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Consider a random sample from a regularly varying distribution function with a finite right endpoint θ and an exponent α of regular variation. The primary interest of the paper is to estimate both the endpoint and the exponent. Since the distribution is semiparametric and the endpoint and the exponent reveal asymptotic properties of the right tail for the distribution, inference can only be based on a few of the largest observations in the sample. The conventional maximum likelihood method can be used to estimate both α and θ, see e.g., Hall (1982) and Drees, Ferreira and de Haan (2004) for the regular case (i.e. $\alpha\ge 2$), and Smith (1987) and Peng and Qi (2009) for the irregular case (i.e. $\alpha\in (1,2)$). Unfortunately a global maximum of the likelihood function doesn't exist if one allows $\alpha\in (0,1]$, and a local maximum exists with probability tending to one only if $\alpha>1$, i.e., a local maximum may not exist for some given sample. In this paper we propose a penalized likelihood method to estimate both parameters. The estimators derived from this new likelihood method exist for all $\alpha>0$ and any sample such that the largest two observations are distinct. We present the asymptotic distributions for the proposed maximum penalized likelihood estimators. A simulation study shows that the proposed method works very well for the irregular case, and has even better finite sample performance than the conventional maximum likelihood method for the regular case.