Methods for Sparse and Low-Rank Recovery under Simplex Constraints

Abstract

The de-facto standard approach of promoting sparsity by means of $\ell_1$-regularization becomes ineffective in the presence of simplex constraints, i.e., the target is known to have non-negative entries summing up to a given constant. The situation is analogous for the use of nuclear norm regularization for low-rank recovery of Hermitian positive semidefinite matrices with given trace. In the present paper, we discuss several strategies to deal with this situation, from simple to more complex. As a starting point, we consider empirical risk minimization (ERM) which turns out to enjoy similar theoretical properties w.r.t. prediction and $\ell_2$-estimation error as $\ell_1$-regularization. In light of this, we argue that ERM combined with a subsequent sparsification step like thresholding represents a sound alternative to the heuristic of using $\ell_1$-regularization after dropping the sum constraint and subsequent normalization. At the next level, we show that any sparsity-promoting regularizer under simplex constraints cannot be convex. A novel sparsity-promoting regularization scheme based on the inverse or negative of the squared $\ell_2$-norm is proposed, which avoids shortcomings of various alternative methods from the literature. Our approach naturally extends to Hermitian positive semidefinite matrices with given trace.