Statistica Sinica Preprint No: SS-2015-0404R2	
Title	Orthogonal Gaussian process models
Manuscript ID	SS-2015-0404R2
URL	http://www.stat.sinica.edu.tw/statistica/
DOI	10.5705/ss.202015.0404
Complete List of Authors	Matthew Plumlee and V. Roshan Joseph
Corresponding Author	Matthew Plumlee
E-mail	mplumlee@umich.edu

Statistica Sinica: Preprint
doi:10.5705/ss.202015.0404

respect to the mean function.

The idea of making the random field orthogonal to the mean function to avoid identifiability problems was proposed in Reich et al. (2006) in the context of spatial random effects modeling. Hodges and Reich (2010) used $m(x) = \beta_0 + \beta_1 x$, for modeling stomach cancer incidence ratio in Solvenia with respect to the socioeconomic scores (x). Their approach achieves orthogonality only at the observed locations, which induces two negatives: (i) the stochastic model has a dependency on the observations and (ii) outside of the observed locations, such as prediction points, there is no orthogonality. Extensions of this work in the spatial statistics literature include Hughes and Haran (2013) and Hanks et al. (2015). Hanks et al. (2015) proposed to make the random field orthogonal to fixed effects over the entire region X, using "conditioning by Kriging" (Rue and Held (2005)) . Ultimately, they were forced to resort to approximation methods circumventing the orthogonality condition to be met only at the observed locations. We show that by carefully choosing a covariance function we can make the random field orthogonal to the mean function over the entire region X. The proposed orthogonal Gaussian process is orthogonal to the mean function. Our approach is computationally attractable and is amenable to both the frequentist and Bayesian frameworks.

2. Orthogonal Gaussian process models

This section discusses the specification of an orthogonal Gaussian process $z_*(\cdot)$ in general. The procedure is as follows: given a covariance function $c(\cdot, \cdot)$, replace it with the covariance