrix)

%%%% (3) Output

% Model.b= beta estimate

% Model.bse= se(beta) estimate
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```
% Model.h= h estimate

% Model.t= time points corresponding to h

%%%% The code for the function 'RRT.m' is given below.


function myModel=RRT(Data)

Wp=Data.Wp; Qp=Data.Qp; v=Data.v; C=Data.C; Y=Data.Y; Z=Data.Z;

m1=min(C(Y==1)); m2=max(C(Y==0)); del=((C<m1 & Y==0) | (C>m2 & Y==1));

C(del==1)=[]; Y(del==1)=[]; Z(del==1,:)=[];

t=unique(C(Y==1)); J=length(t);

AC=C(Y==1); AZ=Z(Y==1,:); nA=length(AC);

BC=C(Y==0); BZ=Z(Y==0,:); nB=length(BC);

AtC=( (t*ones(1,nA)) <= (ones(J,1)*AC') );

BtC=( (t*ones(1,nB)) <= (ones(J,1)*BC') );

tt=(t*ones(1,J)<=ones(J,1)*t');

d=size(Z,2); b=zeros(d,1); h=ones(J,1)/J;

loop=1;  judge=0;

while (loop<500 && judge<1)

    old=[b;h]';

    AeH =exp(AZ*b)*(tt'*h)';  AF =1-(1+v*AeH).^(-1/v);

    AeHC=exp(AZ*b).*(AtC'*h); AFC=1-(1+v*AeHC).^(-1/v);

    BeHC=exp(BZ*b).*(BtC'*h); BFC=1-(1+v*BeHC).^(-1/v);

    AF(AF==1)=1-1e-16; AFC(AFC==1)=1-1e-16; BFC(BFC==1)=1-1e-16;

    AQ=AFC*Qp./(AFC*Qp+Wp*(1-Qp));

    BQ=(1-BFC)*Qp./((1-BFC)*Qp+(1-Wp)*(1-Qp));

    Afij=(1-AF).^(1+v).*(exp(AZ*b)*h').*AtC';

    AN=Afij./(sum(Afij,2)*ones(1,J));

    ANQ=AN.*(AQ*ones(1,J));
```

```
    AWo=-(1+v)*(1-AF).^v.*(exp(AZ*b)*ones(1,J));

    Bwo=-(1-BFC).^v.*exp(BZ*b);

    h=( sum(ANQ)./( -sum((ANQ.*AWo)*double(tt)')- (BtC*(BQ.*Bwo))' ) )';

    AW=-(1+v)*(1-AF).^v.*AeH;

    Bw=-(1-BFC).^v.*BeHC;

    db=AZ'*sum(ANQ,2)+AZ'*(sum( ANQ.*AW,2 ))+BZ'*(BQ.*Bw);

    dAW=v*(1+v)*(1-AF).^(2*v).*AeH.^2+AW;

    dBw=v*(1-BFC).^(2*v).*BeHC.^2+Bw;

    dbb=AZ'*( repmat(sum(ANQ.*dAW,2),1,d).*AZ )+BZ'*( repmat(BQ.*dBw,1,d).*BZ );

    b=b-dbb\db;

    new=[b;h]';

    judge=mean(abs(new-old)<10^(-4));

    loop=loop+1;

end

myModel.b=b;  myModel.h=h; myModel.t=t;

[n,d]=size(Z);

tC=( (t*ones(1,n)) <= (ones(J,1)*C') );

U=C; nU=length(U);

eHui=(tC'*h)*exp(Z*b)';Fui=1-(1+v*eHui).^(-1/v); Fui(Fui==1)=1-1e-16;

dFui=(1-Fui).^(1+v); EYui=Fui*Qp+Wp*(1-Qp);

wetui=(Qp.*dFui.*eHui).^2./(EYui.*(1-EYui));

hd1=(4/(d+2))^(1/(d+4))*n^(-1/(d+4));

hd2=(4/(d+3))^(1/(d+5))*n^(-1/(d+5));

hC=std(C)*hd2; KCuk=exp( -( U*ones(1,n)./hC-ones(nU,1)*C'./hC ).^2 )./hC;

