| Statistica Sinica Preprint No: SS-2015-0408R2 | |
|---|---|
| **Title** | Calibration and Multiple Robustness When Data Are Missing Not At Random |
| **Manuscript ID** | SS-2015-0408R2 |
| **URL** | http://www.stat.sinica.edu.tw/statistica/ |
| **DOI** | 10.5705/ss.202015.0408 |
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| Notice: Accepted version subject to English editing. | |

Statistica Sinica: Preprint
doi:10.5705/ss.202015.0408



# Calibration and Multiple Robustness When Data Are Missing Not At Random

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*Abstract:* In missing data analysis, multiple robustness is a desirable property resulting from the calibration technique. A multiply robust estimator is consistent if any one of the multiple data distribution models and missingness mechanism models is correctly specified. So far in the literature, multiple robustness has only been established when data are missing at random (MAR). We study how to carry out calibration to construct a multiply robust estimator when data are missing not at random (MNAR). With multiple models available, where each model consists of two components, one for data distribution for complete cases and one for missingness mechanism, our proposed estimator is consistent if any one pair of models are correctly specified.

*Key words and phrases:* Calibration; Empirical likelihood; Missing not at random (MNAR); Multiple robustness; Nonignorable nonresponse.

## 1. Introduction

Missing data problems are commonly seen in practice. Depending on the nature of missingness, there are three mechanisms that are widely adopted in the literature: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR) (Rubin (1976)). For MCAR, the missingness depends on neither the observed nor the missing values; for MAR, the missingness depends on the observed but not on the missing values; and for MNAR, the missingness depends on both the observed and the missing values. As the miss-

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ingness mechanism becomes more complex, statistical analysis becomes more difficult. For MCAR, a complete-case analysis ignoring subjects with missing data leads to consistent estimation, and usually gives the best solution in terms of efficiency. Extensive research has been done in the case of MAR, yielding a rich collection of effective methods and interesting results. Much less has been done for MNAR, largely due to the unknown dependence of the missingness on unobserved values, although in many observational studies MNAR is the most likely mechanism.

Calibration is a method originally developed in the sampling survey literature (Deville and Särndal (1992)), where it was used to calibrate the sampling weight so that the weighted average of some auxiliary variables based on the sampled subjects is equal to the known population average. In the survey context calibration has been studied a lot (e.g., Chen and Sitter (1999); Lundström and Särndal (1999); Wu and Sitter (2001); Wu (2003); Chang and Kott (2008); Kim (2009, 2010); Kim and Park (2010); Tan and Wu (2015)). The application of calibration to missing data analysis has attracted considerable research interests recently and has produced many interesting results (e.g., Tan (2006, 2010); Qin and Zhang (2007); Chen et al. (2008); Qin et al. (2008); Han and Wang (2013); Chan and Yam (2014); Han (2014, 2016a, 2016b)). In particular, the estimators in Han and Wang (2013), Chan and Yam (2014) and Han (2014, 2016a, 2016b) are multiply robust, in that they are consistent if any one of the multiple missingness mechanism models and/or data distribution models is correctly specified. Such a robustness property is a significant improvement over the well-known double robustness (e.g. Scharfstein et al. (1999); Bang and Robins (2005); Tsiatis (2006)).

So far multiple robustness has been established and studied only when data are MAR. In this paper, for the estimation of the mean of a response variable

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that is MNAR, we show how to carry out calibration so that multiple robustness can be achieved. Here each model consists of two components, one for the data distribution for complete cases and one for the missingness mechanism. The two components together characterize the whole data distribution. When multiple models are available, our proposed estimator is consistent if any one is correctly specified. Estimating the mean is a common problem in both sampling survey and causal inference, and thus our proposed method is of practical importance.

This paper is organized as follows. Section 2 introduces the notation and gives a review of calibration under MAR. Section 3 covers calibration under MNAR and establishes the multiple robustness property of our proposed estimator. Section 4 contains some simulation results. Some discussion is given in Section 5.

