Sufficient variable screening for ultrahigh-dimensional right censored data via independence measures

Baoying Yang[1], Qingcong Yuan[2,4], and Xiangrong Yin[3]

[1]*Southwest Jiaotong University,* [2]*Miami University,*

[3]*University of Kentucky and* [4]*Biostatistics and Programming*

Supplementary Material

In this supplementary file, section A.1 provides the comprehensive computation of equations (1), (2), and (3); Section A.2 gives an estimation of $G(t|\mathbf{X})$ across various scenarios; Section A.3 presents rigorous proofs for Theorems 1, 2, and 3. Section A. 4 contains additional simulation instances, real data analysis, and corresponding results.

A.1 Detailed Calculation of Equations (3.1), (3.2), and (3.3)

Calculation of Equation (3.1) To obtain that $dcov_\alpha^2(X_\alpha, T) = dcov_\alpha^2(X_\alpha, Y)$, we need to prove that,

$$
\begin{aligned}
S_1 &= E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|T-T'\|_{d_T}\right] = E\left(\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y'\|_{d_Y}\right),\\
S_2 &= E\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|T-T'\|_{d_T}\right] = E\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\|Y-Y'\|_{d_Y},\\
S_3 &= E\left[\frac{\delta\delta'\delta''}{G(T|\mathbf{X})G(T'|\mathbf{X}')G(T''|\mathbf{X}'')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|T-T''\|_{d_T}\right] = E\left(\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y''\|_{d_Y}\right),
\end{aligned}
$$

In detail,

$$
\begin{aligned}
S_1 =& E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|T-T'\|_{d_T}\right)\\
=&E\left[E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|T-T'\|_{d_T} \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=&E\left[\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\left(\frac{\delta\delta'\|T-T'\|_{d_T}}{G(T|\mathbf{X})G(T'|\mathbf{X}')} \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=&E\left[\frac{\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y'\|_{d_T}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(\delta=1,\delta'=1 \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=&E\left[\frac{\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y'\|_{d_T}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(\delta=1 \mid (\mathbf{X},Y)\right) Pr\left(\delta'=1 \mid (\mathbf{X}',Y')\right)\right]\\
=&E\left[\frac{\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y'\|_{d_T}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(C>Y \mid (\mathbf{X},Y)\right) Pr\left(C'>Y' \mid (\mathbf{X}',Y')\right)\right]\\
=&E\left(\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y'\|_{d_T}\right)
\end{aligned}
$$

$$
\begin{aligned}
S_2 =& E\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|T-T'\|_{d_T}\right]\\
=&E\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\left[E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|T-T'\|_{d_T} \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=&E\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\left[\frac{\|Y-Y'\|_{d_T}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(\delta=1,\delta'=1 \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=&E\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} E\|Y-Y'\|_{d_T}
\end{aligned}
$$

$$
\begin{aligned}
S_3 =& E\left(\frac{\delta\delta'\delta''}{G(T|\mathbf{X})G(T'|\mathbf{X}')G(T''|\mathbf{X}'')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|T-T''\|_{d_T}\right)\\
=&E\left[E\left(\frac{\delta\delta''}{G(T|\mathbf{X})G(T'|\mathbf{X}')G(T''|\mathbf{X}'')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|T-T''\|_{d_T} \mid (\mathbf{X},\mathbf{X}',\mathbf{X}'',Y,Y',Y'')\right)\right]\\
=&E\left[\frac{\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y''\|_{d_T}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')G(Y''|\mathbf{X}'')} Pr\left(\delta=1,\delta'=1,\delta''=1 \mid (\mathbf{X},\mathbf{X}',\mathbf{X}'',Y,Y',Y'')\right)\right]\\
=&E\left(\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|Y-Y''\|_{d_T}\right),
\end{aligned}
$$

where $G(t|\mathbf{X}) \overset{\text{def}}{=} P(C>t|\mathbf{X})$ is the survival function of censoring time C.

Calculation of Equation (3.2) To obtain that ${dcov}^{*}_{\alpha}{}^{2}(\mathbf{U},\mathbf{W}) = {dcov_\alpha}^2(\mathbf{U},\mathbf{V})$, we need to prove that,

$$S_1 = E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}}\right) = E\left(\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}}\right),$$

$$S_2 = E\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}}\right] = E\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}},$$

$$S_3 = E\left(\frac{\delta\delta'\delta''}{G(T|\mathbf{X})G(T'|\mathbf{X}')G(T''|\mathbf{X}'')}\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{W}-\mathbf{W}''\|_{d_\mathbf{V}}\right) = E\left(\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}''\|_{d_\mathbf{V}}\right),$$

In detail,

$$\begin{aligned}
S_1 =& E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}}\right)\\
=& E\left[E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}} \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=& E\left[\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}} \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=& E\left[\frac{\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(\delta=1,\delta'=1 \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=& E\left[\frac{\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(\delta=1|(\mathbf{X},Y)\right) Pr\left(\delta'=1 \mid (\mathbf{X}',Y')\right)\right]\\
=& E\left(\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}}\right)
\end{aligned}$$

$$\begin{aligned}
S_2 =& E\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}}\right]\\
=& E\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\left[E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{W}-\mathbf{W}'\|_{d_\mathbf{V}} \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=& E\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\left[\frac{\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')} Pr\left(\delta=1,\delta'=1 \mid (\mathbf{X},\mathbf{X}',Y,Y')\right)\right]\\
=& E\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}} E\|\mathbf{V}-\mathbf{V}'\|_{d_\mathbf{V}}
\end{aligned}$$

$$\begin{aligned}
S_3 =& E\left(\frac{\delta\delta'\delta''}{G(T|\mathbf{X})G(T'|\mathbf{X}')G(T''|\mathbf{X}'')}\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{W}-\mathbf{W}''\|_{d_\mathbf{V}}\right)\\
=& E\left[E\left(\frac{\delta\delta'\delta''}{G(T|\mathbf{X})G(T'|\mathbf{X}')G(T''|\mathbf{X}'')}\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{W}-\mathbf{W}''\|_{d_\mathbf{V}} \mid (\mathbf{X},\mathbf{X}',\mathbf{X}'',Y,Y',Y'')\right)\right]\\
=& E\left[\frac{\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')G(Y''|\mathbf{X}'')} E\left(\delta\delta'\delta''\|\mathbf{W}-\mathbf{W}''\|_{d_\mathbf{V}} \mid (\mathbf{X},\mathbf{X}',\mathbf{X}'',Y,Y',Y'')\right)\right]\\
=& E\left[\frac{\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}''\|_{d_\mathbf{V}}}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')G(Y''|\mathbf{X}'')} Pr\left(\delta=1,\delta'=1,\delta''=1 \mid (\mathbf{X},\mathbf{X}',\mathbf{X}'',Y,Y',Y'')\right)\right]\\
=& E\left(\|\mathbf{U}-\mathbf{U}'\|_{d_\mathbf{U}}\|\mathbf{V}-\mathbf{V}''\|_{d_\mathbf{V}}\right)
\end{aligned}$$

Calculation of Equation (3.3)

The numerator of $S_{1,s}$ is:

$$
\begin{aligned}
&E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}}\mathbf{1}\{T \in [l_{s-1}, l_s)\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}\right] \\
=&E\left[\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}}E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\mathbf{1}\{T \in [l_{s-1}, l_s)\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right)\right]
\end{aligned}
$$

The last term inside the expectation above can be derived as follows:

$$
\begin{aligned}
&E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\mathbf{1}\{T \in [l_{s-1}, l_s)\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right) \\
=&E\left[E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\mathbf{1}\{T \in [l_{s-1}, l_s)\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}|(Y, Y', \mathbf{X}, \mathbf{X}')\right)|((\mathbf{X}, \mathbf{X}'))\right] \\
=&E\left[\frac{1}{G(Y|\mathbf{X})G(Y'|\mathbf{X}')}\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}Pr\left(Y < C, Y' < C'|(Y, Y', \mathbf{X}, \mathbf{X}')\right)|((\mathbf{X}, \mathbf{X}'))\right] \\
=&E\left(\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right),
\end{aligned}
$$

Thus, the numerator of $S_{1,s}$ has the following equivalence, which shows its connection with the noncensored case:

$$
\begin{aligned}
&E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}}\mathbf{1}\{T \in [l_{s-1}, l_s)\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}\right] \\
=&E\left[\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}}E\left(\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right)\right] \\
=&E\left[E\left(\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}}\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right)\right] \\
=&E\left[\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}}\|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}}\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}\right]
\end{aligned}
$$

Following similar proofs as above, the denominator of $S_{1,s}$ is:

$$
\begin{aligned}
&E\left[\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\mathbf{1}\{T \in [l_{s-1}, l_s)\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}\right] \\
=&E\left[E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\mathbf{1}\{T \in [l_s, l_{s+1})\}\mathbf{1}\{T' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right)\right] \\
=&E\left[E\left(\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}|(\mathbf{X}, \mathbf{X}')\right)\right] \\
=&E\left(\mathbf{1}\{Y \in [l_{s-1}, l_s)\}\mathbf{1}\{Y' \in [l_{s-1}, l_s)\}\right) \\
=&Pr\{Y \in [l_{s-1}, l_s)\}Pr\{Y' \in [l_{s-1}, l_s)\}.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
S_{1,s} =& \frac{E\left[\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} \|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}} \mathbf{1}\{Y \in [l_{s-1}, l_s)\} \mathbf{1}\{Y' \in [l_{s-1}, l_s)\}\right]}{Pr\{Y \in [l_{s-1}, l_s)\} Pr\{Y' \in [l_{s-1}, l_s)\}} \\
=& E\left[\|X_\alpha - X'_\alpha\|_{d_{X_\alpha}} \|\mathbf{X}_{-\alpha} - \mathbf{X}'_{-\alpha}\|_{d_{\mathbf{X}_{-\alpha}}} | Y, Y' \in [l_{s-1}, l_s)\right]
\end{aligned}
$$

Following a similar approach, we can prove that $S_{2,s}$ and $S_{3.s}$ have similar results as $S_{1,s}$. Therefore, equation (3.3) holds.

A.2 Estimation for $G(t|\mathbf{X})$

To estimate $G(t|\mathbf{X})$, the simplest approach is to assume that $G(t|\mathbf{X}) = G(t)$. Following Zhou and Zhu (2017), we will explore other cases for estimating $G(t|\mathbf{X})$ and identify the corresponding conditions required for the proof of Theorems.

1. Assume that $G(t|\mathbf{X}) = G(t)$.

Similar assumptions were also employed by He et al. (2013), Song et al. (2014), Zhou and Zhu (2017), and Zhang et al. (2018). With this assumption, estimating $G(t|\mathbf{X})$ amounts to estimating $G(t)$.

2. Instead of using $G(t|\mathbf{X})$, we use $G_\alpha(t|X_\alpha) = Pr(C > t|X_\alpha)$.

This assumption was suggested by He et al. (2013) and Zhou and Zhu (2017). The direct utilization of $G(t|\mathbf{X})$ may encounter computational challenges when dealing with high dimensional data. Additionally, this assumption leads to rank consistency and sure screening properties. In the subsequent discussions, we present various estimators for $G_\alpha(t|X_\alpha)$, depending on the characteristics of X_α. For details, a) Regarding a categorical or discrete random variable X_α with a finite number of potential values, we can estimate $G_\alpha(t|X_\alpha)$ within each level or category of X_α through the classical Kaplan-Meier method. For example, if X_α has H categories, the estimator for each category can be denoted as $\widehat{G}_\alpha(t|X_\alpha = h)$, where $h = 1, 2, \cdots, H$.

b) A continuous random variable X_α can be sliced into H nonoverlapping and nonrandom intervals. Employing similar ideas as in the categorical/discrete case, we can estimate $G_\alpha(t|X_\alpha)$ using the Kaplan-Meier method. We denote the slices as $I_1, \ldots, I_H$, and assume that $G_\alpha(t|X_\alpha = x) = G_\alpha(t|X_\alpha \in I_h)$ for $x \in I_h$. The estimator for each slice can be denoted as $\widehat{G}_\alpha(t|X_\alpha \in I_h)$.

3. It is possible to posit a semiparametric model, such as a proportional hazards model, and then estimate $G(T|\mathbf{X})$ based on the fitted model. This approach is straightforward to employ, and as long as the model is correctly specified, and the resulting estimator of the survival function is consistent. This assumption was suggested by Lu and Li (2011).

4. Assume that $G(T|\mathbf{X}) = G(T|B\mathbf{X})$, where B is a $d \times p$ matrix with $d < n$.

To directly estimate $G(T|\mathbf{X})$, the kernel smoothing method is a useful nonparametric method. However, the kernel smoothing method performs optimally only in low dimensional scenarios. Hence, it is necessary to reduce the dimension first. The simplest approach is double slicing proposed by Li (1991) to reduce the dimension of $\mathbf{X}$. It is equivalent to the usual sliced inverse regression (SIR) method, except that Y is partitioned within each subsample. Although the double slicing SIR is simple to use, it imposes some restrictive conditions and relies on a nonparametric estimation of the conditional survival function of Y given $\mathbf{X}$, which itself is computationally complicated. Xia et al. (2010) developed a dimension reduction method for censored survival data based on kernel estimation of the conditional hazard function, but this method may suffer from the curse of dimensionality when the number of predictors is large.

Once the dimension is reduced from p to r, following Beran (1981) and Dabrowska (1989), we apply d-dimension kernel smoothing on $\widehat{B}_d\mathbf{X}$ to obtain the estimate of $G(t|\widehat{B}_d\mathbf{X})$.

$$G(t|\widehat{B}_d\mathbf{X}) = \prod_{i=1}^{n}\{1 - \frac{w_{ni}(x;h_n)}{\sum_{k=1}^{n} I(T_k \geq T_i)w_{nk}(x;h_n)}\}^{I(T_i \leq t, \delta_i = 0)}$$

with

$$w_{ni}(x;h_n) = \frac{K(h_n^{-1}\widehat{B}_d(\mathbf{X}_i - x))}{\sum_{j=1}^{n} K(h_n^{-1}\widehat{B}_d(\mathbf{X}_j - x))} \quad i = 1, \cdots, n,$$

where K is a density function, and $\{h_n\}$ is the bandwidth.

Furthermore, the conditions required for proving the Theorems are as follows, case by case, in detail:

($C3.1$) If $G(t \mid \mathbf{X}) = G(t)$, then we assume $Pr(t \leq Y \leq C) \geq \tau_0 > 0$ for $t \in [0, T_{\max}]$, where $T_{\max}$ represents the maximum follow-up time. Additionally, $sup\{t : Pr(Y > t) > 0\} \geq sup\{t : pr(C > t) > 0\}$ and $G'_\alpha(t)$, the first derivative of $G_\alpha(t)$, is uniformly bounded.

($C3.2$) If we use $G(t|X_\alpha)$ instead of $G(t \mid \mathbf{X})$, and assume that X_α is categorical or discrete, then we assume $Pr(t \leq Y \leq C \mid X_\alpha = h) \geq \tau_0 > 0$ for $t \in [0, T_{\max}]$, where $T_{\max}$ is the maximum follow-up time. Additionally, $sup\{t : Pr(Y > t \mid X_\alpha = h) > 0\} \geq sup\{t : pr(C > t|X_\alpha = h) > 0\}$ and $G'_\alpha(t \mid X_\alpha = h)$, the first derivative of $G_\alpha(t \mid X_\alpha = h)$, is uniformly bounded.

($C3.3$) If we use $G(t|X_\alpha)$ instead of $G(t \mid \mathbf{X})$, and if X_α is continuous, we slice the range of X_α into H nonoverlapping and nonrandom intervals, denoted as $I_1, \cdots, I_H$, and assume that $G(t|X_\alpha = x) = G(t|X_\alpha \in I_h)$, for $x \in I_h$. Then we assume $Pr(t \leq Y \leq C \mid X_\alpha \in I_h) \geq \tau_0 > 0$ for $t \in [0, T_{\max}]$, where $T_{\max}$ is the maximum follow-up time. Additionally, $sup\{t : Pr(Y > t \mid X_\alpha \in I_h) > 0\} \geq sup\{t : pr(C > t|X_\alpha \in I_h) > 0\}$ and $G'_\alpha(t \mid X_\alpha \in I_h)$, the first derivative of $G_\alpha(t \mid X_\alpha \in I_h)$, is uniformly bounded.

($C3.4$) We assume $\inf_x Pr(t \leq Y \leq C \mid \mathbf{X}) \geq \tau_0 > 0$ for $t \in [0, T_{\max}]$, where $T_{\max}$ is the maximum follow-up time. $G(t \mid \mathbf{X})$ has first derivatives with respect to t, which are uniformly bounded away from infinity, and $G(t \mid \mathbf{X})$ has bounded (uniformly in t) second-order partial derivatives with respect to $\mathbf{X}$. Additionally, $t_0 \leq sup\{t : G(t \mid \mathbf{X}) > 0\} \leq t_1$ uniformly in $\mathbf{X}$ for some positive constants t_0 and t_1, and $sup\{t : Pr(Y > t \mid \mathbf{X}) > 0\} \geq$

$sup\{t : G(t \mid \mathbf{X}) > 0\}$ almost surely for $\mathbf{X}$.

Conditions (C3.1)-(C3.4) are commonly found in the survival analysis literature to ensure that the Kaplan-Meier estimator and its inverse function are well behaved. Conditions (C3.1)-(C3.3) coincide with the conditions in Zhou and Zhu (2017). Condition (C3.4) is an extension of (C3.3). It is similar to $(C6')$ in He et al. (2013), but the dimension of x considered in $(C6')$ in He et al. (2013) is one dimension.

A.3 Proofs of Theorems

Firstly, we establish Theorem 2 and demonstrate that the results hold dimension-free with regard to variables. Subsequently, we establish the marginal conclusion of Theorem 1 as a specific instance of Theorem 2, which can be proven without difficulty. Finally, we prove the corresponding outcome of Theorem 3.

Lemma 1 (Hoeffding (1963), Deviation bound for U-statistics) *Let* $g(\mathbf{U}_1, \cdots, \mathbf{U}_r)$ *be a kernel of the U-statistic* U_n, *where* $U_n = \frac{1}{(n)_r}\sum_{i_r^n} g(\mathbf{U}_{i_1}, \cdots, \mathbf{U}_{i_r})$, $n \geq r$, $(n)_r = \frac{n!}{(n-r)!}$ *and* $\sum_{i_r^n}$ *is taken over all* $r-$*tuples* $i_1, \cdots, i_r$ *drawn without replacement from* $1, \cdots, n$. *If* $a \leq g(\mathbf{U}_1, \cdots, \mathbf{U}_r) \leq b$, *then for any* $t > 0$ *and* $w = [n/r]$ *the largest integer contained in* n/r, *the following bound holds:*

$$P\{U_n - EU_n \geq t\} \leq \exp(-2wt^2/(b-a)^2). \tag{S0.1}$$

If we treat $G(t|\mathbf{X})$ as $G(t)$, then we can follow the lemma:

Lemma 2 (Lo and Singh (1986)) *Under Condition (C3), the Kaplan-Meier estimator* $\widehat{G}(\cdot)$ *satisfies: (1)* $sup_{0\leq t\leq T}|\widehat{G}(t) - G(t)| = O\{(\frac{logn}{n})^{\frac{1}{2}}\}$ *almost surely;*

(2) $\frac{1}{\widehat{G}(t)} - \frac{1}{G(t)} = \frac{1}{nG(t)^2}\sum_{g=1}^{n}\xi(T_g, \delta_g, t) + R_n(t)$, *where* $\xi(T_g, \delta_g, t)$, $g = 1, \cdots, n$ *are i.i.d. random variables with mean zero and* $sup_{0\leq t\leq T}|R_n(t)| = O\{(\frac{logn}{n})^{\frac{3}{4}}\}$ *almost surely;*

(3) $sup_{0\leq t\leq T}|\frac{1}{\widehat{G}(t)} - \frac{1}{G(t)}| = O\{(\frac{logn}{n})^{\frac{1}{2}}\}$ *almost surely.*

If we reduce the dimension of $\mathbf{X}$ to $\widehat{B}_d\mathbf{X}$, following Beran (1981) we apply d-dimension kernel smoothing on $\widehat{B}_d\mathbf{X}$ to obtain the estimate $G_n(t|\widehat{B}_d\mathbf{X})$.