KZik=1; BZik=1;

for dd=1:d

    Zd=Z(:,dd);
```

```
    hZd=std(Zd)*hd2;

    bZd=std(Zd)*hd1;

    KZik=KZik.*exp( -( Zd*ones(1,n)./hZd-ones(n,1)*Zd'./hZd ).^2 )./hZd;

    BZik=BZik.*exp( -( Zd*ones(1,n)./bZd-ones(n,1)*Zd'./bZd ).^2 )./bZd;

end

fCuZi=(KCuk*(KZik)')./n;

fZi=sum(BZik,2)./n;

pCuZi=fCuZi./repmat(fZi',nU,1);

gson=(pCuZi.*wetui)*Z;

gmon=sum(pCuZi.*wetui,2);

g0=gson./repmat(gmon,[1,d]);

eH=exp(Z*b).*(tC'*h); F=1-(1+v*eH).^(-1/v); F(F==1)=1-1e-16;

dF=(1-F).^(1+v); EY=F*Qp+Wp*(1-Qp);

wet0=Qp.*dF.*eH./(EY.*(1-EY));

mb=(Z-g0).*repmat(wet0.*(Y-EY),[1,d]);

BBB=mb'*mb/n;

wet1=(Qp.*dF.*eH).^2./(EY.*(1-EY));

AAA=-(Z-g0)'*(repmat(wet1,[1,d]).*Z)/n;

DDD=diag( AAA\BBB*inv(AAA)' );

myModel.bse=sqrt(DDD/n);
```

S2 Code for related-question RRT estimation

```
%%%% related-question RRT estimation

%%%% (1) Run

% Model=rRRT(Data);
```

S2. CODE FOR RELATED-QUESTION RRT ESTIMATION

```
%%%% (2) Input

% Data.v= 1 if PO model; 0.001 if PH model

% Data.Qp= probability of choosing the sensitive question

% Data.C= survey time (nx1 column vector)

% Data.X= observed response (nx1 column vector)

% Data.Z= covariate (nxd matrix)

%%%% (3) Output

% Model.b= beta estimate

% Model.bse= se(beta) estimate

% Model.h= h estimate

% Model.t= time points corresponding to h

%%%% The code for the function ‘rRRT.m’ is given below.


function myModel=rRRT(Data)

Qp=Data.Qp; v=Data.v; C=Data.C; Y=Data.X; Z=Data.Z;

m1=min(C(Y==1)); m2=max(C(Y==0)); del=((C<m1 & Y==0) | (C>m2 & Y==1));

C(del==1)=[]; Y(del==1)=[]; Z(del==1,:)=[];

t=unique(C(Y==1)); J=length(t);

AC=C(Y==1); AZ=Z(Y==1,:); nA=length(AC);

BC=C(Y==0); BZ=Z(Y==0,:); nB=length(BC);

AtC=( (t*ones(1,nA)) <= (ones(J,1)*AC’) );

BtC=( (t*ones(1,nB)) <= (ones(J,1)*BC’) );

tt=(t*ones(1,J)<=ones(J,1)*t’);

d=size(Z,2); b=zeros(d,1); h=ones(J,1)/J;

loop=1;  judge=0;

while (loop<500 && judge<1)

    old=[b;h]’;
```

```
AeH =exp(AZ*b)*(tt'*h)';  AF =1-(1+v*AeH).^(-1/v); AF(AF==1)=1-1e-16;

BeH =exp(BZ*b)*(tt'*h)';  BF =1-(1+v*BeH).^(-1/v); BF(BF==1)=1-1e-16;

AeHC=exp(AZ*b).*(AtC'*h); AFC=1-(1+v*AeHC).^(-1/v); AFC(AFC==1)=1-1e-16;

BeHC=exp(BZ*b).*(BtC'*h); BFC=1-(1+v*BeHC).^(-1/v); BFC(BFC==1)=1-1e-16;

AQ=AFC*Qp./(AFC*Qp+(1-AFC)*(1-Qp));

BQ=(1-BFC)*Qp./((1-BFC)*Qp+BFC*(1-Qp));

Afij=(1-AF).^(1+v).*(exp(AZ*b)*h').*AtC'; AN=Afij./(sum(Afij,2)*ones(1,J));