## 2. Notation and Review of Calibration Under MAR

Let $Y$ denote the response of interest, $\boldsymbol{X}$ a vector of auxiliary variables that are always observed, and $R$ the indicator of observing $Y$ (i.e., $R = 1$ if $Y$ is observed and $R = 0$ if $Y$ is missing). The quantity of interest is $\mu_0 = E(Y)$. MCAR means that the selection probability $P(R = 1|Y, \boldsymbol{X})$ is a constant, MAR means that $P(R = 1|Y, \boldsymbol{X}) = P(R = 1|\boldsymbol{X})$ only depends on the fully observed $\boldsymbol{X}$, and MNAR means that $P(R = 1|Y, \boldsymbol{X})$ depends on both $Y$ and $\boldsymbol{X}$. We use $\pi(\boldsymbol{X})$ and $\pi(Y, \boldsymbol{X})$ to denote $P(R = 1|Y, \boldsymbol{X})$ under MAR and MNAR, respectively. The observed data are $n$ independent and identically distributed copies of $(R, RY, \boldsymbol{X})$. Let $m = \sum_{i=1}^n R_i$ be the number of complete cases. Without loss of generality, assume that these complete cases are $i = 1, \ldots, m$.

The original calibration estimator in Deville and Särndal (1992) has a weighting structure $\sum_{i=1}^m \hat{w}_i Y_i$, with the weight $\hat{w}_i$ derived through

$$\min_{w_1,\ldots,w_m} \sum_{i=1}^m \pi(\boldsymbol{X}_i)\{n w_i - \pi(\boldsymbol{X}_i)^{-1}\}^2 \text{ subject to } \sum_{i=1}^m w_i \boldsymbol{X}_i = \frac{1}{n}\sum_{i=1}^n \boldsymbol{X}_i.$$

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The calibration constraint above makes the weighted average of $\boldsymbol{X}$ based on complete cases equal to the unweighted average of $\boldsymbol{X}$ based on the whole sample, which consistently estimates the population mean of $\boldsymbol{X}$. The weight $\hat{w}_i$ is derived by minimizing the above discrepancy between $w_i$ and the inverse probability weight $1/\{n\pi(\boldsymbol{X}_i)\}$ subject to the calibration constraint. Many variations of calibration have been proposed in the sampling survey literature, some with different optimization criteria (e.g. Chen and Sitter (1999); Kim (2009, 2010); Tan and Wu (2015)) and some with calibration variables being certain functions of $\boldsymbol{X}$ (e.g. Wu and Sitter (2001)). Most of these variations impose two additional constraints: $w_i > 0$ and $\sum_{i=1}^m w_i = 1$.

Calibration in missing data literature has two major variations. The first one derives $\hat{w}_i$ through

$$\max_{w_1,\ldots,w_m} \prod_{i=1}^m w_i \quad \text{subject to} \quad w_i > 0,\ \sum_{i=1}^m w_i = 1,\ \sum_{i=1}^m w_i \boldsymbol{h}(\boldsymbol{X}_i) = \frac{1}{n}\sum_{i=1}^n \boldsymbol{h}(\boldsymbol{X}_i), \tag{2.1}$$

where $\boldsymbol{h}(\boldsymbol{X})$ comprises user-specified functions of $\boldsymbol{X}$ (e.g. Qin and Zhang (2007); Qin et al. (2008); Han and Wang (2013); Chan and Yam (2014); Han (2014, 2016a, 2016b)). Due to the positivity and sum-to-one constraints, $w_i$ can be viewed as an empirical likelihood (EL) on complete cases. The formulation in (2.1) is the same as that of an EL problem (Owen (1988, 2001); Qin and Lawless (1994)). The second major variation considers an additional EL on incomplete cases (Chen et al. (2008); Tan (2010)):

$$\begin{aligned} &\max_{w_i's, v_j's} \prod_{i=1}^m w_i \prod_{j=m+1}^n v_j \qquad \text{subject to} \\ &w_i > 0,\ \sum_{i=1}^m w_i = 1,\ v_j > 0,\ \sum_{j=m+1}^n v_j = 1, \\ &\sum_{i=1}^m w_i \boldsymbol{h}(\boldsymbol{X}_i) = \sum_{j=m+1}^n v_j \boldsymbol{h}(\boldsymbol{X}_j). \end{aligned} \tag{2.2}$$