Under suitable assumptions, Beran (1981) and Dabrowska (1989) established the consistency of their estimates at rates of convergence (which are slower than the square root of n rate) commonly observed in nonparametric regression. The estimation of the conditional distribution of the response variable, given the covariates in the presence of right censoring, typically use the estimator proposed by Beran (1981). This estimator is often referred to as the conditional Kaplan-Meier estimator, can be viewed as a smooth function of the kernel regression estimator that captures the conditional behavior of the observations (Dabrowska (1989)).

Lemma 3 (Beran (1981), uniform consistency) *Let* $H_{1n}(t|\mathbf{X}=x) = \sum_{i=1}^{n} I(T_i > t, \delta_i = 1)B_{ni}(x)$ *and* $H_{2n}(t|\mathbf{X}=x) = \sum_{i=1}^{n} I(T_i > t)B_{ni}(x)$ *be the strongly consistent estimators of* $H_1(t|x) = P(T > t, \delta = 1|\mathbf{X}=x)$ *and* $H_2(t|x) = P(T > t|\mathbf{X}=x)$*, respectively, i.e., for* $\mu-$*almost all* x, $|H_{in}(t^-|x) - H_i(t^-|x)| \longrightarrow 0$ *a.s. and* $|H_{in}(t^+|x) - H_i(t^+|x)| \longrightarrow 0$ *as* $n \to \infty$ *with* $B_{ni}(x)$ *is a random set of nonnegative weights depending on covariates only. Suppose* $L(z) < sup\{t : H_2(t|\mathbf{X}) > 0\}$*. Then, for* $\mu-$*almost all* x*, as* $n \to \infty$,

$$sup_{0\le t\le L(z)}|G_n(t|x) - G(t|x)| \to 0, \quad a.s.$$

We first prove Theorem 2, and demonstrate that the results are dimension-free of the variables step by step. Subsequently, from this result, we readily deduce the marginal conclusion presented in Theorem 1. To prove these theorems, it is imperative to initially establish Lemma 4. This lemma 4 establishes a uniform bound for the discrepancies between $\widehat{u}_\alpha^*$ and the estimators u_α^* acquired through the screening step, for all $\alpha = 1, \cdots, p$. For random vectors $\mathbf{U}$ and $\mathbf{W}$ with dimensions d_u and d_w respectively, as expounded in Section 2.4 of the main paper, the following lemma ascertains the bound for u_α^* within the context of survival data. Specifically, by setting $\mathbf{U} = X_\alpha$ and $\mathbf{W} = (T, \mathbf{X}_{-\alpha})$, we can establish Theorem 2; whereas assigning $\mathbf{U} = X_\alpha$ and $\mathbf{W} = T$ enables us to substantiate Theorem 1.

Lemma 4 (Bound for u_α^* in survival data) *Under conditions $(C1)$ and $(C3)$, for any $0 < (\nu + \gamma) < \frac{1}{2}$, there exist positive constants $c_1 > 0$ and $c_2 > 0$ such that*

$$\begin{aligned} & Pr\left(|\widehat{u}_\alpha^* - u_\alpha^*| \geq cn^{-\nu}\right) \\ \leq\ & O\left(\left[\exp\left\{-c_1 n^{1-2(\nu+\gamma)}\right\} + n \exp\left(-c_2 n^\gamma \tau_0^2\right)\right]\right). \end{aligned}$$

Proof 1 (of Lemma 4) *By the Cauchy-Schwartz inequality and condition (C3), we have*

$$\begin{aligned} S_1 &= E\left(\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{U} - \mathbf{U}'\|_{d_u}\|\mathbf{W} - \mathbf{W}'\|_{d_w}\right) \\ &\leq \frac{1}{\tau_0^2}\{E[\|\mathbf{U} - \mathbf{U}'\|_{d_u}^2]E[\|\mathbf{W} - \mathbf{W}'\|_{d_w}^2]\}^{1/2} \qquad \text{(S0.2)} \\ &\leq \frac{2}{\tau_0^2}\{E[\|\mathbf{U}\|_{d_u}^2]E[\|\mathbf{W}\|_{d_w}^2]\}^{1/2} = A < \infty \end{aligned}$$

By condition (C1), we have that S_1 is bounded by a constant A, and for any given $\epsilon > 0$, take n large enough such that $\frac{S_1}{n} < 2\epsilon$.

Step 1:

$$\begin{aligned} \widehat{S}_1 =& \frac{1}{n^2}\sum_{i,j=1}^n \frac{\delta_i\delta_j'}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')}\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w}, \\ =& \frac{1}{n^2}\sum_{i,j=1}^n \frac{\delta_i\delta_j'}{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w} \\ &+ \frac{1}{n^2}\sum_{i,j=1}^n \|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w}\delta_i\delta_j'\left(\frac{1}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} - \frac{1}{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}\right), \\ =& \widehat{S}_{11} + \widehat{S}_{12} \end{aligned}$$

For $\widehat{S}_{12}$, we have

$$\begin{aligned} \widehat{S}_{12} =& \frac{1}{n^2}\sum_{i,j=1}^n \|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w}\frac{\delta_i\delta_j'}{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}\left(\frac{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} - 1\right), \\ \leq& max_{i,j}\left|\frac{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} - 1\right|\frac{1}{n^2}\sum_{i,j=1}^n \frac{\delta_i\delta_j'}{G(T_j|\mathbf{X}_j)G(T_j'|\mathbf{X}_j')}\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w} \\ =& max_{i,j}\left|\frac{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} - 1\right|\widehat{S}_{11} \end{aligned}$$

By Condition (C3) and Lemma 2 (or Lemma 3 combined with Taylor expansion), we have that

$$\frac{G(T_i|\mathbf{X}_i)}{\widehat{G}_n(T_i|\mathbf{X}_i)} - 1 = \frac{1}{\widehat{G}(T_i|\mathbf{X}_i)}(G_n(T_i|\mathbf{X}_i) - \widehat{G}(T_i|\mathbf{X}_i)) \to 0,$$

when n goes to infinity. Based on the fact that

$$\frac{AB}{\widehat{A}\widehat{B}} - 1 = (\frac{A}{\widehat{A}} - 1)(\frac{B}{\widehat{B}} - 1) + (\frac{A}{\widehat{A}} - 1) + (\frac{B}{\widehat{B}} - 1)$$

and S_1 is bounded by a constant, we have

$$max_{i,j}\left|\frac{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} - 1\right| S_1 = o_p(1) < \epsilon, \quad as \ \ n \to \infty.$$

Our goal is to obtain the consistency of $\widehat{S}_1$, considering that

$$\begin{aligned}
&Pr(|\widehat{S}_1 - S_1| \geq 6\epsilon) = Pr(\left|\widehat{S}_{11} + max_{i,j}\left|\frac{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} - 1\right|\widehat{S}_{11} - S_1\right| \geq 6\epsilon)\\
\leq& Pr(\left|(o_p(1)+1)(\widehat{S}_{11} - S_1) + cS_1\right| \geq 6\epsilon)\\
\leq& Pr(|\widehat{S}_{11} - S_1| \geq \frac{5\epsilon}{o_p(1)+1})\\
\leq& Pr(|\widehat{S}_{11} - S_1| \geq 4\epsilon)
\end{aligned}$$

To establish the uniform consistency of $\widehat{S}_1$, we need the uniform consistency of $\widehat{S}_{11}$. Define

$$\widehat{S}^*_{11} = \frac{1}{n(n-1)}\sum_{i\neq j}\frac{\delta_i\delta_j'}{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w},$$

which is a usual U-statistic. For $\widehat{S}_{11} = \widehat{S}^*_{11}\frac{n-1}{n}$, it can be easily shown that

$$\begin{aligned}
P(|\widehat{S}_{11} - S_1| \geq 4\epsilon) &= P(|\widehat{S}^*_{11}\frac{n-1}{n} - S_1\frac{n-1}{n} - S_1\frac{1}{n}| \geq 4\epsilon)\\
&\leq P(|\widehat{S}^*_{11} - S_1|\frac{n-1}{n} \geq 4\epsilon - S_1\frac{1}{n})\\
&\leq P(|\widehat{S}^*_{11} - S_1| \geq 2\epsilon).
\end{aligned}$$

Therefore, we show the uniform consistency of $\widehat{S}^*_{11}$. Let $g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}_j', \mathbf{W}_j', \delta_j') = \frac{\delta_i\delta_j'}{G(T_i|\mathbf{X}_i)G(T_j'|\mathbf{X}_j')}\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w}$ be the kernel of $\widehat{S}^*_{11}$. We decompose the kernel function g_1 into two parts $g_1 = g_1\mathbf{1}(g_1 \le M) + g_1\mathbf{1}(g_1 > M)$, where M will be specified later. Note that $\widehat{S}^*_{11}$ can be written as

$$\widehat{S}^*_{11} = \frac{1}{n(n-1)}\sum_{i\neq j} g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\mathbf{1}(g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) \le M)$$
$$+ \frac{1}{n(n-1)}\sum_{i\neq j} g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\mathbf{1}(g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) > M) = \widehat{S}^*_{1,1} + \widehat{S}^*_{1,2}.$$

Accordingly, we decompose S_1 into two parts

$$S_1 = E[g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\mathbf{1}(g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) \le M)]$$
$$+ E[g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\mathbf{1}(g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) > M)] = S_{1,1} + S_{1,2}.$$

Clearly, $\widehat{S}^*_{1,1}$ and $\widehat{S}^*_{1,2}$ are unbiased estimators of $S_{1,1}$ and $S_{1,2}$, respectively.

By using the conclusion of Lemma 1 in this supplement, we can obtain the uniform consistency of the bounded two-order U statistic $\widehat{S}^*_{1,1}$ directly. That is,

$$P\{|\widehat{S}^*_{1,1} - S_{1,1}| \ge \epsilon\} \le 2\exp(-2[n/2]\epsilon^2/M^2) \tag{S0.3}$$

For $S_{1,2}$, with the Cauchy-Schwartz inequality

$$\begin{aligned} S^2_{1,2} &\le E\left\{g^2_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\right\} Pr\left\{g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) > M\right\} \\ &\le E\left\{g^2_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\right\} * I\left\{g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) > M\right\} \end{aligned}$$

Using the fact that $(a^2+b^2)/2 \ge (a+b)^2/4 \ge |ab|$ and condition (C3), we have

$$\begin{aligned} & g^2_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) \\ =\; & \left(\frac{\delta_i\delta'_j}{G(T_i|\mathbf{X}_i)G(T'_j|\mathbf{X}'_j)}\|\mathbf{U}_i - \mathbf{U}'_j\|_{d_u}\|\mathbf{W}_i - \mathbf{W}'_j\|_{d_w}\right)^2 \\ \le\; & \frac{1}{\tau_0^4}\left[(\|\mathbf{U}_i\|_{d_u} + \|\mathbf{U}'_j\|_{d_u})(\|\mathbf{W}_i\|_{d_w} + \|\mathbf{W}'_j\|_{d_w})\right]^2 \\ \le\; & \frac{1}{4\tau_0^4}\left[(\|\mathbf{U}_i\|_{d_u} + \|\mathbf{U}'_j\|_{d_u})^2 + (\|\mathbf{W}_i\|_{d_w} + \|\mathbf{W}'_j\|_{d_w})^2\right]^2 \\ \le\; & \frac{1}{\tau_0^4}\left(\|\mathbf{U}_i\|^2_{d_u} + \|\mathbf{U}'_j\|^2_{d_u} + \|\mathbf{W}_i\|^2_{d_w} + \|\mathbf{W}'_j\|^2_{d_w}\right)^2 \end{aligned}$$

By condition (C1), we have $E(g^2_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)) < \infty$ then $S_{1,2} \le \epsilon/2$ when n is sufficiently large. Consequently,

$$Pr\big(|\widehat{S}^*_{1,2} - S_{1,2}| > \epsilon\big) \;\le\; Pr\big(|\widehat{S}^*_{1,2}| > \epsilon/2\big). \tag{S0.4}$$

It remains to bound the probability $Pr\big(|\widehat{S}^*_{1,2}| > \epsilon/2\big)$. *We observe that the events satisfy*

$$\big\{|\widehat{S}^*_{1,2}| > \epsilon/2\big\} \subseteq \big\{\|\mathbf{U}_i\|^2_{d_u} + \|\mathbf{W}_i\|^2_{d_w} > \frac{M\tau_0^2}{2},\ \textit{for some } 1 \le i \le n\big\}. \qquad \text{(S0.5)}$$

Based on the fact that,

$$\begin{aligned} M &< \widehat{S}^*_{1,2} = \frac{1}{n(n-1)}\sum_{i\neq j} g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j)\mathbf{1}(g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) > M) \\ &\le E(max_{i,j} g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j))\mathbf{1}(g_1(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}'_j, \mathbf{W}'_j, \delta'_j) > M) \\ &\le \frac{1}{\tau_0^2}\left(\|\mathbf{U}_i\|^2_{d_u} + \|\mathbf{U}'_j\|^2_{d_u} + \|\mathbf{W}_i\|^2_{d_w} + \|\mathbf{W}'_j\|^2_{d_w}\right). \end{aligned}$$

This means that if the event $|\widehat{S}^*_{1,2}| > \epsilon/2$ *occurs, the event* $\big\{\|\mathbf{U}_i\|^2_{d_u} + \|\mathbf{W}_i\|^2_{d_w} > \frac{M\tau_0^2}{2},\ \textit{for some } 1 \le i \le n\big\}$ *will occur. This verifies that (S0.5) is true.*

By invoking the above condition $(C1)$ *that* $E\{\exp(s||\mathbf{U}||^2_{d_u})\} < \infty$, $E\{\exp(s||\mathbf{W}||^2_{d_w})\} < \infty$, *and applying Markov's inequality for* $s > 0$, *there must exist a constant* C *such that*

$$\begin{aligned} Pr(|\widehat{S}^*_{1,2}| > \epsilon/2) &\le nPr(\|\mathbf{U}\|^2_{d_u} + \|\mathbf{W}\|^2_{d_w} \ge \frac{M\tau_0^2}{2}) \\ &\le nPr(\|\mathbf{U}\|^2_{d_u} \ge \frac{M\tau_0^2}{4}) + nPr(\|\mathbf{W}\|^2_{d_w} \ge \frac{M\tau_0^2}{4}) \\ &\le nE\left[\frac{\exp(s\|\mathbf{U}\|^2_{d_u})}{\exp(sM\tau_0^2/4)}\right] + nE\left[\frac{\exp(s\|\mathbf{W}\|^2_{d_w})}{\exp(sM\tau_0^2/4)}\right] \end{aligned}$$

Then we have

$$Pr(|\widehat{S}^*_{1,2}| > \epsilon/2) \le 2nC\exp(-sM\tau_0^2/4).$$

If we choose that $M = c_1 n^\gamma$, *with* $0 < \gamma < \frac{1}{2} - \nu$, *then*

$$\begin{aligned} &Pr(|\widehat{S}_{11} - S_1| \ge 4\epsilon) \\ \le& 2nC\exp(-sM\tau_0^2/4) + 2\exp(-2[n/2]\epsilon^2/M^2) \\ =& 2nC\exp(-sc_1 n^\gamma \tau_0^2/4) + 2\exp(-\epsilon^2 n^{1-2\gamma}). \end{aligned} \qquad \text{(S0.6)}$$

Combining (S0.6) and (S0.4), we have

$$\begin{aligned} &Pr\big(|\widehat{S}_1 - S_1| \ge 6\epsilon\big) \\ \le& 2nC\exp(-cn^\gamma\tau_0^2/4) + 2\exp(-\epsilon^2 n^{1-2\gamma}). \end{aligned} \qquad \text{(S0.7)}$$

Step 2. Next, we turn to $\widehat{S}_2$. We write $S_2 = S_{2,1}S_{2,2}$, where $S_{2,1} = E\{\|\mathbf{U} - \mathbf{U}'\|_{d_u}\}$ and $S_{2,2} = E\{\frac{\delta\delta'}{G(T|\mathbf{X})G(T'|\mathbf{X}')}\|\mathbf{W} - \mathbf{W}'\|_{d_w}\}$. Correspondingly, $\widehat{S}_2 = \widehat{S}_{2,1}\widehat{S}_{2,2}$, where $\widehat{S}_{2,1} = n^{-2}\sum_{i\neq j}\sum\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}$, and $\widehat{S}_{2,2} = n^{-2}\sum_{i\neq j}\sum\frac{\delta_i\delta_j'}{\widehat{G}(T_i|\mathbf{X}_i)\widehat{G}(T_j'|\mathbf{X}_j')}\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w}$.

Following arguments for proving (S0.7) in the supplement, we can show that

$$\begin{aligned} &Pr\left(\left|\widehat{S}_{2,1} - S_{2,1}\right| \geq 4\epsilon\right) \leq 2nC\exp(-cn^{\gamma}\tau_0^2/4) + 2\exp(-\epsilon^2 n^{1-2\gamma}), \text{ and} \\ &Pr\left(\left|\widehat{S}_{2,2} - S_{2,2}\right| \geq 4\epsilon\right) \leq 2nC\exp(-cn^{\gamma}\tau_0^2/4) + 2\exp(-\epsilon^2 n^{1-2\gamma}). \end{aligned} \tag{S0.8}$$

Condition $(C1)$ ensures that $S_{2,1} \leq \left\{E(\|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u}^2)\right\}^{1/2} \leq \left\{4E(\|\mathbf{U}\|_{d_u}^2)\right\}^{1/2}$ and $S_{2,2} \leq \left\{E(\|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w}^2)\right\}^{1/2} \leq \left\{4E(\|\mathbf{W}\|_{d_w}^2)\right\}^{1/2}$ are uniformly bounded. That is, $\max\left\{S_{2,1}, S_{2,2}\right\} \leq C$*, for some constant C. Using (S0.8) repeatedly, we can easily prove that*

$$\begin{aligned} Pr\left\{\left|(\widehat{S}_{2,1} - S_{2,1})S_{2,2}\right| \geq \epsilon\right\} &\leq Pr\left(\left|\widehat{S}_{2,1} - S_{2,1}\right| \geq \epsilon/C\right) \\ &\leq 2\exp\left\{-\epsilon^2 n^{1-2\gamma}/(16C^2)\right\} + 2nC\exp\left(-cn^{\gamma}\tau_0^2/4\right), \\ Pr\left(\left|S_{2,1}(\widehat{S}_{2,2} - S_{2,2})\right| \geq \epsilon\right) &\leq Pr\left(\left|\widehat{S}_{2,2} - S_{2,2}\right| \geq \epsilon/C\right) \\ &\leq 2\exp\left\{-\epsilon^2 n^{1-2\gamma}/(16C^2)\right\} + 2nC\exp\left(-cn^{\gamma}\tau_0^2/4\right), \end{aligned} \tag{S0.9}$$

and

$$\begin{aligned} &Pr\left\{\left|(\widehat{S}_{2,1} - S_{2,1})(\widehat{S}_{2,2} - S_{2,2})\right| \geq \epsilon\right\} \\ \leq &Pr\left(\left|\widehat{S}_{2,1} - S_{2,1}\right| \geq \sqrt{\epsilon}\right) + Pr\left(\left|\widehat{S}_{2,2} - S_{2,2}\right| \geq \sqrt{\epsilon}\right) \\ \leq &4\exp\left(-\epsilon n^{1-2\gamma}/16\right) + 4nC\exp\left(-cn^{\gamma}\tau_0^2/4\right). \end{aligned} \tag{S0.10}$$

It follows from Bonferroni's inequality that (S0.9) and (S0.10) imply that,

$$\begin{aligned} &Pr\left(\left|\widehat{S}_2 - S_2\right| \geq 3\epsilon\right) = Pr\left(\left|\widehat{S}_{2,1}\widehat{S}_{2,2} - S_{2,1}S_{2,2}\right| \geq 3\epsilon\right) \\ \leq &Pr\left\{\left|(\widehat{S}_{2,1} - S_{2,1})S_{2,2}\right| \geq \epsilon\right\} + Pr\left\{\left|S_{2,1}(\widehat{S}_{2,2} - S_{2,2})\right| \geq \epsilon\right\} \\ &+ Pr\left\{\left|(\widehat{S}_{2,1} - S_{2,1})(\widehat{S}_{2,2} - S_{2,2})\right| \geq \epsilon\right\} \\ \leq &8\exp\left\{-\epsilon^2 n^{1-2\gamma}/(16C^2)\right\} + 8nC\exp\left(-cn^{\gamma}\tau_0^2/4\right), \end{aligned} \tag{S0.11}$$

where the last inequality holds when ϵ is sufficiently small and C is sufficiently large.

