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# Are Instrumental Variables Really That Instrumental? A Note on Endogeneity Resolution in Regression Models for Comparative Studies

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## Abstract

We provide a justification for why, and when, endogeneity will not cause bias in the interpretation of the coefficients in a regression model. This technique can be a viable alternative to, or even used alongside, the instrumental variable method. We show that when performing any comparative study, it is possible to measure the true change in the coefficients under a broad set of conditions. Our results hold, as long as the product of the covariance structure between the explanatory variables and the covariance between the error term and the explanatory variables are equal, within the same system at different time periods or across multiple systems at the same point in time.

## 1 Introduction

Endogeneity is an econometric issue that arises in regression models, causing the coefficients of the independent (or explanatory) variables to be biased. Despite the problem having been studied extensively in the literature, many researchers are either unaware of the extent of the errors caused by this issue, or they end up looking for solutions that might not be easily found (Avery 2005; Kim & Washington 2006; Gordon 2015; Hill et al. 2021).

Consider the multiple linear regression model, shown using matrix notation, in Equation (1). Here,  $\mathbf{y}$  is a vector of observed values of the dependent variables,  $\mathbf{X}$  is a matrix of the

independent variables,  $\beta$  is the parameter vector to be estimated, and  $\mathbf{u}$  is a vector of the error terms:

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u}. \quad (1)$$

Endogeneity arises when the error term in the regression model and the independent variables are correlated, that is, if  $E[\mathbf{X}^T \mathbf{u}] \neq \mathbf{0}$ .

We are interested in studying this model (Equation 1), with  $m$  and  $n$  independent and identically distributed (i.i.d) samples, before and after an event. For such an event study, let the true coefficients before and after the event be denoted by  $\beta_B$  and  $\beta_A$ , respectively. Clearly,  $\beta_B$  and  $\beta_A$  could be the coefficients for two different systems we are looking to compare. We wish to make inferences on the difference between the coefficients,  $\beta_B - \beta_A$ , under the possible presence of endogeneity.

Our main result (Proposition 1) in Section (2) gives a mathematical justification for the conditions under which endogeneity will not cause significant bias in the interpretation of the regression coefficients. Even if these criteria are not met in their entirety, we will see that endogeneity is mitigated to a great extent. This illustrates that our solution is an ideal way to approach the construction of regression models. Despite the simplicity of our methodology, to the best of our knowledge, no other works in the literature employ a similar approach to either partially combat or completely resolve endogeneity. The relative ease of implementation should allow researchers from almost any discipline to effortlessly adopt our solution when designing their experiments or while utilizing econometric techniques on the data collected. Section (3) considers situations under which our results might hold, including suggestions for further topics that need to be investigated to make this approach more useful. Section (4) discusses the main consequences of our result and concludes the paper.

## 2 Condition for Endogeneity Resolution

With the framework given in Equation (1), we present our primary result below.

**Proposition 1.** *Let the true coefficients of the regression model be denoted by  $\beta_B$  and  $\beta_A$ . The suffixes  $B$  and  $A$  denote values before and after an event, respectively, or across the two different systems under comparison. Suppose we have not been able to completely eliminate*

endogeneity. Then, the biased coefficients are denoted by  $\hat{\beta}_{BE}$  and  $\hat{\beta}_{AE}$ , and  $\mathbf{c}$  is a vector that holds the covariance of the error term with each of the explanatory variables. The change in the coefficient values before and after the event or across the two systems is given by,

$$\mathbf{E} [\hat{\beta}_{AE}] - \mathbf{E} [\hat{\beta}_{BE}] = \beta_A - \beta_B + \mathbf{E} [(\mathbf{X}_A^T \mathbf{X}_A)^{-1} \mathbf{c}_A] - [\mathbf{E} (\mathbf{X}_B^T \mathbf{X}_B)^{-1} \mathbf{c}_B]. \quad (2)$$