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Han (2014) gave a justification of the compatibility of the constraints in (2.1) and (2.2) in the following way. Let $w(\boldsymbol{X}) = 1/\pi(\boldsymbol{X})$ and $v(\boldsymbol{X}) = 1/\{1-\pi(\boldsymbol{X})\}$. It is easy to verify that

$$\begin{aligned} E\left(w(\boldsymbol{X})\left[\boldsymbol{h}(\boldsymbol{X}) - E\{\boldsymbol{h}(\boldsymbol{X})\}\right] \mid R=1\right) &= \mathbf{0} \\ E\left(v(\boldsymbol{X})\left[\boldsymbol{h}(\boldsymbol{X}) - E\{\boldsymbol{h}(\boldsymbol{X})\}\right] \mid R=0\right) &= \mathbf{0}. \end{aligned} \tag{2.3}$$

Then constraints in (2.1) and (2.2) are simply the empirical version of these equalities with expectations replaced by sample averages.

One interesting result produced by calibration is multiple robustness. Suppose that multiple models $\pi^{(j)}(\boldsymbol{X};\boldsymbol{\alpha}^{(j)})$, $j = 1,\ldots,J$, for $\pi(\boldsymbol{X})$ and multiple models $a^{(k)}(\boldsymbol{X};\boldsymbol{\gamma}^{(k)})$, $k = 1,\ldots,K$, for $E(Y \mid \boldsymbol{X})$ are postulated. Han and Wang (2013) proposed to derive $\hat{w}_i$ the same way as in (2.1) with $\boldsymbol{h}(\boldsymbol{X}) = \{\pi^{(1)}(\hat{\boldsymbol{\alpha}}^{(1)}),\ldots,\pi^{(j)}(\hat{\boldsymbol{\alpha}}^{(J)}),a^{(1)}(\hat{\boldsymbol{\gamma}}^{(1)}),\ldots,a^{(K)}(\hat{\boldsymbol{\gamma}}^{(K)})\}^{\mathrm{T}}$, where $\hat{\boldsymbol{\alpha}}^{(j)}$ and $\hat{\boldsymbol{\gamma}}^{(k)}$ are estimators of $\boldsymbol{\alpha}^{(j)}$ and $\boldsymbol{\gamma}^{(k)}$, respectively. The resulting estimator of $\mu_0$, denoted by $\hat{\mu}_{\mathrm{mr}}$, is multiply robust, in that it is consistent if any one of the $J+K$ models is correctly specified. Multiple robustness significantly improves over double robustness on protecting estimation consistency against possible model misspecifications, since doubly robust estimators take $J = K = 1$.

In addition to multiple robustness, like other calibration-based estimators with weights derived from (2.1) or (2.2), $\hat{\mu}_{\mathrm{mr}}$ always falls into the parameter space of $\mu$ due to it being a convex combination of the observed $Y$. This is called the sample boundedness property (Robins et al. (2007); Tan (2010)) and is especially desirable when $Y$ is binary. Another merit of $\hat{\mu}_{\mathrm{mr}}$ is its insensitivity to near-zero values of $\pi^{(j)}(\hat{\boldsymbol{\alpha}}^{(j)})$. The maximization in (2.1) makes the occurrence of extreme weights unlikely even if some estimated values of $\pi(\boldsymbol{X})$ are close to zero. Some numerical evidence of the superior performance of $\hat{\mu}_{\mathrm{mr}}$ in this case can be found in Han (2014).

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## 3. Calibration and Multiple Robustness Under MNAR

### 3.1 Calibration under MNAR

It is seen that, under MAR, the calibration variables used to derive $\hat{\mu}_{\rm mr}$ are models for $\pi(\boldsymbol{X})$ and models for $E(Y \mid \boldsymbol{X})$. Under MNAR, models for $\pi(Y, \boldsymbol{X})$ can no longer serve as calibration variables because $Y$ is missing for some subjects, but models for $E(Y \mid \boldsymbol{X})$ still can. However, the estimator with models for $E(Y \mid \boldsymbol{X})$ as calibration variables will no longer be consistent even if one model is correctly specified, since the proof of consistency in this case requires MAR assumption (Han and Wang (2013)).

