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Title	Bayesian Modeling and Inference for Nonignorably Missing Longitudinal Binary Response Data with Applications to HIV Prevention Trials
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and by Tonelli's theorem, it suffices to show that for each k

$$\prod_{i \in I_c} \int f_y(\mathbf{y}_i^*|z_i, \mathbf{x}_{1i}, \boldsymbol{\epsilon}_i, \boldsymbol{\theta}) d\boldsymbol{\beta} f(\boldsymbol{\epsilon}_i|\alpha, \rho) d\boldsymbol{\epsilon} f(\zeta_k|\tau) d\boldsymbol{\zeta} \pi(\tau) d\tau d\boldsymbol{\alpha} d\rho < \infty.$$

By Chen and Shao (2001), and under (C1) and (C2), there exists a constant K_0 depending only on $\mathbf{X}^*_{obs}$ such that

$$\begin{aligned}
&\prod_{i \in I_c} \int f_y(\mathbf{y}_i^*|z_i, \mathbf{x}_{1i}, \boldsymbol{\epsilon}_i, \boldsymbol{\theta}) d\boldsymbol{\beta} f(\boldsymbol{\epsilon}_i|\alpha, \rho) d\boldsymbol{\epsilon} f(\zeta_k|\tau) d\boldsymbol{\zeta} \pi(\tau) d\tau d\boldsymbol{\alpha} d\rho \\
=& E_{\mathbf{u}} \left(\int \mathbf{1}(\mathbf{X}^*_{\text{obs}} \boldsymbol{\beta} + \tau \boldsymbol{\zeta} + \boldsymbol{\epsilon} \leq \mathbf{u}) d\boldsymbol{\beta} f(\boldsymbol{\epsilon}|\alpha, \rho) d\boldsymbol{\epsilon} f(\zeta_k|\tau) d\boldsymbol{\zeta} \pi(\tau) d\tau d\boldsymbol{\alpha} d\rho \right) \\
=& E_{\mathbf{u}} \left(\int K_0 \|\mathbf{u} - \tau \boldsymbol{\zeta} - \boldsymbol{\epsilon}\|^p d\boldsymbol{\beta} f(\boldsymbol{\epsilon}|\alpha, \rho) d\boldsymbol{\epsilon} f(\zeta_k|\tau) d\boldsymbol{\zeta} \pi(\tau) d\tau d\boldsymbol{\alpha} d\rho \right) \\
\leq& E_{\mathbf{u}} \left(K_0 \|\mathbf{u}\|^p \int f(\boldsymbol{\epsilon}|\alpha, \rho) d\boldsymbol{\epsilon} f(\zeta_k|\tau) d\boldsymbol{\zeta} \pi(\tau) d\tau d\boldsymbol{\alpha} d\rho \right) + \\
& K_0 \int \|\boldsymbol{\zeta}\|^p f(\zeta_k|\tau) d\boldsymbol{\zeta} \tau^p \pi(\tau) d\tau f(\boldsymbol{\epsilon}|\alpha, \rho) d\boldsymbol{\epsilon} d\boldsymbol{\alpha} d\rho \\
& K_0 \int \|\boldsymbol{\epsilon}\|^p f(\boldsymbol{\epsilon}|\alpha, \rho) d\boldsymbol{\epsilon} f(\zeta_k|\tau) d\boldsymbol{\zeta} \pi(\tau) d\tau d\boldsymbol{\alpha} d\rho
\end{aligned}$$

The first and second terms are finite since $\alpha \in (0, 1)$, $\rho \in (-1, 1)$, $\pi(\tau)$ is proper with a finite p^{th} moment, $\zeta_k \overset{i.i.d.}{\sim} N(0, 1)$, and condition C3. Let $\Sigma = \Gamma\Gamma$, where $\Gamma = \Gamma'$. To bound the second term, we carry out a transformation