Step 3. The uniform consistency of $\widehat{S}_3$ remains to be shown. We first study the *following U-statistic:*

$$\begin{aligned}
\widehat{S}_3^* &= \frac{1}{n(n-1)(n-2)} \sum_{i<j<l} \Big\{ \frac{\delta_j'\delta_l''}{\widehat{G}_n(T_j'|\mathbf{X}_j')\widehat{G}_n(T_l''|\mathbf{X}_l'')} \|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u} \|\mathbf{W}_j' - \mathbf{W}_l''\|_{d_w} + \\
& \frac{\delta_j\delta_l''}{\widehat{G}_n(T_j'|\mathbf{X}_j')\widehat{G}_n(T_l''|\mathbf{X}_l'')} \|\mathbf{U}_i - \mathbf{U}_l''\|_{d_u} \|\mathbf{W}_j' - \mathbf{W}_l''\|_{d_w} + \\
& \frac{\delta_i\delta_l''}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_l''|\mathbf{X}_l'')} \|\mathbf{U}_i - \mathbf{U}_j'\|_{d_u} \|\mathbf{W}_i - \mathbf{W}_l''\|_{d_w} + \\
& \frac{\delta_i\delta_l''}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_l''|\mathbf{X}_l'')} \|\mathbf{U}_l'' - \mathbf{U}_j'\|_{d_u} \|\mathbf{W}_i - \mathbf{W}_l''\|_{d_w} + \\
& \frac{\delta_i\delta_j'}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} \|\mathbf{U}_l'' - \mathbf{U}_j'\|_{d_u} \|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w} + \\
& \frac{\delta_i\delta_j'}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T_j'|\mathbf{X}_j')} \|\mathbf{U}_l'' - \mathbf{U}_i\|_{d_u} \|\mathbf{W}_i - \mathbf{W}_j'\|_{d_w} \Big\} \\
&= \frac{6}{n(n-1)(n-2)} \sum_{i<j<l} g_3(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}_j', \mathbf{W}_j', \delta_j'; \mathbf{U}_l'', \mathbf{W}_l'', \delta_l''). \qquad \text{(S0.12)}
\end{aligned}$$

Here, $g_3(\mathbf{U}_i, \mathbf{W}_i, \delta_i; \mathbf{U}_j', \mathbf{W}_j', \delta_j'; \mathbf{U}_l'', \mathbf{W}_l'', \delta_l'')$ *is the kernel of U-statistic* $\widehat{S}_3^*$*. Following the arguments to deal with* $\widehat{S}_1^*$*, we decompose* g_3 *into two parts:* $g_3 = g_3\mathbf{1}(g_3 > M) + g_3\mathbf{1}(g_3 \le M)$*. Accordingly,*

$$\begin{aligned}
\widehat{S}_3^* &= \frac{6}{n(n-1)(n-2)} \sum_{i<j<l} g_3\mathbf{1}(g_3 \le M) + \frac{6}{n(n-1)(n-2)} \sum_{i<j<l} g_3\mathbf{1}(g_3 > M) \\
&= \widehat{S}_{3,1}^* + \widehat{S}_{3,2}^*, \\
S_3 &= E\{g_3\mathbf{1}(g_3 \le M)\} + E\{g_3\mathbf{1}(g_3 > M)\} = S_{3,1} + S_{3,2}.
\end{aligned}$$

Following similar arguments for proving (), we can show that

$$Pr\left(\left|\widehat{S}_{3,1}^* - S_{3,1}\right| \ge \epsilon\right) \le 2\exp\left(-2\epsilon^2 m'/M^2\right), \qquad \text{(S0.13)}$$

where $m' = [n/3]$ *because* $\widehat{S}_{3,1}^*$ *is a third-order U-statistic.*

Then we deal with $\widehat{S}_{3,2}^*$*. We observe that* $g_3(\mathbf{U}_i, \mathbf{W}_i; \mathbf{U}_j', \mathbf{W}_j'; \mathbf{U}_l'', \mathbf{W}_l'') \le \frac{2}{3\tau_0^2}(\|\mathbf{U}_i\|_{d_u}^2 + \|\mathbf{U}_j'\|_{d_u}^2 + \|\mathbf{U}_l''\|_{d_u}^2 + \|\mathbf{W}_i\|_{d_w}^2 + \|\mathbf{W}_j'\|_{d_w}^2 + \|\mathbf{W}_l''\|_{d_w}^2)$*, which will be smaller than* M *if* $\|\mathbf{U}_i\|_{d_u}^2 + \|\mathbf{W}_i\|_{d_w}^2 \le \frac{M\tau_0^2}{2}$*, for all* $1 \le i \le n$*. Thus, for any* $\epsilon > 0$*, the events satisfy*

$$\left\{\left|\widehat{S}_{3,2}^*\right| > \epsilon/2\right\} \subseteq \left\{\|\mathbf{U}_i\|_{d_u}^2 + \|\mathbf{V}_i\|_{d_v}^2 > M\tau_0^2/2, \text{ for some } 1 \le i \le n\right\}.$$

By using similar arguments to prove (S0.6), it follows that

$$Pr\big(\big|\widehat{S}_{3,2}^{*}-S_{3,2}\big|>\epsilon\big)\leq Pr\big(\big|\widehat{S}_{3,2}^{*}\big|>\epsilon/2\big)\leq 2nC\exp(-cM\tau_0^2/4). \tag{S0.14}$$

Then, we combine the results (S0.13) and (S0.14) with $M=c_1n^{\gamma}$ *for some* $0<\gamma<1/2-\nu$ *to obtain that*

$$Pr\left(\left|\widehat{S}_3^{*}-S_3\right|\geq 2\epsilon\right)\ \leq\ 2\exp\left(-2\epsilon^2n^{1-2\gamma}/3\right)+2nC\exp\left(-cn^{\gamma}\tau_0^2/4\right). \tag{S0.15}$$

By the definition of $\widehat{S}_3$,

$$\widehat{S}_3=\frac{(n-1)(n-2)}{n^2}\left\{\widehat{S}_3^{*}+\frac{1}{(n-2)}\widehat{S}_1^{*}\right\}.$$

Thus, using similar techniques to deal with $\widehat{S}_1$, *we can obtain that*

$$\begin{aligned}Pr\left(\left|\widehat{S}_3-S_3\right|\geq 4\epsilon\right)\ &=\ Pr\left\{\left|\frac{(n-1)(n-2)}{n^2}\left(\widehat{S}_3^{*}-S_3\right)-\frac{3n-2}{n^2}S_3\right.\right.\\ &\qquad\left.\left.+\ \frac{n-1}{n^2}\left(\widehat{S}_{11}^{*}-S_1\right)+\frac{n-1}{n^2}S_1\right|\geq 4\epsilon\right\}.\end{aligned}$$

Using similar arguments for dealing with S_1, *we can show that* S_3 *is also uniformly bounded. Taking* n *large enough such that* $\{(3n-2)/n^2\}S_3\leq\epsilon$ *and* $\{(n-1)/n^2\}S_1\leq\epsilon$, *then*

$$\begin{aligned}Pr\big(\big|\widehat{S}_3-S_3\big|\geq 4\epsilon\big)&\leq Pr\big(\big|\widehat{S}_3^{*}-S_3\big|\geq\epsilon\big)+Pr\big\{\big|\widehat{S}_{11}^{*}-S_1\big|\geq\epsilon\big\}\\ &\leq 4\exp\left(-\epsilon^2n^{1-2\gamma}/6\right)+4nC\exp\left(-cn^{\gamma}\tau_0^2/4\right).\end{aligned} \tag{S0.16}$$

The last inequality follows from (S0.7) and (S0.15). This, together with (S0.7), (S0.11) and Bonferroni's inequality, implies

$$\begin{aligned}&Pr\big\{\big|(\widehat{S}_1+\widehat{S}_2-2\widehat{S}_3)-(S_1+S_2-2S_3)\,\big|\geq\epsilon\big\}\\ \leq&Pr\big(\big|\widehat{S}_1-S_1\big|\geq\epsilon/4\big)+Pr\big(\big|\widehat{S}_2-S_2\big|\geq\epsilon/4\big)+Pr\big(\big|\widehat{S}_3-S_3\big|\geq\epsilon/4\big)\\ =&O\left\{\exp\left(-c_1\epsilon^2n^{1-2\gamma}\right)+n\exp\left(-c_2n^{\gamma}\tau_0^2\right)\right\},\end{aligned} \tag{S0.17}$$

for some positive constants c_1 *and* c_2. *The convergence rate of the numerator of* $\widehat{u}_{\alpha}^{*},\alpha=1,\cdots,p$ *is now achieved. Following similar arguments, we can obtain the convergence rate of the denominator. In effect the convergence rate of* $\widehat{u}_{\alpha}^{*}$ *has the same form as (S0.17).*

Proof 2 (of Theorem 2) *Based on the bound of* $\widehat{u}^*$*, where* $\mathbf{U} = X_\alpha$ *and* $\mathbf{W} = (T, \mathbf{X}_{-\alpha})$*, we have*

$$Pr(|\widehat{u}^*_\alpha - u^*_\alpha| \geq \epsilon) \leq O\left\{\exp\left(-c_1\epsilon^2 n^{1-2\gamma}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right)\right\},$$

for $\alpha = 1, 2, \cdots, p$*, with* c *being the positive constants. If we take* $\epsilon = cn^{-\nu}$ *with* $0 < (\nu + \gamma) < \frac{1}{2}$*, then we have*

$$\begin{aligned} &Pr(\max_{1\leq\alpha\leq p} |\widehat{u}^*_\alpha - u^*_\alpha| \geq cn^{-\nu}) \leq p \max_{1\leq\alpha\leq p} Pr(|\widehat{u}^*_\alpha - u^*_\alpha| \geq cn^{-\nu}) \\ \leq& O\left\{p\left[\exp\left(-c_1 n^{1-2(\nu+\gamma)}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right)\right]\right\}. \end{aligned}$$

Denote $\zeta^* = \min_{\alpha\in\mathcal{D}^*} u^*_\alpha - \max_{\alpha\in\bar{\mathcal{D}}^*} u^*_\alpha$*, under condition (C2*),* $\zeta^* \geq 2cn^{-\nu} \geq 0$*, Then*

$$\begin{aligned} &Pr(\max_{\alpha\in\bar{\mathcal{D}}^*} \widehat{u}^*_\alpha < \min_{\alpha\in\mathcal{D}^*} \widehat{u}^*_\alpha) \\ =&1 - Pr(\max_{\alpha\in\bar{\mathcal{D}}^*} \widehat{u}^*_\alpha \geq \min_{\alpha\in\mathcal{D}^*} \widehat{u}^*_\alpha) \\ =&1 - Pr(\max_{\alpha\in\bar{\mathcal{D}}^*} \widehat{u}^*_\alpha - \max_{\alpha\in\bar{\mathcal{D}}^*} u^*_\alpha \geq \min_{\alpha\in\mathcal{D}^*} \widehat{u}^*_\alpha - \min_{\alpha\in\mathcal{D}^*} u^*_\alpha + \zeta^*) \\ \geq&1 - \left[Pr(\max_{\alpha\in\bar{\mathcal{D}}^*} |\widehat{u}^*_\alpha - u^*_\alpha| \geq \zeta^*/2) + Pr(\max_{\alpha\in\mathcal{D}^*} |\widehat{u}^*_\alpha - u^*_\alpha| \geq \zeta^*/2)\right] \\ \geq&1 - O\left\{2p\left[\exp\left(-c_1 n^{1-2(\nu+\gamma)}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right)\right]\right\}. \end{aligned}$$

Next we show that

$$Pr(\mathcal{D}^* \subseteq \widehat{\mathcal{D}}^*) \geq 1 - O(s_n\left[\exp\left(-c_1 n^{1-2(\nu+\gamma)}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right)\right]),$$

where s_n *is the cardinality of* $\mathcal{D}^*$*.*

If $\mathcal{D}^* \nsubseteq \widehat{\mathcal{D}}^*$*, there must exist an* $\alpha \in \mathcal{D}^*$ *such that* $u^*_\alpha \geq 2cn^{-\nu}$*, but* $\widehat{u}^*_\alpha \leq cn^{-\nu}$ *to ensure that* $|u^*_\alpha - \widehat{u}^*_\alpha| \geq cn^{-\nu}$ *for some* $\alpha \in \mathcal{D}^*$*. This means that the event satisfies* $\mathcal{D}^* \nsubseteq \widehat{\mathcal{D}}^* \subseteq \{|u^*_\alpha - \widehat{u}^*_\alpha| \geq cn^{-\nu}$*, for some* $\alpha \in \mathcal{D}^*\}$*. Then we have* $\mathcal{E}_n = \{\max_{\alpha\in\mathcal{D}^*} |u^*_\alpha - \widehat{u}^*_\alpha| \leq cn^{-\nu}\} \subseteq \{\mathcal{D}^* \subseteq \widehat{\mathcal{D}}^*\}$*.*

$$\begin{aligned} Pr(\mathcal{D}^* \subseteq \widehat{\mathcal{D}}^*) &\geq Pr(\mathcal{E}_n) = 1 - Pr(\mathcal{E}_n^c) \\ &= 1 - Pr(\min_{\alpha\in\mathcal{D}^*} |u^*_\alpha - \widehat{u}^*_\alpha| \geq cn^{-\nu}) \\ &= 1 - s_n Pr(|u^*_\alpha - \widehat{u}^*_\alpha| \geq cn^{-\nu}) \\ &\geq 1 - O(s_n\left[\exp\left(-c_1 n^{1-2(\nu+\gamma)}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right)\right]). \end{aligned}$$

To prove Theorem 1, we can use a special case of Theorem 2, with $\mathbf{U} = X_\alpha$, and $\mathbf{W} = T$. It includes the rank consistence of marginal measure $\widehat{u}$, and the sure screening property. The details of Theorem 1 are included in the main paper.

To prove Theorem 3, apart from the additional conditions included in the main manuscript, the following Lemma 5 is also needed:

Lemma 5 (Bound for $u^{\mathcal{J}}_{\alpha,s}$ in survival data) *For slice s, under conditions $(C1)$, $(C3)$, $(C4^{**})$, and $(C5^{**})$, for any $0 < \gamma < \frac{1}{2} - \nu$, there exist positive constants $c_1 > 0$ and $c_2 > 0$ such that*

$$\begin{aligned} & Pr\left(\left|\widehat{u}^{\mathcal{J}}_{\alpha,s} - u^{\mathcal{J}}_{\alpha,s}\right| \geq cn^{-\nu}\right) \\ \leq\ & O\left(\left[\exp\left\{-c_1\tau_0^4 n^{1-2(\nu+\gamma)}\right\} + n\exp\left(-c_2 n^\gamma \tau_0^2\right)\right]\right). \end{aligned}$$

Proof 3 (of Lemma 5) *Similar to the bound proof for $\widehat{S}_1$ in Lemma 4, we can obtain the bound for the numerator of the first term $\widehat{S}_{1,s}$ in $\widehat{u}^{\mathcal{J}}_{\alpha,s}$, and denote it as $\widehat{S}_{1,s,n}$, except that each random variable is in the s-th slice, i.e. $i, j, l \in \mathcal{V}_s$. That is*

$$\begin{aligned} & Pr\left(\left|\widehat{S}_{1,s,n} - S_{1,s,n}\right| \geq 6\epsilon\right) \\ \leq & 2nC\exp(-cn^\gamma\tau_0^2/4) + 2\exp(-\epsilon^2 n^{1-2\gamma}). \end{aligned} \tag{S0.18}$$

Consider the denominator of $\widehat{S}_{1,s}$ in $\widehat{u}^{\mathcal{J}}_{\alpha,s}$, denoted as $\widehat{S}_{1,s,d}$,

$$\begin{aligned} \widehat{S}_{1,s,d} =& \frac{1}{n^2}\sum_{i,j\in\mathcal{V}_s} \frac{\delta_i\delta'_j}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T'_j|\mathbf{X}'_j)} \\ =& \frac{1}{n^2}\sum_{i,j\in\mathcal{V}_s} \frac{\delta_i\delta'_j}{G(T_i|\mathbf{X}_i)G(T'_j|\mathbf{X}'_j)} \\ & + \frac{1}{n^2}\sum_{i,j\in\mathcal{V}_s} \delta_i\delta'_j \left(\frac{1}{\widehat{G}_n(T_i|\mathbf{X}_i)\widehat{G}_n(T'_j|\mathbf{X}'_j)} - \frac{1}{G(T_i|\mathbf{X}_i)G(T'_j|\mathbf{X}'_j)}\right), \\ =& \widehat{S}_{11,d} + \widehat{S}_{12,d} \end{aligned}$$

Under condition $(C3)$, $\widehat{S}_{11,d} \geq \tau_0^2$ has a lower bound, and $\widehat{S}_{12,d} \to 0$ as n is sufficiently *large, so*

$$
\begin{aligned}
&Pr\big(|\widehat{S}_{1,s} - S_{1,s}| \geq 6\epsilon\big)\\
\leq &Pr\big(|\widehat{S}_{1,s,n} - S_{1,s,n}| \geq 6\epsilon\tau_0^2\big)\\
\leq &2nC\exp(-cn^{\gamma}\tau_0^2/4) + 2\exp(-\epsilon^2\tau_0^4 n^{1-2\gamma})\\
\leq &2nC\exp(-c_2 n^{\gamma}\tau_0^2/4) + 2\exp(-c_1\epsilon^2\tau_0^4 n^{1-2\gamma}).
\end{aligned}
\tag{S0.19}
$$

The rest of the proof for Lemma 5 is almost the same as that of Lemma 4 for u_{α}^{*}.