Another look at the calibration in (2.1) when $\boldsymbol{h}(\boldsymbol{X})$ is taken to be models for $E(Y \mid \boldsymbol{X})$ reveals that the third constraint in (2.1) is essentially

$$\sum_{i=1}^{m} w_i E^{(k)}(Y \mid \boldsymbol{X}_i, R_i = 1) = \frac{1}{n}\sum_{i=1}^{n}\Big\{R_i E^{(k)}(Y \mid \boldsymbol{X}_i, R_i = 1) + (1 - R_i) E^{(k)}(Y \mid \boldsymbol{X}_i, R_i = 0)\Big\}, \tag{3.1}$$

with $E^{(k)}(\cdot)$ the expectation under the $k$-th model, because MAR implies that $E(Y \mid \boldsymbol{X})$, $E(Y \mid \boldsymbol{X}, R = 1)$ and $E(Y \mid \boldsymbol{X}, R = 0)$ are all equal. Now under MNAR, we propose to use the calibration constraint as in (3.1).

Modelling $E(Y \mid \boldsymbol{X}, R = 0)$ as needed by (3.1) is difficult since $Y$ is not observed for subjects with $R = 0$. One possible solution is to use the fact that

$$f(Y \mid \boldsymbol{X}, R = 0) \propto f(Y \mid \boldsymbol{X}, R = 1)\frac{1 - \pi(Y, \boldsymbol{X})}{\pi(Y, \boldsymbol{X})}$$

(e.g., Kim and Yu (2011)) to obtain a model for $f(Y \mid \boldsymbol{X}, R = 0)$ by modelling $f(Y \mid \boldsymbol{X}, R = 1)$ and $\pi(Y, \boldsymbol{X})$. Suppose there are multiple pairs of models available: $\{f^{(k)}(Y \mid \boldsymbol{X}, R = 1; \boldsymbol{\gamma}^{(k)}), \pi^{(k)}(Y, \boldsymbol{X}; \boldsymbol{\alpha}^{(k)})\}$, $k = 1, \ldots, K$, each of which determines a model for $f(Y \mid \boldsymbol{X}, R = 0)$. Here $f^{(k)}(Y \mid \boldsymbol{X}, R = 1; \boldsymbol{\gamma}^{(k)})$, $k = 1, \ldots, K$, have to be different, because two models with the same $f^{(k)}(Y \mid \boldsymbol{X}, R = 1; \boldsymbol{\gamma}^{(k)})$ but different $\pi^{(k)}(Y, \boldsymbol{X}; \boldsymbol{\alpha}^{(k)})$ will lead to (3.1) with the same left-hand side but different right-hand sides.

extend the current method to regression analysis with missing response and/or covariates.

## Acknowledgement

We wish to thank the Editor, an associate editor and two reviewers for their valuable comments that have helped to greatly improve the quality of this work. Support for this project was partially provided by the Natural Sciences and Engineering Research Council of Canada.

