Statistica Sinica Preprint No: SS-2016-0224.R2	
Title	Hyper Markov Laws for Correlation Matrices
Manuscript ID	SS-2016-0224.R2
URL	http://www.stat.sinica.edu.tw/statistica/
DOI	10.5705/ss.202016.0224
Complete List of Authors	Jeremy Gaskins
Corresponding Author	Jeremy Gaskins
E-mail	jeremy.gaskins@louisville.edu
Notice: Accepted version subject to English editing.	

Statistica Sinica: Preprint
doi:10.5705/ss.202016.0224

2. Hyper Markov laws for correlation matrices

2.1. Distributions for correlation matrix on a complete graph

Our interest is the correlation matrix $\mathbf{R}$, and we consider the implied distributions of (non-sparse) $\mathbf{R}$ under Wishart and inverse Wishart. Let $\mathcal{R}_p \subset \mathcal{M}_p$ denote the space of $p \times p$ positive definite matrices with unit diagonal, the space of correlation matrices. Using the separation strategy (Barnard et al. (2000)), we write $\mathbf{\Sigma} \in \mathcal{M}_p$ as $\mathbf{DRD}$ where $\mathbf{R} \in \mathcal{R}_p$ is the correlation matrix corresponding to $\mathbf{\Sigma}$ and $\mathbf{D}$ is a diagonal matrix containing the standard deviations.

Let $\mathrm{IW}_p(\delta, \mathbf{\Psi})$ denote the inverse Wishart distribution with $\delta > 0$ and scale $\mathbf{\Psi} \in \mathcal{M}_p$, and $\mathrm{W}_p(\delta, \mathbf{V})$ is the Wishart distribution with $\delta > p - 1$ and $\mathbf{V} \in \mathcal{M}_p$. We describe the marginal distributions of the correlation matrix from these distributions (e.g., Barnard et al. (2000); Zhang et al. (2006)). We denote these by CIW and CW, respectively. Let $\Gamma_p(x) = \pi^{p(p-1)/4} \prod_{j=1}^{p} \Gamma(x + [1-j]/2)$ be the multivariate gamma function, $\mathbf{I}_p$ denote the $p \times p$ identity matrix, and $\mathbf{E}_j$ be the matrix formed by removing column j from $\mathbf{I}_p$. We use $|\cdot|$ to denote both the determinant of a matrix and the cardinality of a set, but the relevant interpretation will be clear from context.

Lemma 1 (CIW distribution)**.** *For* $\mathbf{\Sigma} = \mathbf{DRD} \sim \mathrm{IW}_p(\delta, \mathbf{I}_p)$ *for* $\delta > 0$*, the*