Proof 4 (of Theorem 3) *Define* $u_{\alpha,s}^{**} = \mathcal{I}_{t\in[l_{s-1},l_s)}\{(X_{\alpha}, \mathbf{X}_{-\alpha}) \mid T = t\}$, *for a specific* $t \in [l_{s-1}, l_s)$,

$$
\begin{aligned}
Pr(|\widehat{u}_{\alpha,s}^{\mathcal{J}} - u_{\alpha,s}^{**}| \geq \epsilon) \leq &Pr(|\widehat{u}_{\alpha,s}^{\mathcal{J}} - u_{\alpha,s}^{\mathcal{J}}| \geq \frac{\epsilon}{2}) + Pr(|u_{\alpha,s}^{\mathcal{J}} - u_{\alpha,s}^{**}| \geq \frac{\epsilon}{2})\\
= &Pr(|\widehat{\mathcal{I}}\{(X_{\alpha}, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\} - \mathcal{I}\{(X_{\alpha}, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\}| \geq \frac{\epsilon}{2})\\
&+ Pr(|\mathcal{I}\{(X_{\alpha}, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\} - \mathcal{I}_{t\in[l_{s-1},l_s)}\{(X_{\alpha}, \mathbf{X}_{-\alpha})|T = y\}| \geq \frac{\epsilon}{2})\\
= &I_1 + I_2.
\end{aligned}
$$

In term I_1, *for each* s, $s = 1, \cdots, S$, *the rank consistency of* $\widehat{u}_{\alpha,s}^{\mathcal{J}}$ *can be obtained directly, by replacing* $(T, \mathbf{X}_{-\alpha})$ *in Theorem 2 with* $\mathbf{X}_{-\alpha}$. *That is, under conditions (C1) and (C3),*

$$
I_1 = Pr(|\widehat{u}_{\alpha,s}^{\mathcal{J}} - u_{\alpha,s}^{\mathcal{J}}| \geq \frac{\epsilon}{2}) \leq O\left\{\exp\left(-\frac{c_1}{4}\tau_0^4\epsilon^2 n^{1-2\gamma}\right) + n\exp\left(-c_2 n^{\gamma}\tau_0^2\right)\right\}
$$

For the s*-th sample quantile slice, if the empirical cumulative distribution of* $F_n(t)$ *and the cumulative distribution* $F(t)$ *are sufficiently close, i.e. for any* $\epsilon > 0$, *the event* $A = sup_y|F_n(t) - F(t)| < \frac{\epsilon}{2}$ *occurs, then, we have event* $B = \{\frac{1}{S} - \epsilon \leq P(T \in [l_{s-1}, l_s)) \leq \frac{1}{S} + \epsilon\}$, *and event* B *meets the condition of* $(C5^{**})$. *In fact, for any* $\epsilon > 0$, *if the event* $A = sup_y|F_n(t) - F(t)| < \frac{\epsilon}{2}$, *following Lemma 4 in Mai and Zou (2015), we have*

$$
\begin{aligned}
Pr(l_{s-1} \leq T < l_s) &= Pr(\frac{s-1}{S} \leq F_n(T) < \frac{s}{S})\\
&\leq Pr(\frac{s-1}{S} - \frac{\epsilon}{2} \leq F(T) < \frac{s}{S} + \frac{\epsilon}{2})\\
&= \frac{1}{S} + \epsilon.
\end{aligned}
$$

Similarly, we have

$$\begin{aligned} Pr(l_{s-1} \le T < l_s) &= Pr(F(l_{s-1}) \le F(T) < F(l_s)) \\ &= F(l_s) - F(l_{s-1}) \\ &\ge F_n(l_s) - \frac{\epsilon}{2} - (F_n(l_{s-1}) + \frac{\epsilon}{2}) \\ &= \frac{1}{S} - \epsilon. \end{aligned}$$

Furthermore, we have $Pr(B) \ge Pr(A) \ge 1 - 2\exp(-\frac{1}{2}n\epsilon^2)$ *by the Dvoretzky-Kiefer-Wolfowitz inequality.*

Based on event B*, i.e.* $\frac{1}{S} - \epsilon \le P(T \in [l_{s-1}, l_s)) \le \frac{1}{S} + \epsilon$*, the event* $D = \{|\mathcal{I}\{(X_\alpha, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\} - \mathcal{I}_{t\in[l_{s-1},l_s)}\{(X_\alpha, \mathbf{X}_{-\alpha})|T = t\}| \le \frac{\epsilon}{2}\}$ *occurs, i.e.* $B \subset D$*. Actually, for any* $t \in [l_{s-1}, l_s)$*,*

$$\begin{aligned} &|\mathcal{I}\{(X_\alpha, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\} - \mathcal{I}_{t\in[l_{s-1},l_s)}\{(X_\alpha, \mathbf{X}_{-\alpha})|T = t\}| \\ \le& |\sup_{t\in[l_{s-1},l_s)} \mathcal{I}\{(X_\alpha, \mathbf{X}_{-\alpha})|T = t\} - \inf_{t\in[l_{s-1},l_s)} \mathcal{I}\{(X_\alpha, \mathbf{X}_{-\alpha})|T = t\}| \\ \le& \frac{\epsilon}{2}, \end{aligned}$$

the last inequality follows from the condition $(C5^{**})$*.*

$$\begin{aligned} Pr(D) &= Pr(|\mathcal{I}\{(X_\alpha, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\} - \mathcal{I}_{t\in[l_{s-1},l_s)}\{(X_\alpha, \mathbf{X}_{-\alpha})|T = t\}| \le \frac{\epsilon}{2}) \\ &\ge Pr(B) \ge Pr(A) \ge 1 - 2\exp(-\frac{1}{2}n\epsilon^2) \end{aligned}$$

Therefore, we have that

$$\begin{aligned} I_2 &= Pr(|\mathcal{I}\{(X_\alpha, \mathbf{X}_{-\alpha})|T \in [l_{s-1}, l_s)\} - \mathcal{I}_{t\in[l_{s-1},l_s)}\{(X_\alpha, \mathbf{X}_{-\alpha})|T = t\}| \ge \frac{\epsilon}{2}) \\ &= 1 - Pr(D) \le 2\exp(-\frac{1}{2}n\epsilon^2) \end{aligned}$$

Combining the bounds for I_1 *and* I_2*, we have*

$$Pr(|\widehat{u}^{\mathcal{J}}_{\alpha,s} - u^{**}_{\alpha,s}| \ge \frac{\epsilon}{2}) \le O\left\{\exp\left(-c_1\tau_0^4\epsilon^2 n^{1-2\gamma}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right) + \exp\left(-c_3 n\epsilon^2\right)\right\}$$

Similar to the proof of Theorem 2, if we take $\epsilon = cn^{-\nu}$ *for any* $0 < (\nu + \gamma) < \frac{1}{2}$*, there exists a positive constant* $c > 0$ *such that*

$$Pr(\max_{1\le\alpha\le p} |\widehat{u}^{\mathcal{J}}_{\alpha,s} - u^{**}_{\alpha,s}| \ge cn^{-\nu}) \le O\left\{p\left[\exp\left(-c_1\tau_0^4 n^{1-2(\gamma+\nu)}\right) + n\exp\left(-c_2 n^\gamma \tau_0^2\right) + \exp\left(-c_3 n^{1-2\nu}\right)\right]\right\}$$

Furthermore, we have

$$
\begin{aligned}
&Pr(\max_{1\le\alpha\le p}|\widehat{u}_\alpha^{**}-u_\alpha^{**}|\ge cn^{-\nu})\\
&\le Pr(\max_{1\le\alpha\le p}|\frac{1}{S}\sum_{s=1}^{S}\widehat{u}_{\alpha,s}^{\mathcal{J}}-\frac{1}{S}\sum_{s=1}^{S}u_{\alpha,s}^{\mathcal{J}}|\ge cn^{-\nu})\\
&\le pPr(|\frac{1}{S}\sum_{s=1}^{S}\widehat{u}_{\alpha,s}^{\mathcal{J}}-\frac{1}{S}\sum_{s=1}^{S}u_{\alpha,s}^{\mathcal{J}}|\ge cn^{-\nu})\\
&\le pPr(\max_{1\le s\le S}|\widehat{u}_{\alpha,s}^{\mathcal{J}}-u_{\alpha,s}^{\mathcal{J}}|\ge cn^{-\nu})\\
&= pPr(\max_{1\le s\le S}|\widehat{\mathcal{I}}\{(X_\alpha,\mathbf{X}_{-\alpha})|T\in[l_{s-1},l_s)\}-\mathcal{I}\{(X_\alpha,\mathbf{X}_{-\alpha})|T\in[l_{s-1},l_s)\}|\ge cn^{-\nu})\\
&\le pPr(\sup_{t\in[l_{s-1},l_s)}|\widehat{\mathcal{I}}\{(X_\alpha,\mathbf{X}_{-\alpha})|T\in[l_{s-1},l_s)\}-\mathcal{I}\{(X_\alpha,\mathbf{X}_{-\alpha})|T=t\}|\ge cn^{-\nu})\\
&\le pPr(|\widehat{u}_{\alpha,s}^{\mathcal{J}}-u_{\alpha,s}^{**}|\ge cn^{-\nu})\\
&\le O(p\left[\exp\left(-c_1\tau_0^4 n^{1-2(\gamma+\nu)}\right)+n\exp\left(-c_2 n^{\gamma}\tau_0^2\right)+\exp\left(-c_3 n^{1-2\nu}\right)\right])
\end{aligned}
$$

Under condition ($C2^{**}$), *denote* $\delta=\min_{\alpha\in\mathcal{D}^{**}}u_\alpha^{**}-\max_{\alpha\in\bar{\mathcal{D}}^{**}}u_\alpha^{**}$, *then*

$$
\begin{aligned}
&Pr(\max_{\alpha\in\bar{\mathcal{D}}^{**}}\widehat{u}_\alpha^{**}\ge\min_{\alpha\in\mathcal{D}^{**}}\widehat{u}_\alpha^{**})\\
=&Pr(\max_{\alpha\in\bar{\mathcal{D}}^{**}}\widehat{u}_\alpha^{**}-\max_{\alpha\in\bar{\mathcal{D}}^{**}}u_\alpha^{**}\ge\min_{\alpha\in\mathcal{D}^{**}}\widehat{u}_\alpha^{**}-\min_{\alpha\in\mathcal{D}^{**}}u_\alpha^{**}+\delta)\\
\le&Pr(\max_{\alpha\in\bar{\mathcal{D}}^{**}}|\widehat{u}_\alpha^{**}-u_\alpha^{**}|\ge\delta/2)+Pr(\max_{\alpha\in\mathcal{D}^{**}}|\widehat{u}_\alpha^{**}-u_\alpha^{**}|\ge\delta/2)\\
\le&2p\left[\exp\left(-c_1\tau_0^4 n^{1-2(\gamma+\nu)}\right)+n\exp\left(-c_2 n^{\gamma}\tau_0^2\right)+\exp\left(-c_3 n^{1-2\nu}\right)\right].
\end{aligned}
$$

Under conditions (C1), (C3), ($C2^{**}$), ($C4^{**}$), *and* ($C5^{**}$), *similar to the proof of Theorem 2, we have that*

$$
Pr(\mathcal{D}^{**}\subseteq\widehat{\mathcal{D}}^{**})\ge 1-O(s_n\left[\exp\left(-c_1\tau_0^4 n^{1-2(\gamma+\nu)}\right)+n\exp\left(-c_2 n^{\gamma}\tau_0^2\right)+\exp\left(-c_3 n^{1-2\nu}\right)\right]),
$$

where s_n *is the cardinality of* $\mathcal{D}^{**}$.

A.4 Additional Simulation Examples

In this section, additional simulation examples and real data analysis results are presented.

Table 1 shows the results of Example 1 in the main text with $p = 2000$.

Example 1 *Assuming a general transformation model for survival time* T *described in Song et al. (2014) and Liu et al. (2018), we have* $H(T) = -\beta^\top \mathbf{X} + \epsilon$, *where* $H(t) = \log\{0.5(\exp(2t) - 1)\}$, *and* $\epsilon \sim N(0, 1)$. *Here,* $\beta^\top = (-1, -0.9, \mathbf{0}_6, 0.8, 1, \mathbf{0}_{p-10})$, *with* $\mathbf{0}_p$ *referring to a zero vector of length* p, *such that only four predictors are active.* $\mathbf{X} = (X_1, \ldots, X_p)^\top \sim N(0, \Sigma)$, *and* $\Sigma = (\sigma_{ij})_{p\times p}$ *with* $\sigma_{ij} = 0.8^{|i-j|}$. *The censoring time* $C \sim U(0, 100)$. *We set* $n = 100, 200$, $p = 1000, 2000$. *The results are exhibited in Table 2.*

Example 2 *This example follows Example 2 in main text. Tables (3)-(13) contain the additional simulation results for Example 2 (a) and the simulation results for Example 2 (b)–(d).*

Table 14 represents the identified gene IDs based on different variable screening methods utilized on the training set for the real data analysis presented in the main text.

Example 3 *This is an additional illustration of real data analysis. We applied our proposed procedures to diffuse large-B-cell lymphoma(DLBCL) dataset, which was studied by Rosenwald et al. (2002). To identify genes that have an influence on patient survival risk, DLBCL is one of the most common types of lymphoma in adults in the United*

Table 1: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 1 in main text with $p = 2000$.

	$\mathcal{P}_s$					$\mathcal{P}_a$	$\mathcal{P}_s$					$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	X_5	ALL	X_1	X_2	X_3	X_4	X_5	ALL
	$n = 100, p = 2000, CR \approx 0.019$						$n = 200, p = 2000, CR \approx 0.019$					
CDC	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
CDC_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
CDC_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$QaSIS$	0.935	1.000	0.995	0.990	0.945	0.870	1.000	1.000	1.000	1.000	1.000	1.000
$CSIR$	0.525	0.755	0.775	0.705	0.600	0.325	0.990	0.995	1.000	0.995	0.985	0.970
$RCDCS$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$CRSIS$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$SVSIR$	0.960	1.000	0.995	0.995	0.980	0.945	1.000	1.000	1.000	1.000	1.000	1.000
$CRIS$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
KM	0.990	0.990	1.000	1.000	0.990	0.985	1.000	1.000	1.000	1.000	1.000	1.000
$IPOD$	0.985	1.000	1.000	1.000	0.995	0.980	1.000	1.000	1.000	1.000	1.000	1.000
LQ	0.965	0.980	0.960	0.975	0.975	0.870	1.000	1.000	1.000	1.000	1.000	1.000
$ISIS_C$	0.840	0.888	0.934	0.918	0.824	0.530	0.984	0.976	0.986	0.986	0.978	0.910

States (Rosenwald et al., 2002). The dataset is available from: `http://www-stat.stanford.edu/~tibs/superpc/staudt.html`. *DLBCL data has also been utilized by Zhu et al. (2011), He et al. (2013) and Zhou and Zhu (2017). It consists of survival times for* $n = 240$ *DLBCL patients following chemotherapy, as well as gene expression measurements for* $p = 7399$ *genes obtained from complementary DNA microarrays for each individual patient. The survival rate after standard chemotherapy is approximately* $35 \sim 40\%$*, During the exploratory data analysis, all gene expression levels have been standardized to have a mean of zero and a standard deviation of one. Following Rosenwald et al. (2002), Zhu et al. (2011), He et al. (2013) and Zhou and Zhu (2017), we divided this dataset into a training set with* $n_1 = 160$ *patients and a testing set with the remaining* $n_2 = 80$ *patients. Specifically, various screening procedures were applied to the training data, and select out* $[n_1/\log(n_1)] = 31$ *genes. These selected covariates are denoted by* $\mathbf{X}_{\widehat{\mathcal{D}}}$*. Table 15 in Appendix A.4 summarizes the first 31 screened genes. Gene IDs 2409 and 1969 were chosen by the one-stage screening procedures, while genes with IDs 2308 and 2306 were selected by all two-stage screening procedures. This suggests a strong association between these genes and patient survival times, although they were not chosen by the marginal methods.*

The p-values of the log-rank test can be found in the final column of Table 16. Based on the p-values, all screening methods successfully identify subgroups that exhibit differences in survival times. Table 16 summarizes the C-statistic, along with its standard deviations, and the lower and upper bounds for different methods. The standard deviation (SD) of the C-statistic is obtained using a perturbation resampling method with 200 replicates. Figure 1 illustrates the C-statistic for various marginal methods, one-stage methods and two-stage methods. These methods demonstrate moderate predictive power, as the C-statistic is approximately 0.80 throughout. Notably, the one-stage and two-stage procedures yield *higher C-statistic values compared to the marginal methods, with the one-stage procedure exhibiting particularly promising results. The green dashed lines in the figure are generally similar or above the marginal line. Among all the methods, the CSIR and CRSIS methods exhibit slightly better overall performance. This confirms the importance of focusing on gene IDs 2308,2306, 2409 and 1969, which were selected by the one-stage and two-stage procedures, in future research. Furthermore, the proposed method, which combines the CSIR measure in the marginal stage and the DC-based measure in the one-stage or two-stage, enhances the performance of CSIR in Zhou and Zhu (2017). Specifically,* $CRSIS_2^M$ *stands out as the best among all competitors.*

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Table 2: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 1 in Appendix.

Model	$\mathcal{P}_s$				$\mathcal{P}_a$	Model $\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_9	X_{10}	ALL	X_1	X_2	X_9	X_{10}	ALL
$n = 100, p = 1000, d = [n/log(n)], CR \approx 0.008$						$n = 200, p = 1000, d = [n/log(n)], CR \approx 0.008$				
CDC	1.000	1.000	0.994	1.000	0.994	1.000	1.000	1.000	1.000	1.000
CDC_1	1.000	1.000	0.992	1.000	0.992	1.000	1.000	1.000	1.000	1.000
CDC_2	1.000	1.000	0.992	1.000	0.992	1.000	1.000	1.000	1.000	1.000
$QaSIS$	0.946	0.894	0.810	0.910	0.652	1.000	1.000	0.998	1.000	0.998
$CSIR$	0.734	0.628	0.508	0.630	0.218	0.988	0.968	0.938	0.980	0.900
$RCDCS$	1.000	1.000	0.992	0.998	0.992	1.000	1.000	1.000	1.000	1.000
$CRSIS$	1.000	1.000	0.998	1.000	0.998	1.000	1.000	1.000	1.000	1.000
$SVSIR$	1.000	1.000	0.956	0.992	0.954	1.000	1.000	1.000	1.000	1.000
$CRIS$	1.000	1.000	0.998	1.000	0.998	1.000	1.000	1.000	1.000	1.000
KM	0.990	0.976	0.946	0.988	0.916	1.000	1.000	1.000	1.000	1.000
$IPOD$	0.996	0.982	0.934	0.986	0.904	1.000	1.000	1.000	1.000	1.000
LQ	0.988	0.986	0.964	0.986	0.932	1.000	1.000	1.000	1.000	1.000
$ISIS_C$	0.994	0.974	0.904	0.996	0.872	1.000	1.000	1.000	1.000	1.000
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.008$						$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.008$				
CDC	1.000	1.000	0.995	1.000	0.995	1.000	1.000	1.000	1.000	1.000
CDC_1	1.000	1.000	0.995	1.000	0.995	1.000	1.000	1.000	1.000	1.000
CDC_2	1.000	1.000	0.995	1.000	0.995	1.000	1.000	1.000	1.000	1.000
$QaSIS$	0.920	0.825	0.615	0.825	0.410	1.000	1.000	1.000	1.000	1.000
$CSIR$	0.645	0.555	0.405	0.500	0.110	0.985	0.950	0.865	0.950	0.795
$RCDCS$	1.000	1.000	0.991	1.000	0.991	1.000	1.000	1.000	1.000	1.000
$CRSIS$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$SVSIR$	1.000	1.000	0.920	0.980	0.910	1.000	1.000	1.000	1.000	1.000
$CRIS$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
KM	0.990	0.980	0.945	0.965	0.895	0.995	0.995	0.995	0.995	0.995
$IPOD$	0.990	0.975	0.915	0.950	0.860	1.000	1.000	1.000	1.000	1.000
LQ	0.990	0.990	0.955	0.970	0.905	1.000	1.000	1.000	1.000	1.000
$ISIS_C$	0.990	0.964	0.898	0.970	0.834	1.000	1.000	1.000	1.000	1.000

Table 3: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (a) Continued.