## Appendix

Assume that, for $k = 1, \ldots, K$, (1) the parameter spaces $\mathcal{A}^k$ and $\mathcal{G}^k$ for $\boldsymbol{\alpha}^{(k)}$ and $\boldsymbol{\gamma}^{(k)}$, respectively, are compact; (2) $\pi^{(k)}(Y, \boldsymbol{X}; \boldsymbol{\alpha}^{(k)})$ and $f^{(k)}(Y \mid \boldsymbol{X}, R = 1; \boldsymbol{\gamma}^{(k)})$ are continuous in $\boldsymbol{\alpha}^{(k)}$ and $\boldsymbol{\gamma}^{(k)}$, respectively; (3) $E\{\log f(Y \mid \boldsymbol{X}, R = 1)\} < \infty$ and $E\{\sup_{\boldsymbol{\gamma}^{(k)} \in \mathcal{G}^k} |\log f^{(k)}(Y \mid \boldsymbol{X}, R = 1; \boldsymbol{\gamma}^{(k)})|\} < \infty$; (4) the Kullback–Leibler distance between $f(Y \mid \boldsymbol{X}, R = 1)$ and $f^{(k)}(Y \mid \boldsymbol{X}, R = 1; \boldsymbol{\gamma}^{(k)})$, viewed as a function of $\boldsymbol{\gamma}^{(k)}$, has a unique minimum in $\mathcal{G}^k$; (5) $\pi(Y, \boldsymbol{X})$ is bounded away from 0; (6) $E[\sup_{\boldsymbol{\alpha}^{(k)} \in \mathcal{A}^k} ||\{R/\pi^{(k)}(Y, \boldsymbol{X}; \boldsymbol{\alpha}^{(k)}) - 1\}\boldsymbol{h}(\boldsymbol{X})||] < \infty$, where $\boldsymbol{h}(\boldsymbol{X})$ is user-specified for estimating $\boldsymbol{\alpha}^{(k)}$; (7) the quadratic form of $E[\{R/\pi^{(k)}(Y, \boldsymbol{X}; \boldsymbol{\alpha}^{(k)}) - 1\}\boldsymbol{h}(\boldsymbol{X})]$ with the weighting matrix being the inverse of the covariance matrix of $\{R/\pi^{(k)}(Y, \boldsymbol{X}; \boldsymbol{\alpha}^{(k)}) - 1\}\boldsymbol{h}(\boldsymbol{X})$, viewed as a function of $\boldsymbol{\alpha}^{(k)}$, has a unique minimum in $\mathcal{A}^k$; (8) $a_1^{(k)}(\boldsymbol{X}; \boldsymbol{\gamma}^{(k)})$ and $a_0^{(k)}(\boldsymbol{X}; \boldsymbol{\gamma}^{(k)}, \boldsymbol{\alpha}^{(k)}, L)$ are continuous in $\boldsymbol{\gamma}^{(k)}$ and $(\boldsymbol{\gamma}^{(k)}, \boldsymbol{\alpha}^{(k)})$, respectively; (9) $E\{\sup_{\boldsymbol{\gamma}^{(k)} \in \mathcal{G}^k} |a_1^{(k)}(\boldsymbol{X}; \boldsymbol{\gamma}^{(k)})|\} < \infty$ and $E\{\sup_{\boldsymbol{\gamma}^{(k)} \in \mathcal{G}^k, \boldsymbol{\alpha}^{(k)} \in \mathcal{A}^k} |a_0^{(k)}(\boldsymbol{X}; \boldsymbol{\gamma}^{(k)}, \boldsymbol{\alpha}^{(k)}, L)|\} < \infty$; (10) $E[\sup_{\boldsymbol{\rho} \in \mathcal{P}} \log\{1 + \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{g}(\boldsymbol{\alpha}, \boldsymbol{\gamma}, L)\} \mid R = 1] < \infty$ where $\mathcal{P}$ is the parameter space for $\boldsymbol{\rho}$ and is compact; (11) $E[\sup_{\boldsymbol{\gamma}^{(k)} \in \mathcal{G}^k, \boldsymbol{\alpha}^{(k)} \in \mathcal{A}^k, \boldsymbol{\rho} \in \mathcal{P}} \{Y - a_1^{(k)}(\boldsymbol{X}; \boldsymbol{\gamma}^{(k)})\}/\{1 + \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{g}(\boldsymbol{\alpha}, \boldsymbol{\gamma}, L)\} \mid R = 1] < \infty$.

Remark: (1)-(4) ensure the convergence in probability of $\hat{\boldsymbol{\gamma}}^{(k)}$ (White (1982));

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(1), (2) and (5)-(7) ensure the convergence in probability of $\hat{\boldsymbol{\alpha}}^{(k)}$ (Hall (2005)); (1), (2) and (8)-(11) ensure the weak law of large numbers needed in the proof of Theorem 1 (Newey and McFadden (1994); Schennach (2007)).

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doi:10.5705/ss.202015.0408



Table 5.1: Simulation results based on $n = 200$ and 1000 replications. The number in the name of each estimator indicates which one(s) among the 4 models is (are) used. The results have been multiplied by 100.