Bfij=(1-BF).^(1+v).*(exp(BZ*b)*h').*BtC'; BN=Bfij./(sum(Bfij,2)*ones(1,J));

ANQ=AN.*(AQ*ones(1,J));

BNQb=BN.*((1-BQ)*ones(1,J));

AWo=-(1+v)*(1-AF).^v.*(exp(AZ*b)*ones(1,J)); Awo=-(1-AFC).^v.*exp(AZ*b);

BWo=-(1+v)*(1-BF).^v.*(exp(BZ*b)*ones(1,J)); Bwo=-(1-BFC).^v.*exp(BZ*b);

son1=sum(ANQ);

son2=sum(BNQb);

mon1=-sum((ANQ.*AWo)*double(tt)')- (BtC*(BQ.*Bwo))';

mon2=-sum((BNQb.*BWo)*double(tt)')- (AtC*((1-AQ).*Awo))';

h=( (son1+son2)./( mon1+mon2 ) )';

AW=-(1+v)*(1-AF).^v.*AeH; Aw=-(1-AFC).^v.*AeHC;

BW=-(1+v)*(1-BF).^v.*BeH; Bw=-(1-BFC).^v.*BeHC;

db1=AZ'*sum(ANQ,2)+AZ'*(sum( ANQ.*AW,2 ))+BZ'*(BQ.*Bw);

db2=BZ'*sum(BNQb,2)+BZ'*(sum( BNQb.*BW,2 ))+AZ'*((1-AQ).*Aw);

dAW=v*(1+v)*(1-AF).^(2*v).*AeH.^2+AW; dAw=v*(1-AFC).^(2*v).*AeHC.^2+Aw;

dBW=v*(1+v)*(1-BF).^(2*v).*BeH.^2+BW; dBw=v*(1-BFC).^(2*v).*BeHC.^2+Bw;

dbb1=AZ'*(repmat(sum(ANQ.*dAW,2),1,d).*AZ)+BZ'*(repmat(BQ.*dBw,1,d).*BZ);

dbb2=BZ'*(repmat(sum(BNQb.*dBW,2),1,d).*BZ)+AZ'*(repmat((1-AQ).*dAw,1,d).*AZ);

b=b-(dbb1+dbb2)\(db1+db2);

new=[b;h]';
```

```
    judge=mean(abs(new-old)<10^(-4));

    loop=loop+1;

end

myModel.b=b;  myModel.h=h; myModel.t=t; myModel.loop=loop;

[n,d]=size(Z);

tC=( (t*ones(1,n)) <= (ones(J,1)*C') );

U=C; nU=length(U);

eHui=(tC'*h)*exp(Z*b)';Fui=1-(1+v*eHui).^(-1/v); Fui(Fui==1)=1-1e-16;

dFui=(1-Fui).^(1+v); EYui=Fui*Qp+(1-Fui)*(1-Qp);

wetui=((2*Qp-1).*dFui.*eHui).^2./(EYui.*(1-EYui));

hd1=(4/(d+2))^(1/(d+4))*n^(-1/(d+4));

hd2=(4/(d+3))^(1/(d+5))*n^(-1/(d+5));

hC=std(C)*hd2; KCuk=exp( -( U*ones(1,n)./hC-ones(nU,1)*C'./hC ).^2 )./hC;

KZik=1; BZik=1;

for dd=1:d

    Zd=Z(:,dd);

    hZd=std(Zd)*hd2;

    bZd=std(Zd)*hd1;

    KZik=KZik.*exp( -( Zd*ones(1,n)./hZd-ones(n,1)*Zd'./hZd ).^2 )./hZd;

    BZik=BZik.*exp( -( Zd*ones(1,n)./bZd-ones(n,1)*Zd'./bZd ).^2 )./bZd;

end

fCuZi=(KCuk*(KZik)')./n;

fZi=sum(BZik,2)./n;

pCuZi=fCuZi./repmat(fZi',nU,1);

gson=(pCuZi.*wetui)*Z;

gmon=sum(pCuZi.*wetui,2);

g0=gson./repmat(gmon,[1,d]);
```

```
eH=exp(Z*b).*(tC'*h); F=1-(1+v*eH).^(-1/v); F(F==1)=1-1e-16;

dF=(1-F).^(1+v); EY=F*Qp+(1-F)*(1-Qp);

wet0=(2*Qp-1).*dF.*eH./(EY.*(1-EY));

mb=(Z-g0).*repmat(wet0.*(Y-EY),[1,d]);