	$r = 0.5$					$r = 0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n = 100, p = 1000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.916	0.93	0.914	0.026	0.02	0.678	0.714	0.670	0.028	0.000
CDC_1	0.912	0.926	0.908	0.958	0.734	0.664	0.696	0.654	0.990	0.288
CDC_2	0.912	0.926	0.908	0.990	0.762	0.664	0.698	0.654	0.988	0.290
$QaSIS$	0.562	0.566	0.530	0.314	0.038	0.102	0.106	0.110	0.356	0.000
$CSIR$	0.288	0.334	0.310	0.008	0.000	0.152	0.184	0.178	0.002	0.000
$RCDCS$	0.914	0.926	0.920	0.002	0.000	0.682	0.698	0.692	0.004	0.000
$CRSIS$	0.928	0.932	0.924	0.000	0.000	0.654	0.650	0.636	0.000	0.000
$SVSIR$	0.934	0.946	0.938	0.000	0.000	0.738	0.744	0.742	0.000	0.000
$CRIS$	0.930	0.946	0.944	0.000	0.000	0.770	0.762	0.766	0.000	0.000
KM	0.716	0.722	0.716	0.030	0.002	0.274	0.244	0.264	0.036	0.000
$IPOD$	0.770	0.782	0.774	0.024	0.008	0.338	0.306	0.314	0.044	0.000
LQ	0.616	0.616	0.578	0.064	0.006	0.114	0.096	0.106	0.048	0.000
$ISIS_C$	0.643	0.643	0.596	0.506	0.276	0.216	0.193	0.203	0.330	0.006
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.905	0.870	0.830	0.010	0.000	0.660	0.600	0.560	0.015	0.000
CDC_1	0.905	0.870	0.820	0.940	0.615	0.635	0.585	0.535	0.950	0.170
CDC_2	0.905	0.870	0.820	0.985	0.630	0.635	0.585	0.535	0.965	0.180
$QaSIS$	0.460	0.405	0.390	0.225	0.005	0.095	0.060	0.040	0.205	0.000
$CSIR$	0.210	0.265	0.280	0.000	0.000	0.070	0.120	0.150	0.000	0.000
$RCDCS$	0.870	0.875	0.830	0.000	0.000	0.580	0.655	0.575	0.000	0.000
$CRSIS$	0.905	0.900	0.860	0.000	0.000	0.590	0.575	0.515	0.000	0.000
$SVSIR$	0.900	0.875	0.855	0.000	0.000	0.650	0.695	0.635	0.000	0.000
$CRIS$	0.910	0.890	0.850	0.000	0.000	0.685	0.705	0.655	0.000	0.000
KM	0.660	0.655	0.575	0.030	0.005	0.175	0.180	0.180	0.005	0.000
$IPOD$	0.705	0.680	0.595	0.030	0.005	0.235	0.250	0.180	0.025	0.000
LQ	0.515	0.560	0.470	0.040	0.000	0.090	0.065	0.060	0.050	0.000
$ISIS_C$	0.486	0.486	0.450	0.300	0.103	0.150	0.173	0.140	0.196	0.000
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	1.000	1.000	1.000	0.035	0.035	0.955	0.945	0.900	0.055	0.025
CDC_1	1.000	1.000	1.000	1.000	1.000	0.955	0.945	0.895	1.000	0.805
CDC_2	1.000	1.000	1.000	1.000	1.000	0.955	0.945	0.895	1.000	0.805
$QaSIS$	0.905	0.915	0.890	0.575	0.410	0.195	0.245	0.205	0.655	0.015
$CSIR$	0.595	0.585	0.640	0.005	0.000	0.360	0.330	0.380	0.005	0.000
$RCDCS$	1.000	1.000	1.000	0.000	0.000	0.950	0.940	0.920	0.005	0.005
$CRSIS$	1.000	1.000	0.990	0.000	0.000	0.940	0.895	0.875	0.000	0.000
$SVSIR$	1.000	1.000	0.995	0.000	0.000	0.965	0.960	0.945	0.000	0.000
$CRIS$	1.000	1.000	1.000	0.000	0.000	0.975	0.975	0.950	0.000	0.000
KM	0.965	0.970	0.990	0.100	0.095	0.465	0.510	0.510	0.100	0.010
$IPOD$	0.975	0.975	0.995	0.090	0.085	0.575	0.555	0.560	0.135	0.015
LQ	0.975	0.980	0.975	0.140	0.135	0.220	0.285	0.275	0.170	0.010
$ISIS_C$	0.986	0.983	0.980	0.980	0.956	0.586	0.516	0.583	0.780	0.226
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.480	0.530	0.380	0.005	0.005	0.415	0.415	0.320	0.015	0.000
CDC_1	0.465	0.505	0.355	0.670	0.060	0.400	0.405	0.315	0.765	0.030
CDC_2	0.465	0.505	0.355	0.975	0.085	0.400	0.410	0.315	0.960	0.040
$QaSIS$	0.095	0.100	0.100	0.030	0.000	0.030	0.090	0.065	0.090	0.000
$CSIR$	0.210	0.305	0.330	0.000	0.000	0.075	0.130	0.155	0.000	0.000
$RCDCS$	0.835	0.820	0.740	0.000	0.000	0.535	0.570	0.495	0.005	0.000
$CRSIS$	0.87	0.85	0.76	0.00	0.00	0.525	0.520	0.475	0.000	0.000
$SVSIR$	0.890	0.865	0.840	0.000	0.000	0.635	0.660	0.615	0.000	0.000
$CRIS$	0.880	0.875	0.815	0.000	0.000	0.680	0.715	0.635	0.000	0.000
KM	0.645	0.670	0.575	0.025	0.000	0.180	0.180	0.175	0.005	0.000
$IPOD$	0.665	0.670	0.570	0.025	0.000	0.230	0.250	0.180	0.025	0.000
LQ	0.475	0.520	0.460	0.045	0.005	0.105	0.050	0.050	0.050	0.000
$ISIS_C$	0.393	0.410	0.343	0.253	0.046	0.126	0.140	0.123	0.183	0.000
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.750	0.675	0.820	0.010	0.005	0.770	0.780	0.760	0.040	0.010
CDC_1	0.750	0.670	0.820	0.920	0.320	0.765	0.775	0.760	0.995	0.430
CDC_2	0.750	0.670	0.825	1.000	0.360	0.765	0.775	0.760	1.000	0.435
$QaSIS$	0.515	0.425	0.510	0.380	0.045	0.150	0.210	0.210	0.555	0.005
$CSIR$	0.650	0.670	0.695	0.005	0.000	0.365	0.365	0.395	0.005	0.000
$RCDCS$	1.000	0.995	0.985	0.000	0.000	0.955	0.925	0.895	0.005	0.005
$CRSIS$	1.000	0.995	0.990	0.000	0.000	0.925	0.860	0.860	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.000	0.000	0.970	0.955	0.945	0.000	0.000
$CRIS$	1.000	1.000	1.000	0.000	0.000	0.965	0.975	0.950	0.000	0.000
KM	0.965	0.965	0.975	0.085	0.080	0.465	0.505	0.515	0.105	0.015
$IPOD$	0.975	0.970	0.980	0.095	0.090	0.550	0.545	0.550	0.130	0.015
LQ	0.975	0.960	0.955	0.130	0.115	0.260	0.285	0.255	0.180	0.000
$ISIS_C$	0.966	0.980	0.963	0.953	0.923	0.526	0.493	0.520	0.756	0.166

Table 4: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (b) with $p = 1000$.

	$r=0.5$					$r=0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n=100, p=1000, d=[n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.930	0.940	0.942	0.034	0.024	0.728	0.750	0.744	0.054	0.014
CDC_1	0.916	0.940	0.936	0.952	0.772	0.720	0.734	0.724	0.980	0.372
CDC_2	0.916	0.940	0.936	0.974	0.784	0.720	0.734	0.724	0.984	0.372
$QaSIS$	0.674	0.664	0.632	0.142	0.034	0.280	0.284	0.256	0.150	0.004
$CSIR$	0.316	0.330	0.312	0.002	0.000	0.142	0.166	0.134	0.002	0.000
$RCDCS$	0.916	0.926	0.922	0.046	0.034	0.688	0.714	0.704	0.076	0.018
$CRSIS$	0.932	0.956	0.936	0.000	0.000	0.680	0.646	0.664	0.000	0.000
$SVSIR$	0.958	0.958	0.944	0.020	0.016	0.782	0.788	0.776	0.028	0.002
$CRIS$	0.958	0.968	0.954	0.024	0.020	0.794	0.798	0.790	0.026	0.006
KM	0.780	0.774	0.788	0.030	0.010	0.400	0.432	0.386	0.058	0.002
$IPOD$	0.810	0.814	0.816	0.026	0.010	0.448	0.470	0.428	0.056	0.006
LQ	0.658	0.640	0.636	0.028	0.004	0.180	0.176	0.198	0.012	0.002
$ISIS_C$	0.796	0.733	0.773	0.676	0.323	0.370	0.320	0.350	0.440	0.020
$n=200, p=1000, d=[n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	1.000	1.000	1.000	0.124	0.124	0.986	0.983	0.968	0.199	0.175
CDC_1	1.000	1.000	1.000	1.000	1.000	0.986	0.982	0.968	1.000	0.937
CDC_2	1.000	1.000	1.000	1.000	1.000	0.986	0.982	0.968	1.000	0.937
$QaSIS$	0.978	0.982	0.978	0.394	0.372	0.624	0.578	0.588	0.498	0.120
$CSIR$	0.696	0.714	0.736	0.004	0.000	0.328	0.374	0.396	0.002	0.000
$RCDCS$	1.000	1.000	0.998	0.212	0.212	0.972	0.962	0.964	0.376	0.338
$CRSIS$	1.000	1.000	1.000	0.006	0.006	0.968	0.958	0.944	0.006	0.006
$SVSIR$	1.000	1.000	1.000	0.076	0.076	0.994	0.992	0.990	0.102	0.100
$CRIS$	1.000	1.000	1.000	0.066	0.066	0.996	0.992	0.988	0.090	0.088
KM	0.996	0.986	0.982	0.080	0.078	0.798	0.774	0.776	0.136	0.062
$IPOD$	0.998	0.990	0.990	0.086	0.084	0.832	0.824	0.848	0.150	0.084
LQ	0.974	0.974	0.952	0.062	0.052	0.568	0.560	0.538	0.046	0.012
$ISIS_C$	0.976	0.966	0.980	0.993	0.920	0.670	0.670	0.596	0.870	0.273
$n=200, p=1000, d=[n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.830	0.830	0.824	0.048	0.034	0.630	0.636	0.614	0.084	0.026
CDC_1	0.820	0.824	0.822	0.866	0.494	0.622	0.636	0.610	0.844	0.218
CDC_2	0.820	0.824	0.822	1	0.552	0.622	0.636	0.610	1.000	0.242
$QaSIS$	0.738	0.754	0.736	0.228	0.122	0.352	0.398	0.366	0.240	0.010
$CSIR$	0.708	0.738	0.750	0.004	0.000	0.334	0.382	0.394	0.002	0.000
$RCDCS$	1.000	0.998	0.998	0.186	0.186	0.950	0.934	0.940	0.332	0.266
$CRSIS$	1.000	0.998	1.000	0.006	0.006	0.944	0.932	0.908	0.004	0.004
$SVSIR$	1.000	1.000	0.998	0.078	0.078	0.990	0.984	0.980	0.112	0.106
$CRIS$	1.000	1.000	1.000	0.066	0.066	0.984	0.978	0.980	0.084	0.080
KM	0.992	0.988	0.978	0.078	0.076	0.780	0.770	0.782	0.142	0.068
$IPOD$	0.992	0.990	0.994	0.084	0.082	0.830	0.816	0.844	0.144	0.080
LQ	0.974	0.976	0.958	0.074	0.058	0.532	0.566	0.538	0.058	0.014
$ISIS_C$	0.986	0.986	0.973	0.976	0.926	0.660	0.676	0.593	0.886	0.223

Figure 1: Estimated C statistic for DLBCL data with different methods

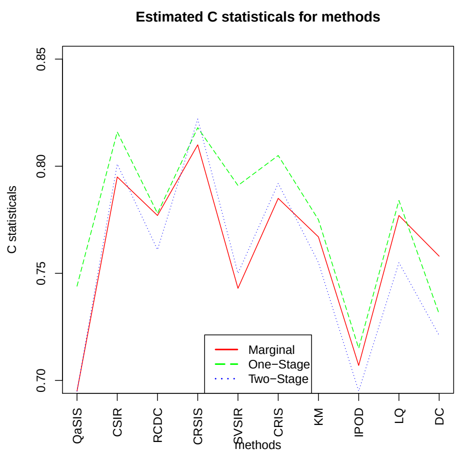

Table 5: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (b) with $p = 2000$

	$r = 0.5$					$r = 0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.930	0.920	0.880	0.010	0.005	0.625	0.730	0.600	0.010	0.000
CDC_1	0.930	0.905	0.855	0.950	0.695	0.620	0.725	0.575	0.955	0.245
CDC_2	0.930	0.905	0.855	0.945	0.685	0.620	0.725	0.575	0.965	0.235
$QaSIS$	0.570	0.580	0.535	0.065	0.000	0.190	0.200	0.215	0.100	0.000
$CSIR$	0.190	0.255	0.290	0.000	0.000	0.060	0.120	0.105	0.000	0.000
$RCDCS$	0.895	0.885	0.830	0.015	0.005	0.560	0.630	0.560	0.020	0.005
$CRSIS$	0.930	0.940	0.875	0.000	0.000	0.590	0.600	0.525	0.000	0.000
$SVSIR$	0.930	0.930	0.870	0.000	0.000	0.670	0.765	0.645	0.000	0.000
$CRIS$	0.930	0.940	0.885	0.005	0.000	0.705	0.780	0.695	0.000	0.000
KM	0.690	0.725	0.590	0.015	0.000	0.275	0.325	0.240	0.020	0.000
$IPOD$	0.740	0.760	0.605	0.015	0.000	0.295	0.360	0.295	0.025	0.000
LQ	0.580	0.505	0.480	0.020	0.000	0.115	0.115	0.125	0.010	0.000
$ISIS_C$	0.760	0.736	0.700	0.5766667	0.273	0.300	0.296	0.280	0.313	0.013
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	1.000	1.000	1.000	0.070	0.070	0.980	0.950	0.935	0.145	0.125
CDC_1	1.000	1.000	1.000	1.000	1.000	0.980	0.950	0.935	1.000	0.870
CDC_2	1.000	1.000	1.000	1.000	1.000	0.980	0.950	0.935	1.000	0.870
$QaSIS$	0.955	0.945	0.955	0.265	0.230	0.500	0.485	0.485	0.325	0.040
$CSIR$	0.585	0.635	0.650	0.010	0.000	0.290	0.270	0.340	0.000	0.000
$RCDCS$	1.000	1.000	1.000	0.125	0.125	0.945	0.955	0.920	0.240	0.205
$CRSIS$	1.000	1.000	1.000	0.005	0.005	0.955	0.915	0.915	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.050	0.050	0.980	0.975	0.955	0.080	0.065
$CRIS$	1.000	1.000	1.000	0.050	0.050	0.990	0.985	0.970	0.040	0.035
KM	0.990	0.990	0.970	0.050	0.050	0.670	0.710	0.690	0.095	0.040
$IPOD$	1.000	0.985	0.985	0.055	0.055	0.730	0.785	0.775	0.100	0.045
LQ	0.965	0.970	0.950	0.055	0.050	0.410	0.435	0.455	0.020	0.000
$ISIS_C$	0.966	0.983	0.980	1.000	0.933	0.600	0.626	0.623	0.816	0.233
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.505	0.540	0.455	0.005	0.000	0.230	0.325	0.205	0.010	0.000
CDC_1	0.495	0.520	0.430	0.475	0.060	0.230	0.325	0.200	0.420	0.010
CDC_2	0.495	0.520	0.430	0.925	0.085	0.230	0.325	0.200	0.950	0.010
$QaSIS$	0.180	0.185	0.115	0.025	0.000	0.110	0.060	0.080	0.050	0.000
$CSIR$	0.195	0.270	0.310	0.000	0.000	0.080	0.130	0.105	0.000	0.000
$RCDCS$	0.865	0.875	0.780	0.015	0.005	0.485	0.580	0.445	0.025	0.000
$CRSIS$	0.895	0.915	0.820	0.000	0.000	0.515	0.555	0.405	0.000	0.000
$SVSIR$	0.915	0.915	0.855	0.005	0.000	0.640	0.755	0.605	0.005	0.000
$CRIS$	0.920	0.910	0.885	0.005	0.000	0.690	0.730	0.645	0.005	0.000
KM	0.700	0.745	0.565	0.015	0.000	0.250	0.345	0.225	0.020	0.000
$IPOD$	0.740	0.765	0.600	0.015	0.000	0.285	0.355	0.300	0.025	0.000
LQ	0.605	0.495	0.445	0.015	0.000	0.135	0.135	0.150	0.015	0.000
$ISIS_C$	0.720	0.696	0.630	0.483	0.186	0.250	0.267	0.230	0.273	0.006
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.805	0.745	0.82 0	0.035	0.020	0.585	0.540	0.505	0.055	0.015
CDC_1	0.805	0.740	0.820	0.845	0.400	0.580	0.530	0.500	0.795	0.140
CDC_2	0.805	0.740	0.820	1.000	0.455	0.580	0.530	0.505	1.000	0.145
$QaSIS$	0.650	0.625	0.675	0.095	0.025	0.305	0.265	0.230	0.145	0.000
$CSIR$	0.585	0.670	0.685	0.010	0.000	0.290	0.280	0.340	0.000	0.000
$RCDCS$	1.000	1.000	0.995	0.100	0.100	0.930	0.920	0.880	0.205	0.150
$CRSIS$	1.000	1.000	1.000	0.005	0.005	0.905	0.850	0.865	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.050	0.050	0.980	0.980	0.940	0.060	0.050
$CRIS$	1.000	1.000	1.000	0.045	0.045	0.980	0.975	0.950	0.040	0.030
KM	0.985	0.975	0.975	0.060	0.060	0.675	0.690	0.685	0.095	0.040
$IPOD$	1.000	0.980	0.985	0.055	0.055	0.735	0.760	0.760	0.095	0.045
LQ	0.965	0.965	0.945	0.040	0.030	0.470	0.435	0.435	0.045	0.000
$ISIS_C$	0.986	0.970	0.966	1.000	0.926	0.620	0.580	0.583	0.796	0.173