| | | $L = 5$ | | | $L = 10$ | | | $L = 20$ | |
|---|---|---|---|---|---|---|---|---|---|
| Estimator | Bias | RMSE | MAE | Bias | RMSE | MAE | Bias | RMSE | MAE |
| IPW-1 | 0 | 23 | 15 | | | | | | |
| IPW-2 | 41 | 68 | 35 | | | | | | |
| IM-1 | -8 | 18 | 13 | -8 | 18 | 12 | -8 | 18 | 12 |
| IM-2 | -50 | 54 | 49 | -50 | 54 | 49 | -50 | 54 | 50 |
| IM-3 | 41 | 44 | 41 | 41 | 44 | 41 | 41 | 44 | 41 |
| IM-4 | 65 | 67 | 65 | 65 | 67 | 64 | 65 | 67 | 65 |
| MR-1 | -7 | 19 | 13 | -8 | 19 | 13 | -8 | 19 | 13 |
| MR-2 | -28 | 40 | 26 | -27 | 42 | 26 | -28 | 40 | 26 |
| MR-3 | 41 | 44 | 41 | 41 | 44 | 41 | 41 | 44 | 41 |
| MR-4 | 64 | 67 | 65 | 64 | 67 | 64 | 64 | 67 | 65 |
| MR-12 | 12 | 31 | 20 | 12 | 31 | 20 | 12 | 32 | 21 |
| MR-14 | -8 | 23 | 14 | -8 | 23 | 14 | -8 | 23 | 14 |
| MR-23 | 50 | 54 | 50 | 50 | 54 | 50 | 50 | 54 | 50 |
| MR-34 | 40 | 43 | 40 | 40 | 43 | 41 | 40 | 43 | 41 |

RMSE: root mean square error. MAE: median absolute error. IPW: the inverse probability weighted estimator. IM: the imputation estimator. MR: the proposed estimator.

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doi:10.5705/ss.202015.0408



Table 5.2: Simulation results based on $n = 500$ and 1000 replications. The number in the name of each estimator indicates which one(s) among the 4 models is (are) used. The results have been multiplied by 100.

| | | $L = 5$ | | | $L = 10$ | | | $L = 20$ | |
|---|---|---|---|---|---|---|---|---|---|
| Estimator | Bias | RMSE | MAE | Bias | RMSE | MAE | Bias | RMSE | MAE |
| IPW-1 | 0 | 15 | 11 | | | | | | |
| IPW-2 | 39 | 49 | 36 | | | | | | |
| IM-1 | -8 | 13 | 9 | -8 | 13 | 9 | -8 | 13 | 9 |
| IM-2 | -51 | 52 | 50 | -51 | 53 | 51 | -51 | 52 | 51 |
| IM-3 | 41 | 42 | 41 | 41 | 42 | 41 | 41 | 42 | 41 |
| IM-4 | 65 | 66 | 65 | 65 | 66 | 65 | 65 | 66 | 65 |
| MR-1 | -8 | 14 | 9 | -8 | 14 | 9 | -8 | 13 | 10 |
| MR-2 | -35 | 44 | 33 | -35 | 42 | 33 | -35 | 43 | 33 |
| MR-3 | 41 | 42 | 41 | 41 | 42 | 41 | 41 | 42 | 41 |
| MR-4 | 65 | 66 | 65 | 65 | 66 | 65 | 65 | 66 | 65 |
| MR-12 | 12 | 24 | 17 | 13 | 23 | 17 | 13 | 23 | 17 |
| MR-14 | -8 | 15 | 10 | -8 | 15 | 10 | -8 | 15 | 10 |
| MR-23 | 51 | 53 | 51 | 51 | 53 | 51 | 51 | 53 | 51 |
| MR-34 | 41 | 42 | 41 | 42 | 47 | 41 | 41 | 42 | 41 |

RMSE: root mean square error. MAE: median absolute error. IPW: the inverse probability weighted estimator. IM: the imputation estimator. MR: the proposed estimator.

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Table 5.3: Performance of the bootstrapping method based on $n = 200$, $L = 5$ and 500 replications. The bootstrapping resampling size is 100. The number in the name of each estimator indicates which one(s) among the 4 models is (are) used.

| Estimator | Bias | SE-EMP | SE-B | CP-B (%) |
|---|---|---|---|---|
| MR-1 | -0.07 | 0.19 | 0.18 | 91.8 |
| MR-2 | -0.28 | 0.28 | 0.28 | 84.4 |
| MR-3 | 0.41 | 0.17 | 0.16 | 24.2 |
| MR-4 | 0.64 | 0.20 | 0.19 | 6.2 |
| MR-12 | 0.12 | 0.27 | 0.27 | 92.2 |
| MR-14 | -0.07 | 0.22 | 0.24 | 95.6 |
| MR-23 | 0.51 | 0.21 | 0.22 | 28.6 |
| MR-34 | 0.41 | 0.17 | 0.16 | 25.8 |

SE-EMP: empirical standard error; SE-B: averaged bootstrapping standard error; CP-B: coverage probability of the 95% confidence interval based bootstrapping standard errors.