BBB=mb'*mb/n;

wet1=((2*Qp-1).*dF.*eH).^2./(EY.*(1-EY));

AAA=-(Z-g0)'*(repmat(wet1,[1,d]).*Z)/n;

DDD=diag( AAA\BBB*inv(AAA)' );

myModel.bse=sqrt(DDD/n);
```

S3 Analysis of TSCS data

The following table shows the estimated proportions of males and females having extramarital relations by years since marriage, which are also depicted in Figure 3 of the main paper.

Table S1. The estimates of proportions of males and females having extramarital relations by years since marriage.

	PH		PO			PH		PO	
Year	Male	Female	Male	Female	Year	Male	Female	Male	Female
0.5	0.065	0.019	0.042	0.010	25.0	0.422	0.149	0.424	0.147
1.0	0.104	0.032	0.073	0.018	26.0	0.422	0.149	0.424	0.148
1.5	0.138	0.043	0.102	0.026	27.0	0.423	0.149	0.425	0.148
2.0	0.166	0.052	0.128	0.033	27.5	0.423	0.149	0.430	0.150
2.5	0.194	0.062	0.155	0.041	28.0	0.423	0.149	0.431	0.151
3.0	0.219	0.070	0.180	0.049	29.0	0.423	0.149	0.431	0.151
3.5	0.242	0.078	0.204	0.057	30.0	0.423	0.149	0.431	0.151
4.0	0.252	0.082	0.219	0.062	30.5	0.423	0.149	0.431	0.151
4.5	0.255	0.083	0.230	0.066	31.0	0.423	0.149	0.431	0.151
5.0	0.258	0.084	0.240	0.069	31.5	0.423	0.149	0.431	0.151
6.0	0.276	0.091	0.258	0.076	32.0	0.423	0.149	0.431	0.151
7.0	0.288	0.095	0.273	0.081	33.0	0.423	0.149	0.431	0.151
7.5	0.313	0.105	0.295	0.089	34.0	0.423	0.149	0.431	0.151
8.0	0.338	0.114	0.317	0.098	35.0	0.423	0.149	0.431	0.151
9.0	0.359	0.123	0.336	0.106	36.0	0.423	0.149	0.431	0.151
10.0	0.382	0.132	0.357	0.115	37.0	0.423	0.149	0.431	0.151
10.5	0.391	0.136	0.369	0.121	38.0	0.423	0.149	0.431	0.151
11.0	0.397	0.138	0.379	0.125	39.0	0.423	0.149	0.431	0.151
12.0	0.402	0.141	0.387	0.129	40.0	0.423	0.149	0.431	0.151
13.0	0.402	0.141	0.388	0.130	41.0	0.423	0.149	0.431	0.151
14.0	0.404	0.141	0.393	0.132	42.0	0.423	0.149	0.431	0.151
15.0	0.405	0.142	0.396	0.134	43.0	0.423	0.149	0.431	0.151
16.0	0.405	0.142	0.401	0.136	44.0	0.423	0.149	0.431	0.151
17.0	0.414	0.145	0.410	0.140	45.0	0.423	0.149	0.431	0.151
18.0	0.422	0.149	0.418	0.144	46.0	0.423	0.149	0.431	0.151
19.0	0.422	0.149	0.420	0.145	47.0	0.423	0.149	0.431	0.151
20.0	0.422	0.149	0.420	0.145	48.0	0.423	0.149	0.431	0.151
20.5	0.422	0.149	0.423	0.147	49.0	0.423	0.149	0.431	0.151
21.0	0.422	0.149	0.424	0.147	50.0	0.423	0.149	0.431	0.151
22.0	0.422	0.149	0.424	0.147	52.0	0.423	0.149	0.431	0.151
23.0	0.422	0.149	0.424	0.147	53.0	0.423	0.149	0.431	0.151
24.0	0.422	0.149	0.424	0.147	54.0	0.423	0.149	0.431	0.151
24.5	0.422	0.149	0.424	0.147	57.0	0.423	0.149	0.431	0.151