Table 6: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (c) with $p = 1000$

	$r=0.5$					$r=0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n=100, p=1000, d=[n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.910	0.908	0.916	0.016	0.012	0.688	0.682	0.646	0.060	0.006
CDC_1	0.906	0.894	0.910	0.952	0.688	0.676	0.66	0.628	0.994	0.306
CDC_2	0.906	0.894	0.910	1.000	0.728	0.676	0.66	0.628	1.000	0.306
$QaSIS$	0.754	0.678	0.696	0.006	0.002	0.256	0.230	0.190	0.006	0.000
$CSIR$	0.452	0.474	0.452	0.000	0.000	0.212	0.244	0.230	0.000	0.000
$RCDCS$	0.966	0.970	0.954	0.002	0.002	0.688	0.736	0.682	0.000	0.000
$CRSIS$	0.990	0.988	0.986	0.000	0.000	0.840	0.836	0.824	0.000	0.000
$SVSIR$	0.988	0.980	0.980	0.004	0.004	0.864	0.866	0.870	0.000	0.000
$CRIS$	0.988	0.982	0.972	0.004	0.004	0.788	0.796	0.786	0.000	0.000
KM	0.842	0.852	0.830	0.004	0.000	0.326	0.300	0.280	0.004	0.000
$IPOD$	0.854	0.866	0.844	0.002	0.002	0.318	0.306	0.282	0.004	0.000
LQ	0.904	0.908	0.904	0.006	0.004	0.388	0.394	0.348	0.002	0.000
$ISIS_C$	0.800	0.790	0.773	0.993	0.440	0.300	0.290	0.300	0.630	0.010
$n=200, p=1000, d=[n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	1.000	1.000	0.998	0.006	0.006	0.966	0.974	0.964	0.064	0.046
CDC_1	1.000	1.000	0.998	1.000	0.998	0.966	0.970	0.964	1.000	0.910
CDC_2	1.000	1.000	0.998	1.000	0.998	0.966	0.970	0.964	1.000	0.910
$QaSIS$	0.992	0.990	0.990	0.036	0.036	0.650	0.662	0.632	0.004	0.000
$CSIR$	0.880	0.922	0.918	0.000	0.000	0.576	0.628	0.606	0.000	0.000
$RCDCS$	1.000	1.000	1.000	0.024	0.024	0.952	0.952	0.950	0.000	0.000
$CRSIS$	1.000	1.000	1.000	0.024	0.024	0.978	0.978	0.972	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.020	0.020	0.988	0.990	0.990	0.000	0.000
$CRIS$	1.000	1.000	1.000	0.022	0.022	0.972	0.966	0.968	0.000	0.000
KM	1.000	0.996	0.996	0.026	0.026	0.724	0.726	0.742	0.010	0.004
$IPOD$	0.998	0.998	0.998	0.032	0.032	0.732	0.748	0.746	0.010	0.008
LQ	1.000	1.000	1.000	0.046	0.046	0.844	0.820	0.832	0.010	0.006
$ISIS_C$	1.000	0.996	0.996	1.000	0.993	0.776	0.793	0.786	0.823	0.710
$n=200, p=1000, d=[n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.630	0.594	0.654	0.054	0.004	0.544	0.548	0.572	0.114	0.000
CDC_1	0.624	0.590	0.646	0.792	0.202	0.534	0.544	0.564	0.834	0.196
CDC_2	0.624	0.590	0.646	1.000	0.276	0.536	0.544	0.564	1.000	0.256
$QaSIS$	0.516	0.530	0.512	0.034	0.004	0.240	0.246	0.214	0.008	0.000
$CSIR$	0.926	0.950	0.944	0.000	0.000	0.648	0.688	0.662	0.000	0.000
$RCDCS$	1.000	0.998	1.000	0.020	0.020	0.890	0.898	0.892	0.000	0.000
$CRSIS$	1.000	1.000	1.000	0.004	0.004	0.964	0.954	0.946	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.012	0.012	0.986	0.984	0.980	0.000	0.000
$CRIS$	1.000	1.000	1.000	0.008	0.008	0.956	0.956	0.954	0.000	0.000
KM	0.994	0.998	0.992	0.030	0.030	0.660	0.678	0.668	0.006	0.002
$IPOD$	0.994	1.000	0.996	0.028	0.028	0.676	0.692	0.694	0.006	0.006
LQ	1.000	1.000	1.000	0.044	0.044	0.800	0.782	0.788	0.006	0.006
$ISIS_C$	0.996	1.000	1.000	1.000	0.996	0.786	0.823	0.783	0.856	0.687

Table 7: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (c) with $p = 2000$

	$r = 0.5$					$r = 0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.865	0.855	0.830	0.015	0.000	0.570	0.585	0.540	0.035	0.000
CDC_1	0.865	0.835	0.825	0.935	0.550	0.560	0.565	0.535	0.965	0.155
CDC_2	0.865	0.835	0.825	1.000	0.575	0.560	0.565	0.535	1.000	0.155
$QaSIS$	0.625	0.535	0.630	0.010	0.010	0.135	0.130	0.145	0.000	0.000
$CSIR$	0.350	0.365	0.420	0.000	0.000	0.165	0.195	0.220	0.000	0.000
$RCDCS$	0.940	0.940	0.950	0.000	0.000	0.590	0.645	0.595	0.000	0.000
$CRSIS$	0.985	0.990	0.980	0.000	0.000	0.810	0.800	0.770	0.000	0.000
$SVSIR$	0.980	0.980	0.975	0.000	0.000	0.795	0.825	0.780	0.000	0.000
$CRIS$	0.975	0.975	0.980	0.000	0.000	0.730	0.730	0.725	0.000	0.000
KM	0.800	0.770	0.775	0.010	0.010	0.265	0.225	0.200	0.000	0.000
$IPOD$	0.800	0.795	0.760	0.015	0.005	0.270	0.220	0.220	0.000	0.000
LQ	0.860	0.865	0.850	0.005	0.000	0.345	0.305	0.255	0.000	0.000
$ISIS_C$	0.776	0.743	0.713	0.993	0.360	0.236	0.276	0.246	0.677	0.010
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	1.000	1.000	0.995	0.010	0.010	0.950	0.925	0.935	0.045	0.025
CDC_1	1.000	1.000	0.995	1.000	0.995	0.950	0.925	0.925	1.000	0.835
CDC_2	1.000	1.000	0.995	1.000	0.995	0.950	0.925	0.925	1.000	0.835
$QaSIS$	0.975	0.985	0.995	0.020	0.015	0.505	0.530	0.580	0.000	0.000
$CSIR$	0.835	0.825	0.845	0.000	0.000	0.535	0.520	0.560	0.000	0.000
$RCDCS$	1.000	1.000	1.000	0.005	0.005	0.945	0.945	0.955	0.000	0.000
$CRSIS$	1.000	1.000	1.000	0.005	0.005	0.985	0.980	0.980	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.010	0.010	0.990	0.990	0.980	0.000	0.000
$CRIS$	1.000	1.000	1.000	0.015	0.015	0.980	0.970	0.975	0.000	0.000
KM	0.980	0.990	0.985	0.015	0.015	0.610	0.655	0.635	0.005	0.000
$IPOD$	0.995	1.000	0.995	0.010	0.010	0.640	0.650	0.635	0.000	0.000
LQ	1.000	1.000	1.000	0.020	0.020	0.790	0.785	0.810	0.000	0.000
$ISIS_C$	1.000	0.996	0.993	1.000	0.990	0.773	0.783	0.770	0.837	0.673
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.355	0.365	0.290	0.015	0.000	0.205	0.230	0.210	0.040	0.000
CDC_1	0.350	0.345	0.290	0.415	0.020	0.200	0.220	0.205	0.425	0.005
CDC_2	0.350	0.345	0.290	0.965	0.045	0.200	0.215	0.205	0.925	0.015
$QaSIS$	0.065	0.055	0.045	0.005	0.000	0.030	0.025	0.035	0.000	0.000
$CSIR$	0.445	0.440	0.515	0.000	0.000	0.235	0.225	0.310	0.000	0.000
$RCDCS$	0.860	0.880	0.870	0.000	0.000	0.500	0.450	0.440	0.000	0.000
$CRSIS$	0.950	0.945	0.930	0.000	0.000	0.660	0.680	0.615	0.000	0.000
$SVSIR$	0.975	0.970	0.960	0.000	0.000	0.760	0.775	0.720	0.000	0.000
$CRIS$	0.940	0.930	0.950	0.000	0.000	0.670	0.660	0.625	0.000	0.000
KM	0.755	0.700	0.715	0.010	0.010	0.215	0.200	0.190	0.000	0.000
$IPOD$	0.755	0.695	0.710	0.005	0.000	0.250	0.205	0.175	0.000	0.000
LQ	0.790	0.780	0.795	0.010	0.000	0.275	0.285	0.230	0.000	0.000
$ISIS_C$	0.593	0.623	0.550	0.980	0.143	0.220	0.230	0.213	0.653	0.006
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.570	0.455	0.590	0.025	0.000	0.470	0.475	0.505	0.090	0.000
CDC_1	0.570	0.455	0.585	0.760	0.115	0.465	0.480	0.505	0.800	0.115
CDC_2	0.570	0.45	0.585	1.000	0.170	0.470	0.475	0.505	1.000	0.165
$QaSIS$	0.350	0.305	0.320	0.010	0.000	0.180	0.145	0.185	0.000	0.000
$CSIR$	0.900	0.870	0.915	0.000	0.000	0.575	0.600	0.650	0.000	0.000
$RCDCS$	1.000	1.000	1.000	0.000	0.000	0.875	0.820	0.870	0.000	0.000
$CRSIS$	1.000	1.000	1.000	0.005	0.005	0.925	0.925	0.940	0.000	0.000
$SVSIR$	1.000	1.000	1.000	0.000	0.000	0.985	0.990	0.980	0.000	0.000
$CRIS$	1.000	0.995	1.000	0.000	0.000	0.950	0.960	0.955	0.000	0.000
KM	0.990	0.985	0.990	0.015	0.015	0.580	0.550	0.565	0.005	0.000
$IPOD$	0.990	0.990	0.990	0.015	0.015	0.560	0.575	0.575	0.000	0.000
LQ	1.000	0.995	1.000	0.020	0.020	0.735	0.735	0.735	0.005	0.005
$ISIS_C$	0.993	0.980	0.983	1.000	0.956	0.756	0.756	0.720	0.833	0.583

Table 8: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (d) with $p = 1000$.

	$r=0.5$					$r=0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
	$n=100, p=1000, d=[n/log(n)], CR \approx 0.01 \sim 0.03$									
CDC	0.804	0.786	0.792	0.016	0.002	0.558	0.558	0.548	0.052	0.000
CDC_1	0.788	0.770	0.778	0.400	0.196	0.548	0.536	0.536	0.514	0.076
CDC_2	0.788	0.770	0.778	0.888	0.422	0.548	0.534	0.536	0.922	0.132
$QaSIS$	0.300	0.256	0.224	0.008	0.000	0.154	0.126	0.130	0.018	0.000
$CSIR$	0.346	0.396	0.368	0.012	0.000	0.180	0.222	0.226	0.062	0.000
$RCDCS$	0.910	0.894	0.904	0.006	0.004	0.640	0.622	0.602	0.042	0.006
$CRSIS$	0.952	0.948	0.952	0.004	0.002	0.764	0.766	0.756	0.042	0.020
$SVSIR$	0.930	0.930	0.938	0.000	0.000	0.722	0.702	0.706	0.014	0.002
$CRIS$	0.924	0.930	0.938	0.004	0.002	0.724	0.690	0.696	0.044	0.008
KM	0.754	0.756	0.770	0.004	0.000	0.314	0.298	0.272	0.018	0.000
$IPOD$	0.610	0.578	0.576	0.008	0.004	0.228	0.232	0.204	0.014	0.000
LQ	0.778	0.774	0.776	0.004	0.002	0.420	0.394	0.372	0.016	0.000
$ISIS_C$	0.480	0.490	0.470	0.933	0.100	0.256	0.240	0.286	0.656	0.020
	$n=200, p=1000, d=[n/log(n)], CR \approx 0.01 \sim 0.03$									
CDC	0.990	0.992	0.992	0.010	0.010	0.926	0.900	0.904	0.048	0.028
CDC_1	0.990	0.992	0.992	0.832	0.812	0.920	0.896	0.902	0.892	0.664
CDC_2	0.990	0.992	0.992	1.000	0.974	0.920	0.896	0.902	1.000	0.754
$QaSIS$	0.624	0.612	0.636	0.002	0.002	0.456	0.378	0.428	0.026	0.006
$CSIR$	0.762	0.808	0.792	0.020	0.012	0.526	0.560	0.552	0.066	0.004
$RCDCS$	1.000	1.000	1.000	0.000	0.000	0.950	0.946	0.952	0.034	0.026
$CRSIS$	1.000	1.000	1.000	0.000	0.000	0.990	0.992	0.986	0.032	0.030
$SVSIR$	1.000	1.000	1.000	0.000	0.000	0.976	0.978	0.980	0.010	0.010
$CRIS$	1.000	1.000	1.000	0.000	0.000	0.970	0.980	0.978	0.040	0.036
KM	0.984	0.984	0.986	0.002	0.002	0.732	0.748	0.734	0.018	0.002
$IPOD$	0.894	0.902	0.906	0.014	0.008	0.512	0.458	0.484	0.036	0.000
LQ	0.990	0.998	1.000	0.000	0.000	0.874	0.872	0.856	0.020	0.008
$ISIS_C$	0.970	0.956	0.976	1.000	0.923	0.803	0.806	0.823	0.853	0.733
	$n=200, p=1000, d=[n/log(n)], CR \approx 0.1 \sim 0.3$									
CDC	0.762	0.744	0.762	0.016	0.000	0.700	0.688	0.686	0.092	0.008
CDC_1	0.756	0.738	0.762	0.868	0.390	0.696	0.682	0.678	0.872	0.316
CDC_2	0.756	0.738	0.762	1	0.430	0.698	0.682	0.678	1.000	0.372
$QaSIS$	0.400	0.372	0.366	0.018	0.002	0.274	0.224	0.276	0.034	0.002
$CSIR$	0.766	0.800	0.786	0.026	0.014	0.532	0.552	0.546	0.082	0.008
$RCDCS$	1.000	0.998	1.000	0.000	0.000	0.948	0.942	0.946	0.032	0.024
$CRSIS$	1.000	1.000	1.000	0.000	0.000	0.988	0.990	0.988	0.032	0.030
$SVSIR$	1.000	1.000	1.000	0.000	0.000	0.974	0.976	0.980	0.010	0.010
$CRIS$	1.000	1.000	1.000	0.002	0.002	0.972	0.976	0.976	0.054	0.046
KM	0.986	0.988	0.990	0.002	0.002	0.728	0.744	0.732	0.016	0.002
$IPOD$	0.940	0.930	0.958	0.008	0.008	0.588	0.574	0.600	0.038	0.000
LQ	0.994	1.000	0.998	0.000	0.000	0.870	0.872	0.854	0.016	0.004
$ISIS_C$	0.990	0.983	0.997	1.000	0.973	0.783	0.816	0.786	0.836	0.713

Table 9: Simulation results of $\mathcal{P}_s$ and $\mathcal{P}_a$ for Example 2 (d) with $p = 2000$

	$r = 0.5$					$r = 0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.685	0.740	0.745	0.005	0.000	0.395	0.475	0.425	0.035	0.000
CDC_1	0.675	0.735	0.740	0.320	0.135	0.390	0.460	0.400	0.410	0.025
CDC_2	0.675	0.735	0.740	0.850	0.300	0.390	0.460	0.400	0.890	0.060
$QaSIS$	0.150	0.135	0.160	0.010	0.000	0.100	0.075	0.105	0.010	0.000
$CSIR$	0.240	0.280	0.330	0.015	0.000	0.125	0.175	0.160	0.035	0.000
$RCDCS$	0.830	0.855	0.815	0.005	0.005	0.470	0.555	0.435	0.035	0.000
$CRSIS$	0.940	0.920	0.885	0.005	0.005	0.690	0.695	0.625	0.045	0.010
$SVSIR$	0.910	0.885	0.850	0.000	0.000	0.615	0.640	0.575	0.005	0.005
$CRIS$	0.905	0.885	0.855	0.005	0.005	0.590	0.635	0.555	0.030	0.000
KM	0.675	0.685	0.600	0.000	0.000	0.175	0.240	0.195	0.005	0.000
$IPOD$	0.525	0.480	0.460	0.010	0.000	0.120	0.175	0.125	0.010	0.000
LQ	0.665	0.630	0.670	0.000	0.000	0.290	0.320	0.235	0.010	0.000
$ISIS_C$	0.350	0.416	0.386	0.906	0.043	0.216	0.223	0.206	0.690	0.013
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.01 \sim 0.03$										
CDC	0.985	1.000	0.965	0.000	0.000	0.835	0.900	0.830	0.025	0.015
CDC_1	0.985	1.000	0.965	0.775	0.750	0.830	0.890	0.825	0.845	0.525
CDC_2	0.985	1.000	0.965	1.000	0.950	0.830	0.890	0.825	1.000	0.620
$QaSIS$	0.505	0.490	0.510	0.000	0.000	0.360	0.285	0.305	0.010	0.000
$CSIR$	0.660	0.695	0.710	0.015	0.010	0.445	0.410	0.490	0.085	0.000
$RCDCS$	1.000	1.000	0.985	0.000	0.000	0.885	0.890	0.910	0.015	0.010
$CRSIS$	1.000	0.995	0.995	0.000	0.000	0.965	0.975	0.950	0.015	0.010
$SVSIR$	1.000	1.000	0.995	0.000	0.000	0.945	0.945	0.940	0.015	0.015
$CRIS$	1.000	1.000	0.995	0.000	0.000	0.935	0.945	0.950	0.015	0.015
KM	0.960	0.955	0.965	0.005	0.005	0.660	0.640	0.615	0.000	0.000
$IPOD$	0.850	0.835	0.820	0.010	0.005	0.405	0.380	0.390	0.005	0.000
LQ	1.000	0.990	0.990	0.000	0.000	0.825	0.805	0.780	0.005	0.000
$ISIS_C$	0.956	0.933	0.920	1.000	0.840	0.810	0.843	0.800	0.893	0.710
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.370	0.425	0.340	0.015	0.000	0.280	0.260	0.280	0.045	0.000
CDC_1	0.360	0.415	0.310	0.460	0.015	0.270	0.245	0.270	0.420	0.005
CDC_2	0.360	0.415	0.310	0.855	0.030	0.270	0.245	0.270	0.885	0.010
$QaSIS$	0.090	0.070	0.050	0.000	0.000	0.045	0.055	0.030	0.000	0.000
$CSIR$	0.265	0.310	0.335	0.015	0.000	0.135	0.185	0.165	0.035	0.000
$RCDCS$	0.830	0.840	0.805	0.000	0.000	0.455	0.540	0.435	0.035	0.000
$CRSIS$	0.935	0.905	0.885	0.005	0.000	0.670	0.680	0.600	0.050	0.015
$SVSIR$	0.915	0.875	0.835	0.000	0.000	0.620	0.625	0.565	0.010	0.005
$CRIS$	0.910	0.875	0.840	0.015	0.010	0.595	0.615	0.540	0.065	0.005
KM	0.680	0.695	0.605	0.000	0.000	0.175	0.240	0.195	0.005	0.000
$IPOD$	0.495	0.540	0.450	0.005	0.000	0.115	0.180	0.130	0.000	0.000
LQ	0.730	0.665	0.685	0.000	0.000	0.305	0.320	0.255	0.010	0.000
$ISIS_C$	0.607	0.623	0.537	0.973	0.146	0.206	0.223	0.216	0.596	0.006
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
CDC	0.695	0.730	0.690	0.005	0.000	0.650	0.585	0.595	0.065	0.000
CDC_1	0.680	0.730	0.685	0.855	0.310	0.650	0.570	0.585	0.810	0.200
CDC_2	0.680	0.730	0.685	1.000	0.325	0.650	0.570	0.585	1.000	0.240
$QaSIS$	0.250	0.305	0.270	0.000	0.000	0.185	0.205	0.140	0.005	0.000
$CSIR$	0.650	0.705	0.720	0.020	0.010	0.435	0.425	0.485	0.085	0.000
$RCDCS$	1.000	0.995	0.985	0.000	0.000	0.885	0.890	0.900	0.015	0.010
$CRSIS$	1.000	0.995	0.995	0.000	0.000	0.965	0.965	0.945	0.015	0.010
$SVSIR$	1.000	1.000	0.995	0.000	0.000	0.940	0.945	0.940	0.015	0.015
$CRIS$	1.000	0.995	0.995	0.015	0.015	0.920	0.940	0.935	0.040	0.035
KM	0.975	0.965	0.975	0.005	0.005	0.660	0.645	0.615	0.000	0.000
$IPOD$	0.930	0.895	0.915	0.005	0.000	0.475	0.445	0.475	0.005	0.000
LQ	1.000	0.995	0.995	0.000	0.000	0.815	0.800	0.780	0.005	0.000
$ISIS_C$	0.993	0.986	0.983	1.000	0.963	0.760	0.776	0.736	0.813	0.660

Table 10: Combination of existing methods and $CSVS_1$, $CSVS_2$ in Example 2 (a) continuous.