Because we are considering only the difference in the coefficients, we end up measuring the true change in the coefficients, as long as

$$\mathbf{E} [(\mathbf{X}_B^T \mathbf{X}_B)^{-1} \mathbf{c}_B] = \mathbf{E} [(\mathbf{X}_A^T \mathbf{X}_A)^{-1} \mathbf{c}_A]. \quad (3)$$

*Proof.* The result follows by taking the difference and then the expectations of the coefficient estimators when there is endogeneity. □

What our result suggests is that as long as the product of the covariance structure between the explanatory variables (inverse of the covariance matrix) and the covariance between the error term and the explanatory variables have equal expectations before and after the event (or across the two systems), we end up measuring the actual change between the coefficients. This ensures that we are using the correct values to understand how any system has changed across time, or how two different systems can be evaluated at the same point in time. This condition is trivially satisfied when  $\mathbf{E} [(\mathbf{X}_B^T \mathbf{X}_B)^{-1} \mathbf{c}_B] = \mathbf{E} [(\mathbf{X}_A^T \mathbf{X}_A)^{-1} \mathbf{c}_A] = 0$ .

### 3 Embarking on Endogeneity Resolution Improvements

Endogeneity can occur under three scenarios: 1) if the dependent variables can influence the explanatory variables, and vice versa; 2) some key variables are omitted in the regression model; and 3) there are errors in the measurement of the variables. Endogeneity arises in almost any study, as a silent malice, because it is difficult to completely rule out omitted variables and measurement errors. The first scenario, in which the dependent variables can

influence the explanatory variables, and vice versa, is generally observable. Most conventional models require additional information for the coefficients to be consistently estimated.

The most common workaround for endogeneity is to use an additional variable that is uncorrelated with the error term (instrument exogeneity), but is correlated with the explanatory variable (instrument relevance) in the original model. This is known as the instrumental variables regression method (Cragg & Donald 1993; Verbeek 2008; Semadeni, Withers & Trevis Certo 2014). The main difficulty with this option is locating valid instruments that satisfy the conditions of relevance and exogeneity (Miguel, Satyanath & Sergenti 2004; Baser 2009; Sovey & Green 2011). When the conditions from our result are satisfied, we do not have to look for instrumental variables, which in many instances are not easy to find.

Below, we list lines of investigation we are currently working on. These can help to make our result more robust as well as shed further light on the scenarios under which it would be more applicable.

1. Based on the type of dependency, between the covariance of the error term with each of the explanatory variables and the covariance structure between the explanatory variables, endogeneity will no longer be an issue. For example, using Equation (3), we could have that  $\mathbf{E} \left[ (\mathbf{X}_B^T \mathbf{X}_B)^{-1} \mathbf{c}_B \right] = \mathbf{E} \left[ (\mathbf{X}_B^T \mathbf{X}_B)^{-1} \right] \mathbf{E} [\mathbf{c}_B]$  and  $\mathbf{E} \left[ (\mathbf{X}_A^T \mathbf{X}_A)^{-1} \mathbf{c}_A \right] = \mathbf{E} \left[ (\mathbf{X}_A^T \mathbf{X}_A)^{-1} \right] \mathbf{E} [\mathbf{c}_A]$ . The requirement from our main result holds if the expectations are equal individually,  $\mathbf{E} [\mathbf{c}_B] = \mathbf{E} [\mathbf{c}_A]$  and  $\mathbf{E} \left[ (\mathbf{X}_B^T \mathbf{X}_B)^{-1} \right] = \mathbf{E} \left[ (\mathbf{X}_A^T \mathbf{X}_A)^{-1} \right]$ . Even if the expectations of the variables are not equal, as long as the variations in one of them is countered by variations in the other, so that they are equal when taken together, as given by the expression  $\mathbf{E} \left[ (\mathbf{X}_B^T \mathbf{X}_B)^{-1} \right] \mathbf{E} [\mathbf{c}_B] = \mathbf{E} \left[ (\mathbf{X}_A^T \mathbf{X}_A)^{-1} \right] \mathbf{E} [\mathbf{c}_A]$ , our results will hold. Many other dependency structures are possible and form fascinating avenues for further research.
2. It would be a fruitful line of inquiry to study Equation (3) and its related properties under each of the three cases under which endogeneity arises, and to derive expressions, including tests, that indicate whether our results hold for each situation.
3. It is quite possible that equality might turn out to be a difficult criterion to satisfy. It

