## Long-memory Log-linear Zero-inflated Generalized Poisson Autoregression for Covid-19 Pandemic Modeling Supplementary Materials

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Figure A displays the time series plots of two exogenous variables (i.e., policy index and temperature) in VNM, GBR, and BRA. It shows that the policy index displays a quick increasing trend first and keeps relatively stable at 60-80 indice for VNM, while there exists a lightly decreasing trend for GBR and BRA from around March 2021.

Figures B-D display the h = 1, 2, 7 and 14-day ahead forecasting using the LFIGX(1, d, 1)model for VNM, ITA, and USA, respectively. It shows that the LFIGX(1, d, 1) model delivers accurate forecasting to the three datasets, although the forecast gets worse slightly along with the forecast horizons. Even for the longer period forecasting of 7-days, the LFIGX(1,d,1) model still captures the dynamics of the daily new cases series well with



Figure A: The policy index and temperature at VNM, GBR, and BRA.

good prediction, while 14-days ahead forecast is worse than other horizons with a delayed prediction and underestimation of the last wave peak for ITA and USA.



Figure B: The *h*-day ahead forecasting using the LFIGX(1, d, 1) model with h = 1, 2, 7 and 14 for ITA.



Figure C: The *h*-day ahead forecasting using the LFIGX(1, d, 1) model with h = 1, 2, 7 and 14 for VNM.



Figure D: The *h*-day-ahead forecasting using the LFIGX(1, d, 1) model with h = 1, 2, 7 and 14 for USA.

Moreover, as pointed by one referee, for USA, analyzing USA as one region could be problematic given the large land area of this country, we have also analyzed the temperature and policy impacts of two selected states of USA: TX (Texas) and CA (California). In specific, for temperature variable, the fact that higher temperature at summer months and cool weather at winter semesters is stable to the whole country, although the temperature is not "exactly same". In fact, we have realized the problem of large land area of USA, we selected four states from USA: CA (California), NJ (New Jersey), TX (Texas), MN (Minnesota). We collected the average temperature of the four states and take the average to use in the main paper.

The temperature of the four states and the average of four states over the same period in the paper are shown in Figure E. We can find certain similar seasonality pattern of the temperature at four states, though they do exist magnitude difference.



Figure E: The average temperature of the four states respectively, and the average among four states over the same data period in USA.

We do observe some differences in the effect of policy index at different states or considering the whole USA country. As illustration, we use the data from the two states, TX and CA, respectively and conduct in-sample analysis to investigate the effects of exogenous covariates. Table A presents the estimated coefficients of policy index and temperature variables using the first 500 observations of USA, TX, and CA from 23 Jan 2020, 26 Jan 2023 and 5 Mar 2023, respectively. It shows that for the longer horizons of h = 7 and 14, higher temperatures have a positive effect on the number of new incidences of Covid-19, indicating that warm weather tends to cause an increase in Covid-19 cases after 1 and 2 weeks in both TX and CA. As for the policy index, USA and CA display a stronger delayed effect with a larger magnitude at h = 14 compared to an immediate effect at h = 1, 2. For TX, both the immediate effect and delayed effect of policies after 2 weeks of implementation are significant.

Table A: The estimated coefficients for policy index and temperature variable with h = 1, 2, 7 and 14 using the first 500 observations of data in USA, TX and CA.

Horizon	USA		TX		CA	
h	Policy index	Temperature	Policy index	Temperature	Policy index	Temperature
1	0.002	0.000	0.039	-0.003	-0.005	0.002
2	0.015	0.001	0.062	-0.005	-0.003	0.001
7	-0.004	0.002	0.014	0.003	0.006	0.003
14	0.021	0.006	0.035	0.002	0.010	0.006

Although some similarities and differences are both observed by using single-state data and the whole country data for USA, we use the latter in the main paper as an illustration, and put the state-level results in the Supplementary Material for reference.