Statistica Sinica: Preprint
doi:10.5705/ss.202016.0536

The functional linear model extends the notion of linear regression to the case where the response and covariates are iid elements of an infinite dimensional Hilbert space. The unknown to be estimated is a Hilbert-Schmidt operator, whose inverse is by definition unbounded, rendering the problem of inference ill-posed. In this paper, we consider the more general context where the sample of response/covariate pairs forms a weakly dependent stationary process in the respective product Hilbert space: simply stated, the case where we have a regression between functional time series. We consider a general framework of potentially nonlinear processes, expoiting recent advances in the spectral analysis of time series. This allows us to quantify the inherent ill-posedness, and motivate a frequency domain Tikhonov regularisation technique. Our main result is the rate of convergence for the corresponding estimators of the regression coefficients, the latter forming a summable sequence in the space of Hilbert-Schmidt operators. In a sense, our main result can be seen as a generalisation of the classical functional linear model rates, to the case of time series, and rests only upon Brillinger-type mixing conditions. It is seen that, just as the covariance operator eigenstructure plays a central role in the independent case, so does the spectral density operator's eigenstructure in the dependent case. While the analysis becomes considerably more involved in the dependent case, the rates are strikingly comparable to those of the iid case, but at the expense of an additional factor caused by the necessity to estimate the spectral density operator at a nonparametric rate.