would be helpful to show that there might be convergence of the relevant parameter values under various assumptions on the distributions. Specifically, we would examine the convergence of the left and right sides of Equation (3). If convergence results cannot be achieved for some cases, depending on the empirical context, approximations can be performed keeping some error limits in mind. These could be upper or lower bounds on the difference between the two sides of Equation (3).

4. In some instances, it is likely that the true coefficients might be equal,  $\beta_B = \beta_A$ . Any test to verify the equality of the coefficients requires the condition in Equation (3), because we are taking the differences and then the expectations of the coefficient estimators to arrive at this result. This is necessary even if the mean difference between the coefficient estimators is zero.
5. When comparing two different systems, it is quite clear how we should estimate the relevant variables in the left and right sides of Equation (3). However, while appraising the same system across different points, the concepts of before and after need clarification. This involves coming up with ways to split the data sample. The context under which the data have been collected should provide some guidelines, but mathematical suggestions on how to do this should not be ruled out, and can be pursued further.
6. A simulation-based study to justify the theory we have put forward would be worthwhile. Supplementing such a simulation-based study with actual data would be a good next step.

Much work can be done in this space toward developing a stronger theoretical foundation supporting the effectiveness of this approach under relatively general assumptions on the structure of endogeneity, or in terms of coming up with tests that can identify whether certain assumptions hold before starting the analysis. Needless to say, some subjective decisions have to be made, but that is the case with all econometric modeling, testing, and interpretation.

## 4 Does Endogeneity Matter? Endogeneity Does Matter, But Only Sometimes!

We have considered the issue of endogeneity in regression models. We have aimed to keep our paper as concise as possible, yet with sufficient detail to keep it self contained, so that the results are immediately useful to researchers across different disciplines. We have shown that endogeneity will not cause major bias in the interpretation of the coefficients, as long as the product of the covariance structure between the explanatory variables and the covariance between the error term and the explanatory variables are equal in magnitude, within the same system at different points in time, or across different systems at the same point in time. When performing any comparative study (Clasen 2004; Hantrais 2008), the conditions we have outlined for the resolution of endogeneity are usually satisfied. This suggests that if we are not able to find valid instrumental variables, which is the most commonly used approach when endogeneity is suspected to be present, we need to consider designing our study as a comparative analysis. Even if we have reasonably good instruments, we can have a comparative angle in our study to supplement the results because this alleviates the effect of omitted variables and errors in measurement, which are usually difficult to detect.

Clearly, a comparative study can assess the same system at different time periods, or it can be across multiple systems at the same point in time. The main prescription from our result is that, by conducting a comparative analysis, we can measure the true change in the coefficients. Another justification for designing studies with a comparative angle is that having an anchor point, or a frame of reference for comparison, results in better decision making, as opposed to making absolute judgments in isolation (Kahneman 1992; Gavirneni & Xia 2009).

Despite endogeneity having been extensively studied, and numerous complicated methods having been put forth to deal with it, simple alternatives such as those presented here would be extremely useful for researchers. This will allow better use of the data they have collected from their experiments, and even the design of better experiments so that the resulting data can be analyzed with minimal errors. Our simple workaround works quite well under a broad set of conditions, especially when valid instruments are difficult to find. More importantly, even if endogeneity is not completely removed, owing to suitable conditions

not being satisfied, it can be partly eliminated in a much broader set of cases.

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