Statistica Sinica Preprint No: SS-2016-0427R1	
Title	Derivative Principal Components for Representing the Time Dynamics of Longitudinal and Functional Data
Manuscript ID	SS-2016-0427R1
URL	http://www.stat.sinica.edu.tw/statistica/
DOI	10.5705/ss.202016.0427
Complete List of Authors	Xiongtao Dai Hans-Georg Müller and Wenwen Tao
Corresponding Author	Xiongtao Dai
E-mail	dai@ucdavis.edu

Statistica Sinica: Preprint
doi:10.5705/ss.202016.0427

Proposition 1. *Under the conditions of Theorem 1,*

$$\sup_{t\in\mathcal{T}}|\hat{\mu}(t)-\mu(t)| = O(a_n) \quad a.s.,$$

$$\sup_{s,t\in\mathcal{T}}|\hat{G}(s,t)-G(s,t)| = O(a_n+b_n) \quad a.s.,$$

$$\sup_{s,t\in\mathcal{T}}|\hat{G}^{(1,0)}(s,t)-G^{(1,0)}(s,t)| = O(a_n+b_n) \quad a.s.,$$

$$|\hat{\sigma}^2-\sigma^2| = O(a_n+b_n) \quad a.s.$$

Lemma 1 (Lemma A.3, Facer and Müller (2003))**.** *Let $\mathbf{A}\in\mathcal{M}_m(\mathbb{R})$ be invertible. For all $\mathbf{B}\in\mathcal{M}_m(\mathbb{R})$ such that*

$$\|\mathbf{A}-\mathbf{B}\| < \frac{1}{2\|\mathbf{A}^{-1}\|},$$

$\mathbf{B}^{-1}$ always exists and there exists a constant $0<c<\infty$ such that

$$\|\mathbf{B}^{-1}-\mathbf{A}^{-1}\| \le c\|\mathbf{A}^{-1}\|^2\|\mathbf{A}-\mathbf{B}\|.$$

To prove the first statement of Theorem 1, note

$$\begin{aligned}
|\hat{\xi}_{ik,1}-\tilde{\xi}_{ik,1}| &= \hat{\boldsymbol{\zeta}}_{ik}^T\hat{\boldsymbol{\Sigma}}_{\mathbf{Y}_i}^{-1}(\mathbf{Y}_i-\hat{\boldsymbol{\mu}}_i)-\boldsymbol{\zeta}_{ik}^T\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}(\mathbf{Y}_i-\boldsymbol{\mu}_i)\\
&= \hat{\boldsymbol{\zeta}}_{ik}^T(\hat{\boldsymbol{\Sigma}}_{\mathbf{Y}_i}^{-1}-\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1})(\mathbf{Y}_i-\hat{\boldsymbol{\mu}}_i)+(\hat{\boldsymbol{\zeta}}_{ik}-\boldsymbol{\zeta}_{ik})^T\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}(\mathbf{Y}_i-\hat{\boldsymbol{\mu}}_i)\\
&\quad+\hat{\boldsymbol{\zeta}}_{ik}^T\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}(\boldsymbol{\mu}_i-\hat{\boldsymbol{\mu}}_i)-(\hat{\boldsymbol{\zeta}}_{ik}-\boldsymbol{\zeta}_{ik})^T\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}(\boldsymbol{\mu}_i-\hat{\boldsymbol{\mu}}_i)\\
&\le \left\|\hat{\boldsymbol{\zeta}}_{ik}\right\|\left\|\hat{\boldsymbol{\Sigma}}_{\mathbf{Y}_i}^{-1}-\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}\right\|\|\mathbf{Y}_i-\hat{\boldsymbol{\mu}}_i\|+\left\|\hat{\boldsymbol{\zeta}}_{ik}-\boldsymbol{\zeta}_{ik}\right\|\|\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}\|\,\|\mathbf{Y}_i-\hat{\boldsymbol{\mu}}_i\|\\
&\quad+\left\|\hat{\boldsymbol{\zeta}}_{ik}\right\|\|\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}\|\,\|\boldsymbol{\mu}_i-\hat{\boldsymbol{\mu}}_i\|+\left\|\hat{\boldsymbol{\zeta}}_{ik}-\boldsymbol{\zeta}_{ik}\right\|\|\boldsymbol{\Sigma}_{\mathbf{Y}_i}^{-1}\|\,\|\boldsymbol{\mu}_i-\hat{\boldsymbol{\mu}}_i\|. \qquad (27)
\end{aligned}$$