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Title	Nearly Unstable Processes: A Prediction Perspective
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such as (1.7). Thus, would the relationship

$$R_B(b) \to \begin{cases} 2\sigma^2, \text{ as } b \to \infty\,, \\ \\ 4\sigma^2, \text{ as } b \to 0\,, \end{cases} \tag{1.10}$$

remain valid? The lower half of (1.10) does remain valid for $b \to 0$, the upper half fails to hold and $R_B(b)$ converges to σ^2 as $b \to \infty$; see (2.15) and (2.17) of Section 2.

This discrepancy in the upper half of (1.10) suggests that $1 - (b/n)$ may be converging to unity too rapidly and as a result, $R_B(b)$ does not attain the limiting value [illegible] $\rho)$ of the stationary case. In Section 3.1, we derive the limiting [illegible] (β, b) [illegible] $(\mathrm{MSPE}_B - \sigma^2)$ for the general near unit-root model, (1.7) with $\rho_n = 1 - b/n^\beta, 0 < \beta \leq 1$ and $b > 0$. No single β directly connects $\Lambda_1(\beta, b)$ from $2\sigma^2$ to $4\sigma^2$, but our r[illegible] that a connection can be established through two critical values of β, $\beta = 1/2$ and $\beta = 1$, that connect $\Lambda_1(\beta, b)$ for the stationary and intermediate states, and for the intermediate and unit-root states, respectively: $\lim_{b\to\infty} \Lambda_1(1/2, b) = 2\sigma^2$, $\lim_{b\to 0} \Lambda_1(1/2, b) = \sigma^2$, $\lim_{b\to\infty} \Lambda_1(1, b) = \lim_{b\to\infty} R_B(b) = \sigma^2$, and $\lim_{b\to 0} \Lambda_1(1, b) = \lim_{b\to 0} R_B(b) = 4\sigma^2$. This provides an alternative finite sample approximation, $\Lambda_1($[illegible]n[illegible]$(1 - \rho))$, for $n(\mathrm{MSPE}_B - \sigma^2)$ when $n(1 - \rho)$ stays far away from 0.

Although the EV predictor also encounters the MSPE jump at $\rho = 1$ in the sense that

$$\lim_{\rho\to 1} R_A^o(\alpha, \rho) \neq R_A^o(\alpha, 1), \tag{1.11}$$

it can be eliminated by $R_A(\alpha, b)$ which satisfies, for any $0 < \alpha < \infty$,

$$\lim_{\rho\to 1} R_A^o(\alpha, \rho) = \lim_{b\to\infty} R_A(\alpha, b), \quad R_A^o(\alpha, 1) = \lim_{b\to 0} R_A(\alpha, b). \tag{1.12}$$

To deepen our understanding of the EV predictor in the near unit-root region, we obtain an asymptotic expression for MSPE_A in the general near unit-root model. This result leads to an alternative finite sample approximation of $n^{\min\{1,2/\alpha\}}(\mathrm{MSPE}_A - \sigma^2)$, which notably improves upon $R_A(\alpha, n(1 - \rho))$ when $n(1 - \rho)$ is relatively large.