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Generally the Likelihood Ratio statistic Λ for standard hypotheses is asymptotically χ^2-distributed, and Bartlett adjustment improves the χ^2-approximation to its asymptotic distribution in the sense of third-order asymptotics. However, if the parameter of interest is on the boundary of the parameter space, \citet{Self and Liang (1987)} show that the limiting distribution of Λ is a mixture of χ^2-distributions. For such ``nonstandard setting of hypotheses", the present paper develops the third-order asymptotic theory for a class of test statistics $\cal {S}$, which includes the Likelihood Ratio and the Wald statistic in the case of observations generated from a general stochastic processes, providing widely applicable results. In particular, it is shown that Λ is Bartlett adjustable despite its nonstandard asymptotic distribution. Although the Wald statistic W is not Bartlett adjustable, a nonlinear adjustment is provided for W which greatly improves the χ^2- approximation to its distribution and allows a subsequent Bartlett-adjustment. Numerical studies confirm the benefits of the adjustments on the accuracy and on the power of tests whose statistics belong to $\cal {S}$.