Model	$r = 0.5$					$r = 0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$		$\mathcal{P}_s$		$\mathcal{P}_a$	
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
$QaSIS_1^M$	0.075	0.090	0.095	0.660	0.000	0.030	0.085	0.065	0.790	0.000
$QaSIS_2^M$	0.075	0.090	0.090	0.975	0.000	0.030	0.085	0.065	0.960	0.000
$CSIR_1^M$	0.210	0.305	0.315	0.660	0.020	0.070	0.125	0.125	0.765	0.000
$CSIR_2^M$	0.210	0.305	0.315	0.975	0.035	0.070	0.125	0.125	0.960	0.000
$RCDCS_1^M$	0.835	0.815	0.735	0.660	0.310	0.520	0.570	0.485	0.765	0.090
$RCDCS_2^M$	0.835	0.815	0.735	0.975	0.450	0.520	0.575	0.485	0.965	0.110
$CRSIS_1^M$	0.865	0.850	0.760	0.660	0.385	0.510	0.510	0.455	0.765	0.080
$CRSIS_2^M$	0.865	0.850	0.760	0.975	0.550	0.510	0.515	0.455	0.960	0.095
$SVSIR_1^M$	0.870	0.860	0.835	0.660	0.425	0.620	0.650	0.595	0.765	0.190
$SVSIR_2^M$	0.870	0.860	0.835	0.975	0.605	0.620	0.655	0.595	0.965	0.230
$CRIS_1^M$	0.850	0.870	0.815	0.660	0.395	0.670	0.695	0.630	0.765	0.215
$CRIS_2^M$	0.850	0.870	0.815	0.975	0.580	0.670	0.700	0.630	0.960	0.270
KM_1^M	0.615	0.655	0.550	0.665	0.120	0.170	0.155	0.160	0.770	0.005
KM_2^M	0.615	0.655	0.550	0.975	0.185	0.170	0.155	0.160	0.960	0.005
$IPOD_1^M$	0.660	0.665	0.565	0.670	0.125	0.220	0.245	0.175	0.775	0.010
$IPOD_2^M$	0.660	0.665	0.565	0.975	0.190	0.220	0.245	0.175	0.960	0.010
LQ_1^M	0.445	0.485	0.455	0.660	0.040	0.105	0.050	0.050	0.775	0.000
LQ_2^M	0.445	0.485	0.455	0.975	0.060	0.105	0.050	0.050	0.965	0.000
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
$QaSIS_1^M$	0.510	0.410	0.505	0.930	0.140	0.145	0.205	0.180	0.995	0.010
$QaSIS_2^M$	0.505	0.410	0.510	1.000	0.150	0.145	0.205	0.185	1.000	0.010
$CSIR_1^M$	0.640	0.650	0.690	0.925	0.225	0.355	0.360	0.395	0.995	0.060
$CSIR_2^M$	0.640	0.650	0.695	1.000	0.250	0.355	0.360	0.400	1.000	0.060
$RCDCS_1^M$	1.000	0.995	0.985	0.915	0.900	0.950	0.925	0.890	0.995	0.770
$RCDCS_2^M$	1.000	0.995	0.985	1.000	0.980	0.950	0.925	0.890	1.000	0.775
$CRSIS_1^M$	1.000	0.995	0.99	0.915	0.900	0.920	0.850	0.855	0.995	0.645
$CRSIS_2^M$	1.000	0.995	0.99	1.000	0.985	0.920	0.850	0.855	1.000	0.650
$SVSIR_1^M$	1.000	1.000	0.995	0.915	0.910	0.965	0.945	0.945	0.995	0.860
$SVSIR_2^M$	1.000	1.000	0.995	1.000	0.995	0.965	0.945	0.945	1.000	0.865
$CRIS_1^M$	1.000	1.000	1.000	0.920	0.920	0.965	0.975	0.945	0.995	0.880
$CRIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.965	0.975	0.945	1.000	0.885
KM_1^M	0.965	0.965	0.965	0.930	0.845	0.455	0.475	0.500	0.995	0.115
KM_2^M	0.965	0.965	0.965	1.000	0.915	0.455	0.475	0.500	1.000	0.115
$IPOD_1^M$	0.975	0.970	0.970	0.930	0.845	0.550	0.530	0.530	0.995	0.140
$IPOD_2^M$	0.975	0.970	0.970	1.000	0.915	0.550	0.530	0.530	1.000	0.140
LQ_1^M	0.975	0.960	0.955	0.930	0.825	0.250	0.285	0.255	0.995	0.000
LQ_2^M	0.975	0.960	0.955	1.000	0.890	0.250	0.285	0.260	1.000	0.000

Table 11: Combination of existing methods and $CSVS_1$, $CSVS_2$ in Example 2 (b).

	$r=0.5$					$r=0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n=100, p=1000, d=[n/log(n)], CR\approx 0.1\sim 0.3$										
$QaSIS_1^M$	0.736	0.748	0.734	0.856	0.434	0.356	0.410	0.364	0.840	0.068
$QaSIS_2^M$	0.736	0.748	0.734	1.000	0.468	0.356	0.41	0.364	1.00	0.072
$CSIR_1^M$	0.704	0.730	0.738	0.848	0.332	0.324	0.372	0.388	0.814	0.054
$CSIR_2^M$	0.704	0.730	0.738	1.000	0.376	0.324	0.372	0.388	1.000	0.056
$RCDCS_1^M$	1.000	0.998	0.998	0.872	0.868	0.948	0.934	0.938	0.856	0.714
$RCDCS_2^M$	1.000	0.998	0.998	1.000	0.996	0.948	0.934	0.938	1.000	0.828
$CRSIS_1^M$	1.000	1.000	1.000	0.852	0.852	0.948	0.93	0.912	0.818	0.668
$CRSIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.948	0.93	0.912	1.000	0.810
$SVSIR_1^M$	1.000	1.000	0.998	0.860	0.858	0.988	0.980	0.978	0.818	0.778
$SVSIR_2^M$	1.000	1.000	0.998	1.000	0.998	0.988	0.980	0.978	1.000	0.946
$CRIS_1^M$	1.000	1.000	1.000	0.860	0.860	0.984	0.978	0.98	0.818	0.772
$CRIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.984	0.978	0.98	1.000	0.942
KM_1^M	0.992	0.986	0.974	0.86	0.824	0.768	0.766	0.778	0.828	0.366
KM_2^M	0.992	0.986	0.974	1.00	0.954	0.768	0.766	0.778	1.000	0.438
$IPOD_1^M$	0.992	0.988	0.986	0.852	0.824	0.824	0.794	0.832	0.834	0.448
$IPOD_2^M$	0.992	0.988	0.986	1.000	0.966	0.824	0.794	0.832	1.000	0.534
LQ_1^M	0.974	0.974	0.958	0.862	0.780	0.528	0.554	0.528	0.82	0.188
LQ_2^M	0.974	0.974	0.958	1.000	0.912	0.528	0.554	0.528	1.00	0.218
$n=100, p=2000, d=[n/log(n)], CR\approx 0.1\sim 0.3$										
$QaSIS_1^M$	0.160	0.165	0.095	0.460	0.015	0.115	0.055	0.075	0.405	0.000
$QaSIS_2^M$	0.160	0.165	0.095	0.925	0.020	0.115	0.055	0.075	0.955	0.000
$CSIR_1^M$	0.195	0.260	0.300	0.455	0.005	0.080	0.120	0.105	0.395	0.000
$CSIR_2^M$	0.195	0.260	0.300	0.925	0.015	0.080	0.120	0.105	0.950	0.000
$RCDCS_1^M$	0.855	0.865	0.775	0.470	0.295	0.460	0.555	0.435	0.405	0.050
$RCDCS_2^M$	0.855	0.865	0.775	0.925	0.525	0.460	0.555	0.435	0.950	0.100
$CRSIS_1^M$	0.880	0.915	0.820	0.455	0.325	0.505	0.535	0.410	0.395	0.045
$CRSIS_2^M$	0.880	0.915	0.820	0.925	0.610	0.505	0.535	0.410	0.950	0.115
$SVSIR_1^M$	0.910	0.915	0.855	0.460	0.340	0.630	0.735	0.585	0.395	0.115
$SVSIR_2^M$	0.910	0.915	0.855	0.925	0.660	0.630	0.735	0.585	0.950	0.265
$CRIS_1^M$	0.910	0.910	0.865	0.465	0.335	0.665	0.705	0.635	0.400	0.125
$CRIS_2^M$	0.910	0.910	0.865	0.925	0.655	0.665	0.705	0.635	0.950	0.275
KM_1^M	0.700	0.745	0.560	0.465	0.155	0.225	0.330	0.225	0.410	0.005
KM_2^M	0.700	0.745	0.560	0.925	0.265	0.225	0.330	0.225	0.950	0.020
$IPOD_1^M$	0.720	0.750	0.595	0.465	0.165	0.270	0.350	0.295	0.405	0.015
$IPOD_2^M$	0.720	0.750	0.595	0.925	0.275	0.270	0.350	0.295	0.950	0.040
LQ_1^M	0.595	0.490	0.430	0.475	0.100	0.135	0.125	0.150	0.410	0.010
LQ_2^M	0.595	0.490	0.430	0.925	0.160	0.135	0.125	0.150	0.950	0.010
$n=200, p=2000, d=[n/log(n)], CR\approx 0.1\sim 0.3$										
$QaSIS_1^M$	0.635	0.605	0.665	0.825	0.275	0.300	0.265	0.240	0.795	0.025
$QaSIS_2^M$	0.635	0.605	0.665	1.000	0.305	0.300	0.265	0.245	1.000	0.030
$CSIR_1^M$	0.585	0.655	0.680	0.825	0.200	0.280	0.270	0.340	0.780	0.015
$CSIR_2^M$	0.585	0.655	0.680	1.000	0.250	0.280	0.270	0.345	1.000	0.020
$RCDCS_1^M$	0.995	1.000	0.995	0.845	0.845	0.930	0.910	0.875	0.800	0.605
$RCDCS_2^M$	0.995	1.000	0.995	1.000	0.990	0.930	0.910	0.875	1.000	0.745
$CRSIS_1^M$	1.000	0.995	1.000	0.825	0.820	0.900	0.860	0.860	0.785	0.530
$CRSIS_2^M$	1.000	0.995	1.000	1.000	0.995	0.900	0.860	0.860	1.000	0.680
$SVSIR_1^M$	1.000	1.000	1.000	0.840	0.840	0.980	0.970	0.935	0.790	0.700
$SVSIR_2^M$	1.000	1.000	1.000	1.000	1.000	0.980	0.970	0.935	1.000	0.890
$CRIS_1^M$	1.000	1.000	1.000	0.840	0.840	0.980	0.970	0.950	0.785	0.705
$CRIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.980	0.970	0.950	1.000	0.900
KM_1^M	0.985	0.975	0.975	0.835	0.790	0.670	0.675	0.680	0.780	0.280
KM_2^M	0.985	0.975	0.975	1.000	0.940	0.670	0.675	0.680	1.000	0.350
$IPOD_1^M$	1.000	0.980	0.985	0.835	0.805	0.720	0.745	0.750	0.785	0.330
$IPOD_2^M$	1.000	0.980	0.985	1.000	0.965	0.720	0.745	0.750	1.000	0.430
LQ_1^M	0.960	0.960	0.940	0.830	0.725	0.465	0.425	0.420	0.785	0.095
LQ_2^M	0.960	0.960	0.940	1.000	0.870	0.465	0.425	0.420	1.000	0.110

Table 12: Combination of existing methods and $CSVS_1$, $CSVS_2$ in Example 2 (c).

Model			$r = 0.5$					$r = 0.8$		
		$\mathcal{P}_s$			$\mathcal{P}_a$		$\mathcal{P}_s$			$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
			$n = 200, p = 1000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$							
$QaSIS_1^M$	0.380	0.320	0.360	0.796	0.052	0.176	0.176	0.192	0.826	0.014
$QaSIS_2^M$	0.376	0.318	0.358	1.000	0.056	0.178	0.174	0.190	1.000	0.014
$CSIR_1^M$	0.852	0.888	0.874	0.79	0.518	0.594	0.634	0.640	0.846	0.232
$CSIR_2^M$	0.852	0.888	0.874	1.00	0.660	0.596	0.632	0.638	1.000	0.272
$RCDCS_1^M$	0.998	0.992	0.998	0.782	0.770	0.880	0.888	0.880	0.840	0.574
$RCDCS_2^M$	0.998	0.992	0.998	1.000	0.988	0.880	0.888	0.880	1.000	0.694
$CRSIS_1^M$	1.000	1.000	1.000	0.784	0.784	0.962	0.952	0.944	0.84	0.728
$CRSIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.962	0.952	0.944	1.00	0.872
$SVSIR_1^M$	1.000	1.000	1.000	0.786	0.786	0.976	0.978	0.970	0.828	0.764
$SVSIR_2^M$	1.000	1.000	1.000	1.000	1.000	0.976	0.978	0.970	1.000	0.924
$CRIS_1^M$	1.000	1.000	1.000	0.790	0.790	0.960	0.968	0.95	0.846	0.756
$CRIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.960	0.968	0.95	1.000	0.896
KM_1^M	0.980	0.986	0.984	0.786	0.746	0.636	0.642	0.622	0.832	0.238
KM_2^M	0.980	0.986	0.984	1.000	0.954	0.638	0.64	0.622	1.000	0.286
$IPOD_1^M$	0.980	0.988	0.992	0.788	0.752	0.630	0.676	0.632	0.834	0.246
$IPOD_2^M$	0.980	0.988	0.992	1.000	0.960	0.632	0.674	0.632	1.000	0.294
LQ_1^M	0.986	0.996	0.996	0.784	0.766	0.792	0.782	0.752	0.828	0.404
LQ_2^M	0.986	0.996	0.996	1.000	0.98	0.792	0.782	0.752	1.000	0.486
			$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$							
$QaSIS_1^M$	0.035	0.020	0.060	0.410	0.000	0.030	0.015	0.040	0.415	0.000
$QaSIS_2^M$	0.020	0.015	0.050	0.965	0.000	0.020	0.0150	0.040	0.915	0.000
$CSIR_1^M$	0.325	0.340	0.380	0.425	0.030	0.170	0.175	0.210	0.435	0.010
$CSIR_2^M$	0.320	0.335	0.375	0.965	0.045	0.165	0.175	0.210	0.915	0.015
$RCDCS_1^M$	0.734	0.748	0.732	0.434	0.168	0.395	0.410	0.350	0.425	0.035
$RCDCS_2^M$	0.732	0.748	0.730	0.960	0.384	0.395	0.405	0.350	0.920	0.065
$CRSIS_1^M$	0.875	0.860	0.760	0.420	0.265	0.535	0.560	0.475	0.430	0.055
$CRSIS_2^M$	0.875	0.860	0.755	0.970	0.575	0.540	0.555	0.475	0.915	0.145
$SVSIR_1^M$	0.586	0.602	0.590	0.418	0.094	0.395	0.410	0.350	0.425	0.035
$SVSIR_2^M$	0.586	0.600	0.590	0.920	0.184	0.395	0.405	0.350	0.920	0.065
$CRIS_1^M$	0.835	0.82	0.845	0.430	0.255	0.565	0.565	0.52	0.475	0.060
$CRIS_2^M$	0.835	0.82	0.845	0.965	0.570	0.565	0.560	0.52	0.925	0.135
KM_1^M	0.640	0.59	0.565	0.415	0.080	0.150	0.200	0.130	0.415	0.005
KM_2^M	0.635	0.59	0.560	0.965	0.175	0.150	0.195	0.130	0.915	0.010
$IPOD_1^M$	0.650	0.565	0.54	0.015	0.000	0.170	0.175	0.155	0.415	0.005
$IPOD_2^M$	0.650	0.565	0.54	0.965	0.165	0.170	0.170	0.155	0.915	0.010
LQ_1^M	0.685	0.64	0.645	0.005	0.000	0.245	0.260	0.175	0.420	0.010
LQ_2^M	0.685	0.64	0.645	0.965	0.235	0.245	0.255	0.175	0.915	0.025
			$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$							
$QaSIS_1^M$	0.220	0.180	0.185	0.760	0.015	0.090	0.130	0.145	0.800	0.005
$QaSIS_2^M$	0.215	0.170	0.185	1.000	0.015	0.095	0.120	0.145	1.000	0.005
$CSIR_1^M$	0.780	0.795	0.800	0.760	0.400	0.505	0.500	0.590	0.820	0.120
$CSIR_2^M$	0.780	0.795	0.800	1.000	0.500	0.505	0.495	0.590	1.000	0.170
$RCDCS_1^M$	0.990	0.990	0.980	0.765	0.740	0.845	0.790	0.830	0.815	0.440
$RCDCS_2^M$	0.990	0.990	0.980	1.000	0.960	0.845	0.780	0.830	1.000	0.555
$CRSIS_1^M$	0.995	0.990	0.980	0.760	0.730	0.950	0.915	0.940	0.810	0.660
$CRSIS_2^M$	0.995	0.990	0.980	1.000	0.965	0.950	0.910	0.940	1.000	0.820
$SVSIR_1^M$	1.000	1.000	0.995	0.760	0.760	0.945	0.925	0.940	0.800	0.665
$SVSIR_2^M$	1.000	1.000	0.995	1.000	0.995	0.945	0.925	0.940	1.000	0.835
$CRIS_1^M$	1.000	1.000	0.995	0.770	0.770	0.940	0.925	0.920	0.810	0.640
$CRIS_2^M$	1.000	1.000	0.995	1.000	0.995	0.940	0.925	0.920	1.000	0.815
KM_1^M	0.965	0.975	0.945	0.760	0.680	0.580	0.540	0.555	0.800	0.175
KM_2^M	0.965	0.975	0.945	1.000	0.890	0.585	0.535	0.555	1.000	0.225
$IPOD_1^M$	0.970	0.975	0.970	0.760	0.705	0.580	0.555	0.580	0.800	0.175
$IPOD_2^M$	0.970	0.975	0.970	1.000	0.915	0.585	0.550	0.580	1.000	0.240
LQ_1^M	0.995	0.975	0.990	0.760	0.735	0.755	0.720	0.710	0.805	0.330
LQ_2^M	0.995	0.975	0.990	1.000	0.960	0.755	0.715	0.710	1.000	0.415

Table 13: Combination of existing methods and $CSVS_1$, $CSVS_2$ in Example 2 (d)

