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3.2 Asymptotic frequentist properties with deterministic inputs under linear models

In permutation tests, the data to be permuted are regarded as random, and an exchangeability assumption like Assumption 2 is needed to guarantee the exact control of the type-I error (Romano (1989); Welch (1990); Good (2005)). For the q-value, the input values to be permuted may be deterministic and for such cases, it does not possess exact frequentist properties. However, this subsection provides an interesting result that for linear models with deterministic inputs, the q-value possesses *asymptotic* frequentist properties under some regularity conditions. Hence, it can be viewed as a p-value from an *approximated* permutation test.

Consider the linear model

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_d x_d + \varepsilon, \tag{7}$$

where $\beta_0, \beta_1, \ldots, \beta_d \in \mathbb{R}$ are unknown parameters and ε is zero-mean random error. Using the data at n points, we have the matrix form

$$\boldsymbol{y} = \boldsymbol{\beta} \tag{8}$$

where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_d)'$, $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_n)'$ and the input matrix $\mathbf{X} = (x_{ij})_{i=1,\ldots,n, j=1,\ldots,d} = (\mathbf{z}_1, \ldots, \mathbf{z}_d)$ is standardized such that for $j = 1, \ldots, d$,

$$\sum_{i=1}^{n} x_{ij} = 0, \quad \sum_{i=1}^{n} x_{ij}^2 = n. \tag{9}$$

We consider the asymptotics for fixed d and $\mathbf{X}$ of full column rank.

In this case, the null hypothesis (5) reduces to

$$H_0 : \ \beta_1 = 0. \tag{10}$$