We propose a nonparametric method to explicitly model and represent the derivatives of smooth underlying trajectories for longitudinal data. This representation is based on a direct Karhunen-Loeve expansion of the unobserved derivatives and leads to the notion of derivative principal components, which complement functional principal components, one of the most popular tools of functional data analysis. The proposed derivative principal components can be obtained for irregularly spaced and sparsely observed longitudinal data, as typically encountered in biomedical studies, as well as for noisy functional data measured on regular and dense grids. We compare the proposed representations for derivatives with alternative approaches in simulation settings and also in a Wallaby growth curve application. It emerges that representations by derivative principal components recover the underlying derivatives more accurately in various settings. In another application example, we demonstrate the utility of derivative principal components for the classification of wheat spectra, where the spectral measurements are densely and regularly spaced. In this application, derivative principal components are found to be more predictive for the protein content of wheat than the conventional functional principal components, and to provide improved classification performance. Consistency results and asymptotic convergence rates for the proposed estimates of derivative principal components and other components of the model are derived under mild conditions, using tools from functional data analysis.