	$r = 0.5$					$r = 0.8$				
	$\mathcal{P}_s$				$\mathcal{P}_a$	$\mathcal{P}_s$				$\mathcal{P}_a$
	X_1	X_2	X_3	X_4	ALL	X_1	X_2	X_3	X_4	ALL
$n = 100, p = 1000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
$QaSIS_1^M$	0.400	0.366	0.362	0.866	0.048	0.268	0.218	0.274	0.872	0.026
$QaSIS_2^M$	0.400	0.366	0.362	1.000	0.052	0.270	0.218	0.274	1.000	0.026
$CSIR_1^M$	0.756	0.792	0.780	0.872	0.412	0.526	0.540	0.530	0.886	0.152
$CSIR_2^M$	0.756	0.792	0.780	1.000	0.470	0.526	0.540	0.530	1.000	0.176
$RCDCS_1^M$	0.998	0.998	1.000	0.866	0.864	0.942	0.94	0.94	0.876	0.728
$RCDCS_2^M$	0.998	0.998	1.000	1.000	0.996	0.942	0.94	0.94	1.000	0.832
$CRSIS_1^M$	1.000	1.000	1.000	0.866	0.866	0.986	0.990	0.984	0.874	0.836
$CRSIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.986	0.990	0.984	1.000	0.960
$SVSIR_1^M$	1.000	1.000	1.000	0.866	0.866	0.972	0.974	0.976	0.870	0.800
$SVSIR_2^M$	1.000	1.000	1.000	1.000	1.000	0.972	0.974	0.976	1.000	0.922
$CRIS_1^M$	1.000	1.000	1.000	0.866	0.866	0.968	0.974	0.976	0.878	0.804
$CRIS_2^M$	1.000	1.000	1.000	1.000	1.000	0.968	0.974	0.976	1.000	0.918
KM_1^M	0.986	0.988	0.988	0.866	0.834	0.718	0.736	0.726	0.870	0.344
KM_2^M	0.986	0.988	0.988	1.000	0.966	0.718	0.736	0.726	1.000	0.394
$IPOD_1^M$	0.938	0.926	0.956	0.866	0.720	0.574	0.56	0.586	0.874	0.182
$IPOD_2^M$	0.938	0.926	0.956	1.000	0.826	0.574	0.56	0.586	1.000	0.208
LQ_1^M	0.994	0.998	0.998	0.866	0.856	0.866	0.870	0.836	0.870	0.558
LQ_2^M	0.994	0.998	0.998	1.000	0.990	0.866	0.870	0.836	1.000	0.640
$n = 100, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
$QaSIS_1^M$	0.070	0.075	0.090	0.895	0.000	0.035	0.035	0.050	0.905	0.000
$QaSIS_2^M$	0.070	0.075	0.090	1.000	0.000	0.035	0.035	0.050	1.000	0.000
$CSIR_1^M$	0.260	0.305	0.335	0.895	0.020	0.115	0.170	0.165	0.910	0.010
$CSIR_2^M$	0.260	0.305	0.335	1.000	0.030	0.115	0.170	0.165	1.000	0.010
$RCDCS_1^M$	0.825	0.830	0.790	0.455	0.245	0.430	0.520	0.425	0.400	0.020
$RCDCS_2^M$	0.825	0.830	0.790	0.850	0.450	0.430	0.520	0.425	0.880	0.075
$CRSIS_1^M$	0.935	0.905	0.855	0.460	0.340	0.655	0.655	0.595	0.420	0.110
$CRSIS_2^M$	0.935	0.905	0.855	0.850	0.630	0.655	0.655	0.595	0.880	0.240
$SVSIR_1^M$	0.905	0.860	0.830	0.455	0.315	0.600	0.610	0.540	0.400	0.070
$SVSIR_2^M$	0.905	0.860	0.830	0.850	0.565	0.600	0.610	0.540	0.875	0.170
$CRIS_1^M$	0.895	0.870	0.835	0.460	0.320	0.580	0.605	0.520	0.425	0.075
$CRIS_2^M$	0.895	0.870	0.835	0.850	0.565	0.580	0.605	0.520	0.885	0.165
KM_1^M	0.640	0.640	0.600	0.895	0.190	0.170	0.205	0.125	0.905	0.005
KM_2^M	0.640	0.640	0.600	1.000	0.215	0.170	0.205	0.125	1.000	0.005
$IPOD_1^M$	0.490	0.495	0.445	0.890	0.075	0.110	0.180	0.110	0.905	0.005
$IPOD_2^M$	0.490	0.495	0.445	1.000	0.090	0.110	0.180	0.110	1.000	0.005
LQ_1^M	0.770	0.710	0.700	0.890	0.310	0.245	0.305	0.215	0.905	0.045
LQ_2^M	0.770	0.710	0.700	1.000	0.350	0.245	0.305	0.215	1.000	0.045
$n = 200, p = 2000, d = [n/log(n)], CR \approx 0.1 \sim 0.3$										
$QaSIS_1^M$	0.235	0.210	0.310	0.990	0.005	0.175	0.165	0.170	0.980	0.010
$QaSIS_2^M$	0.235	0.210	0.315	1.000	0.005	0.175	0.165	0.170	1.000	0.010
$CSIR_1^M$	0.690	0.720	0.745	0.990	0.345	0.460	0.420	0.465	0.980	0.100
$CSIR_2^M$	0.690	0.720	0.750	1.000	0.350	0.460	0.420	0.470	1.000	0.105
$RCDCS_1^M$	1.000	0.995	0.985	0.855	0.840	0.880	0.890	0.900	0.815	0.600
$RCDCS_2^M$	1.000	0.995	0.985	1.000	0.980	0.880	0.890	0.900	1.000	0.710
$CRSIS_1^M$	1.000	0.995	0.995	0.855	0.845	0.965	0.965	0.940	0.815	0.720
$CRSIS_2^M$	1.000	0.995	0.995	1.000	0.990	0.965	0.965	0.940	1.000	0.875
$SVSIR_1^M$	1.000	1.000	0.995	0.855	0.850	0.940	0.945	0.940	0.815	0.705
$SVSIR_2^M$	1.000	1.000	0.995	1.000	0.995	0.940	0.945	0.940	1.000	0.830
$CRIS_1^M$	1.000	0.995	0.995	0.860	0.850	0.920	0.935	0.935	0.815	0.680
$CRIS_2^M$	1.000	0.995	0.995	1.000	0.990	0.920	0.935	0.935	1.000	0.805
KM_1^M	0.975	0.980	0.970	0.990	0.935	0.600	0.610	0.585	0.980	0.255
KM_2^M	0.975	0.980	0.970	1.000	0.940	0.600	0.610	0.585	1.000	0.265
$IPOD_1^M$	0.905	0.905	0.895	0.990	0.720	0.470	0.450	0.430	0.980	0.115
$IPOD_2^M$	0.905	0.905	0.895	1.000	0.725	0.470	0.450	0.430	1.000	0.120
LQ_1^M	1.000	1.000	0.995	0.990	0.985	0.815	0.765	0.755	0.980	0.500
LQ_2^M	1.000	1.000	0.995	1.000	0.995	0.815	0.765	0.755	1.000	0.515

Table 14: Selected genes ID based on different screening methods.

QaSIS	4398	8136	3122	1190	2968	9287	8380	2344	530	8908	5336	6364	5828	575	7523	3455	1987	9068	7241	7634	483	4066
QaSIS_1^M	4398	8136	3122	1190	2968	9287	8380	2344	530	8908	5336	6364	5828	575	7523	3455	1987	9068	7241	7634	6231	491
QaSIS_2^M	4398	8136	3122	1190	2968	9287	8380	2344	530	8908	5336	6364	5828	575	7523	3455	1987	9068	7241	7634	2425	6594
CSIR	1804	870	7951	5746	4054	6666	7027	5894	7207	1000	9479	1746	5756	7366	4118	9074	2720	168	8460	6395	947	5975
CSIR_1^M	1804	870	7951	5746	4054	6666	7027	5894	7207	1000	9479	1746	5756	7366	4118	9074	2720	168	8460	6395	6231	491
CSIR_2^M	1804	870	7951	5746	4054	6666	7027	5894	7207	1000	9479	1746	5756	7366	4118	9074	2720	168	8460	6395	2425	6594
RCDCS	3633	5746	8312	2397	3846	5960	7295	1425	7207	3342	8150	7583	1951	7895	1977	2620	1000	1453	8727	6649	2429	870
RCDCS_1^M	3633	5746	8312	2397	3846	5960	7295	1425	7207	3342	8150	7583	1951	7895	1977	2620	1000	1453	8727	6649	5926	447
RCDCS_2^M	3633	5746	8312	2397	3846	5960	7295	1425	7207	3342	8150	7583	1951	7895	1977	2620	1000	1453	8727	6649	2292	6278
CRSIS	659	4873	2956	5746	6017	2353	1977	1000	5960	8150	1460	7895	2397	4805	9786	3633	1211	8814	6649	2720	10375	8312
CRSIS_1^M	659	4873	2956	5746	6017	2353	1977	1000	5960	8150	1460	7895	2397	4805	9786	3633	1211	8814	6649	2720	5926	447
CRSIS_2^M	659	4873	2956	5746	6017	2353	1977	1000	5960	8150	1460	7895	2397	4805	9786	3633	1211	8814	6649	2720	2292	6278
SVSIR	1000	870	8312	5149	3846	9485	10375	947	7559	1785	2966	6567	1512	2167	5894	1631	10567	3575	7528	8523	3041	2041
SVSIR_1^M	1000	870	8312	5149	3846	9485	10375	947	7559	1785	2966	6567	1512	2167	5894	1631	10567	3575	7528	8523	5926	447
SVSIR_2^M	1000	870	8312	5149	3846	9485	10375	947	7559	1785	2966	6567	1512	2167	5894	1631	10567	3575	7528	8523	2292	6278
CRIS	7674	7207	8326	1746	8460	7215	7366	10209	3342	3739	9065	1425	4081	3633	2367	8024	4833	7684	2335	9074	9912	5131
CRIS_1^M	7674	7207	8326	1746	8460	7215	7366	10209	3342	3739	9065	1425	4081	3633	2367	8024	4833	7684	2335	9074	5926	447
CRIS_2^M	7674	7207	8326	1746	8460	7215	7366	10209	3342	3739	9065	1425	4081	3633	2367	8024	4833	7684	2335	9074	2292	6278
KM	3012	9148	8814	2720	8727	9700	7469	5975	1746	3633	7951	56	1460	5746	2956	10096	5149	168	3033	2731	5303	4136
KM_1^M	3012	9148	8814	2720	8727	9700	7469	5975	1746	3633	7951	56	1460	5746	2956	10096	5149	168	3033	2731	6231	491
KM_2^M	3012	9148	8814	2720	8727	9700	7469	5975	1746	3633	7951	56	1460	5746	2956	10096	5149	168	3033	2731	2425	6594
IPOD	8727	3633	10375	7559	9700	9074	5450	9479	5241	8312	3033	8487	6923	9653	6440	3353	6055	3684	3805	947	1327	5821
IPOD_1^M	8727	3633	10375	7559	9700	9074	5450	9479	5241	8312	3033	8487	6923	9653	6440	3353	6055	3684	3805	947	6231	491
IPOD_2^M	8727	3633	10375	7559	9700	9074	5450	9479	5241	8312	3033	8487	6923	9653	6440	3353	6055	3684	3805	947	2425	6594
LQ	9399	5975	870	8814	4118	1460	2720	1327	1746	2531	5756	5969	10567	675	7469	2041	8326	9367	9279	9283	3633	8523
LQ_1^M	9399	5975	870	8814	4118	1460	2720	1327	1746	2531	5756	5969	10567	675	7469	2041	8326	9367	9279	9283	6231	491
LQ_2^M	9399	5975	870	8814	4118	1460	2720	1327	1746	2531	5756	5969	10567	675	7469	2041	8326	9367	9279	9283	2425	6594
CDC	5926	447	10375	1986	7515	6941	7592	4018	1357	659	9913	3787	3187	2892	2409	2213	9449	6095	8640	2353	7322	5226
CDC_1^M	5926	447	10375	1986	7515	6941	7592	4018	1357	659	9913	3787	3187	2892	2409	2213	9449	6095	8640	2353	7322	5226
CDC_2^M	5926	447	10375	1986	7515	6941	7592	4018	1357	659	9913	3787	3187	2892	2409	2213	9449	6095	8640	2353	2292	6278
ISIS_c	581	635	1781	1977	2001	2595	2605	3327	4585	4704	4741	4982	5653	5719	5772	6980	7335	9257	9353	9953	10027	10090

Table 15: Selected gene IDs for DLBCL data based on different screening methods.

$QaSIS$	5849	871	6217	6145	1021	5957	126	129	1614	1140	2521	2517	6196	2338	6976	616	1328	1163	2598	1361	7118	5980	2252	2501	5289	1155	2154	3202	2588	6904	799
$QaSIS_1^M$	5849	871	6217	6145	1021	5957	126	129	1614	1140	2521	2517	6196	2338	6976	616	1328	1163	2598	1361	7118	5980	2252	2501	5289	1155	2154	3202	2588	2409	1969
$QaSIS_2^M$	5849	871	6217	6145	1021	5957	126	129	1614	1140	2521	2517	6196	2338	6976	616	1328	1163	2598	1361	7118	5980	2252	2501	5289	1155	2154	3202	2588	2308	2306
$CSIR_1^M$	1831	2579	6365	1825	1994	3822	4131	7098	1456	3820	3821	1841	6134	2239	1660	4202	1819	3801	2541	6133	1639	1662	1671	2532	3799	1664	1681	1740	3802	3813	80
$CSIR_1^M$	1831	2579	6365	1825	1994	3822	4131	7098	1456	3820	3821	1841	6134	2239	1660	4202	1819	3801	2541	6133	1639	1662	1671	2532	3799	1664	1681	1740	3802	2409	1969
$CSIR_2^M$	1831	2579	6365	1825	1994	3822	4131	7098	1456	3820	3821	1841	6134	2239	1660	4202	1819	3801	2541	6133	1639	1662	1671	2532	3799	1664	1681	1740	3802	2308	2306
$RCDCS$	2579	50	4131	1825	1994	3787	1819	2479	1097	958	394	1965	3430	2576	2672	1023	2532	1086	4232	1029	3822	7251	1832	1831	7055	1841	6903	1995	4148	4733	5775
RCDCS_1^M	2579	50	4131	1825	1994	3787	1819	2479	1097	958	394	1965	3430	2576	2672	1023	2532	1086	4232	1029	3822	7251	1832	1831	7055	1841	6903	1995	4148	2409	1969
RCDCS_2^M	2579	50	4131	1825	1994	3787	1819	2479	1097	958	394	1965	3430	2576	2672	1023	2532	1086	4232	1029	3822	7251	1832	1831	7055	1841	6903	1995	4148	2308	2306
CRSIS	1819	1825	2672	1456	290	1994	394	2576	2674	2671	1841	50	368	5775	1986	2479	2673	6365	2958	7251	3470	3367	4131	2532	1029	3821	1965	2570	5621	1900	3806
CRSIS_1^M	1819	1825	2672	1456	290	1994	394	2576	2674	2671	1841	50	368	5775	1986	2479	2673	6365	2958	7251	3470	3367	4131	2532	1029	3821	1965	2570	5621	2409	1969
CRSIS_2^M	1819	1825	2672	1456	290	1994	394	2576	2674	2671	1841	50	368	5775	1986	2479	2673	6365	2958	7251	3470	3367	4131	2532	1029	3821	1965	2570	5621	2308	2306
SVSIR	4131	6134	3822	3787	2902	4232	6135	4193	6133	4149	4202	3832	3820	3821	6903	4148	4192	7227	1192	3824	3774	4416	3799	4183	3810	4150	3775	6193	4200	3825	5026
SVSIR_1^M	4131	6134	3822	3787	2902	4232	6135	4193	6133	4149	4202	3832	3820	3821	6903	4148	4192	7227	1192	3824	3774	4416	3799	4183	3810	4150	3775	6193	4200	2409	1969
SVSIR_2^M	4131	6134	3822	3787	2902	4232	6135	4193	6133	4149	4202	3832	3820	3821	6903	4148	4192	7227	1192	3824	3774	4416	3799	4183	3810	4150	3775	6193	4200	2308	2306
CRIS	2579	1456	2532	2425	7019	2672	2958	2569	7251	1653	80	2182	1664	290	1986	1662	2694	6740	5970	1612	2106	2959	2426	6625	1874	7380	1148	1663	5621	50	3146
CRIS_1^M	2579	1456	2532	2425	7019	2672	2958	2569	7251	1653	80	2182	1664	290	1986	1662	2694	6740	5970	1612	2106	2959	2426	6625	1874	7380	1148	1663	5621	2409	1969
CRIS_2^M	2579	1456	2532	2425	7019	2672	2958	2569	7251	1653	80	2182	1664	290	1986	1662	2694	6740	5970	1612	2106	2959	2426	6625	1874	7380	1148	1663	5621	2308	2306
KM	1653	2255	4149	4193	2239	1235	1831	4277	5655	6519	1830	4131	5715	1572	6498	133	2579	2569	2442	4148	2155	2532	1775	1664	3787	1645	50	1832	3917	5599	1669
KM_1^M	1653	2255	4149	4193	2239	1235	1831	4277	5655	6519	1830	4131	5715	1572	6498	133	2579	2569	2442	4148	2155	2532	1775	1664	3787	1645	50	1832	3917	2409	1969
KM_2^M	1653	2255	4149	4193	2239	1235	1831	4277	5655	6519	1830	4131	5715	1572	6498	133	2579	2569	2442	4148	2155	2532	1775	1664	3787	1645	50	1832	3917	2308	2306
IPOD	6519	1653	1830	1831	6733	773	1645	1832	4149	5361	3540	2005	4048	2569	2239	4733	4133	4131	3707	2155	4148	2156	7281	5655	6338	2079	4036	4823	2511	5654	5334
IPOD_1^M	6519	1653	1830	1831	6733	773	1645	1832	4149	5361	3540	2005	4048	2569	2239	4733	4133	4131	3707	2155	4148	2156	7281	5655	6338	2079	4036	4823	2511	2409	1969
IPOD_2^M	6519	1653	1830	1831	6733	773	1645	1832	4149	5361	3540	2005	4048	2569	2239	4733	4133	4131	3707	2155	4148	2156	7281	5655	6338	2079	4036	4823	2511	2308	2306
LQ	2579	6134	5655	1994	4131	1819	5298	6956	1086	5364	1023	4148	50	4149	958	5775	4277	2532	4232	281	1831	6903	1985	4202	2674	1830	2902	3832	3787	1671	1596
LQ_1^M	2579	6134	5655	1994	4131	1819	5298	6956	1086	5364	1023	4148	50	4149	958	5775	4277	2532	4232	281	1831	6903	1985	4202	2674	1830	2902	3832	3787	2409	1969
LQ_2^M	2579	6134	5655	1994	4131	1819	5298	6956	1086	5364	1023	4148	50	4149	958	5775	4277	2532	4232	281	1831	6903	1985	4202	2674	1830	2902	3832	3787	2308	2306

Table 16: The C-statistic for DLBCL data with different screening methods.

	C-statistic	standard error	lower bound of C	upper bound of C	p value
$QaSIS$	0.695	0.084	0.531	0.859	0.00132
$QaSIS_1^M$	0.744	0.090	0.567	0.921	4.281e-05
$QaSIS_2^M$	0.697	0.074	0.553	0.842	0.000342
$CSIR$	0.795	0.068	0.662	0.928	9.179e-09
$CSIR_1^M$	0.816	0.078	0.663	0.969	4.438e-08
$CSIR_2^M$	0.801	0.077	0.651	0.951	9.321e-10
$RCDCS$	0.777	0.078	0.625	0.929	3.069e-06
$RCDCS_1^M$	0.778	0.070	0.641	0.915	7.637e-06
$RCDCS_2^M$	0.761	0.067	0.630	0.892	2.146e-06
$CRSIS$	0.810	0.082	0.650	0.971	2.240e-10
$CRSIS_1^M$	0.818	0.070	0.681	0.954	3.463e-14
$CRSIS_2^M$	0.822	0.062	0.699	0.944	1.217e-08
$SVSIR$	0.743	0.071	0.604	0.883	2.458e-05
$SVSIR_1^M$	0.791	0.075	0.644	0.938	3.239e-05
$SVSIR_2^M$	0.750	0.077	0.599	0.901	1.886e-05
$CRIS$	0.785	0.079	0.630	0.939	3.451e-06
$CRIS_1^M$	0.805	0.076	0.657	0.953	1.095e-08
$CRIS_2^M$	0.792	0.099	0.598	0.986	8.451e-05
KM	0.767	0.075	0.621	0.913	0.000438
KM_1^M	0.775	0.074	0.630	0.920	2.211e-06
KM_2^M	0.755	0.073	0.612	0.898	1.188e-06
$IPOD$	0.707	0.071	0.567	0.846	0.00043
$IPOD_1^M$	0.715	0.086	0.546	0.883	7.912e-06
$IPOD_2^M$	0.695	0.074	0.550	0.839	0.00034
LQ	0.777	0.069	0.641	0.913	1.081e-05
LQ_1^M	0.784	0.084	0.619	0.950	1.243e-08
LQ_2^M	0.755	0.069	0.621	0.890	2.106e-06